MODERN ARITHMETIC

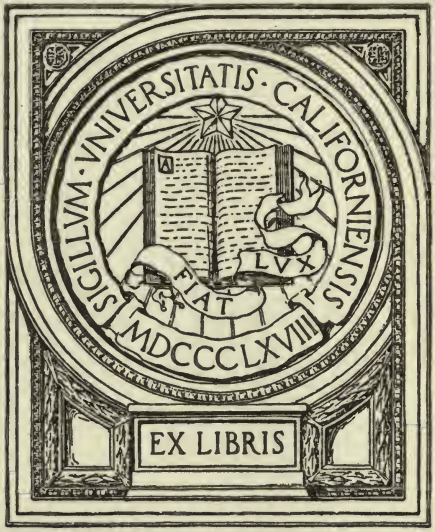
WITH GRAPHIC & PRACTICAL EXERCISES

H. SYDNEY JONES

WITH ANSWERS

860

IN MEMORIAM
FLORIAN CAJORI



Florian Cajori

2755-37

Digitized by the Internet Archive
in 2007 with funding from
Microsoft Corporation

http://www.archive.org/details/modernarithmetic00jonerich

A MODERN ARITHMETIC

MACMILLAN AND CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
ATLANTA · SAN FRANCISCO

THE MACMILLAN CO. OF CANADA, LTD
TORONTO

A MODERN ARITHMETIC

WITH GRAPHIC & PRACTICAL EXERCISES

BY

H. SYDNEY JONES, M.A.

LATE SCHOLAR OF CHRIST'S COLLEGE, CAMBRIDGE
HEADMASTER OF CHELTENHAM GRAMMAR SCHOOL

MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON
1909

First Edition 1908.
Reprinted 1909.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS
BY ROBERT MACLEHOSE AND CO. LTD.

QA103
J63

PREFACE

During the past few years various suggestions for the improved teaching of arithmetic have been made by the Board of Education, by committees of the British Association and the Mathematical Association, and by some public examining bodies. Though arithmetic was included in the school curriculum chiefly for utilitarian reasons, the abuse of examinations led to the introduction of tiresome and complicated calculations of no service in practical life and of little educational value. These undesirable developments are, however, now disappearing, and, in common with other mathematical subjects, the scope, substance and method of arithmetic are being placed upon a more reasonable basis.

In this book an attempt has been made to provide a course of work in which the essential parts of arithmetic are dealt with in the spirit of recent recommendations. The author has endeavoured to produce a practicable as well as a practical course which mathematical teachers may introduce without any violent dislocation of existing practice and with the knowledge that the requirements of the chief examining bodies, which determine to a large extent the nature of the teaching in schools, have not been disregarded. Any efficient teaching of arithmetic must aim at (1) a clear conception of units of quantities involved in calculations, (2) accuracy, (3) quickness

in the manipulation of numbers, (4) cultivation of the reasoning faculties. These principles have determined the character of the exercises throughout the book.

Among other distinguishing features of the volume may be mentioned:

(1) Decimals are introduced at an early stage and before vulgar fractions are explained, with a view to teach students to think in decimals. This introduction is effected by means of practical exercises in measurement in the Metric System and by a judicious extension of the idea of Place Value.

(2) Practical measurements and other exercises, which can be done in ordinary class rooms, are provided at convenient intervals throughout the book and are accompanied by numerous graphic exercises, where these are likely to assist the formation of correct ideas. All exercises have been designed to provide material for mathematical deduction, and will, it is believed, convey sound notions of concrete quantities. These sections of the book are so arranged, however, that the practical exercises can be omitted without any loss of continuity.

(3) The arithmetical exercises illustrate in most cases actual calculations met with in the experience of life. Easy illustrative and stimulating problems precede those which are purely numerical; many of the latter are included to train the student to become able to perform the fundamental operations of arithmetic in a sub-conscious manner.

(4) Mental and Oral exercises, intended to introduce new ideas and to stimulate the habit of ready calculation, are included in each Section.

(5) Early exercises work out exactly; but, as the knowledge of the student extends, approximations are introduced, and he is led in various ways to know how to estimate the degree of accuracy possible from the data with which he is provided.

In this way approximate methods of working lead, not to loose, but to more accurate trains of thought.

(6) Graded Revision Exercises are inserted at various stages of progress, and specimen Examination Papers are included to show the student what may be expected of him by public examining bodies.

(7) The practical, graphic and purely arithmetical exercises are arranged on one common and logical plan, with the view of giving a concrete and connected knowledge of numerical calculations.

No apology is necessary for the introduction of a section on averages and elementary statistics; and it is hoped that the treatment of Stocks may make that subject more real and interesting. Probably the graphic method of calculating volumes and the use of logarithmic graphs have not been used in all schools; some teachers will be surprised at the accuracy of the former and the field of application of both methods.

Part I. covers the subjects prescribed for the Oxford and Cambridge Junior Local examinations in arithmetic, and other examinations of about the same scope: the Second Part will be found to meet, not only the requirements of the more conservative examining bodies, but also those of the Army, Civil Service and other examiners keenly alive to the change taking place in mathematical teaching.

The author wishes here to express his thanks to the various examining bodies which have courteously sanctioned the use of their examination papers, and in particular the Oxford Local Delegates, the Cambridge Local Syndics, the Senates of London and Birmingham Universities, the College of Preceptors, the London Chamber of Commerce, and the Controller of H.M. Stationery Office, in whom is vested the

copyright of papers set by the Civil Service Commissioners and the Board of Education.

It is very difficult for the author adequately to express his gratitude to Prof. R. A. Gregory and Mr. A. T. Simmons for the most unremitting care and thought which they have expended on the MSS. and proof sheets of this work; while he must add that the conception of the work was theirs in the first instance.

H. SYDNEY JONES.

CHELTENHAM, *August* 1908.

CONTENTS

PART I

SECTION		PAGE
1.	NOTATION AND NUMERATION, ...	1
2.	ADDITION, ...	4
	Use of Signs. Addition by Steps. Addition of Long Columns.	
3.	SUBTRACTION, ...	12
4.	MULTIPLICATION, ...	18
	Multiplication by single digits. Multiplication by factors. General Case. Squared and Cubed Numbers.	
5.	DIVISION, ...	29
	Division by Factors. Division by the General Method. Italian Method. Division true to the Nearest Unit.	
6.	FACTORS, ...	37
7.	ARITHMETICAL AVERAGES, ...	40
8.	BRITISH MONEY SYSTEMS, ...	53
9.	COMPOUND ADDITION, ...	56
10.	COMPOUND SUBTRACTION, ...	58
11.	COMPOUND MULTIPLICATION, ...	61
12.	COMPOUND DIVISION, ...	65

SECTION PAGE

13. THE MEASUREMENT OF LENGTH, ... 74

14. REDUCTION, ... 75

15. ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF LENGTHS, ... 79

16. REDUCTION IN THE METRIC OR DECIMAL SYSTEM, . 85

17. INTRODUCTION OF DECIMAL NOTATION, . . . 88

18. ADDITION AND SUBTRACTION OF METRIC LENGTHS, 91

19. MULTIPLICATION AND DIVISION OF DECIMALS BY A SINGLE FIGURE, ... 95

20. MULTIPLICATION OF DECIMALS, ... 98
General Case.

21. DIVISION OF DECIMALS, ... 107
General Case.

22. DIVISION TRUE TO A CERTAIN NUMBER OF DECIMAL PLACES, ... 110

23. BRITISH MEASURES OF LENGTH (*Continued*), . . 113

24. AREA, ... 116

25. BRITISH MEASURES OF AREA, ... 117
Units of Area.

26. METRIC MEASURES OF AREA, ... 118
Measurement of Irregular Areas.

27. DECIMAL COINAGE, ... 130

28. VOLUMES, ... 132

29. PROBLEMS ON MEASURES OF CAPACITY, . . . 139

30. MEASUREMENT OF WEIGHT, ... 141

31. MEASUREMENT OF TIME, ... 147

SECTION PAGE

32. VULGAR FRACTIONS, ... 157
Graphic Illustrations of Vulgar Fractions.

33. IMPROPER FRACTIONS AND MIXED NUMBERS, . . 160
One Quantity as a Vulgar Fraction of another.

34. EQUIVALENT FRACTIONS, ... 162
Equivalent Fractions Illustrated Graphically.

35. ADDITION AND SUBTRACTION OF FRACTIONS, . . 166

36. CONVERSION OF FRACTIONS, ... 169
Conversion of a Decimal into a Vulgar Fraction. Conversion of a Vulgar Fraction into a Decimal. Recurring Decimals.

37. MULTIPLICATION AND DIVISION OF VULGAR FRACTIONS, ... 172
Multiplication by a Whole Number. Division by a Whole Number. Multiplication of one Fraction by another. Division of one Fraction by another.

38. GREATEST COMMON MEASURE, ... 182
Graphical Introduction to General Method. General Method.

39. LEAST COMMON MULTIPLE, ... 191
General Method.

40. PROPORTION, ... 197

41. PERCENTAGES, ... 207
Scale Drawing.

42. CALCULATIONS AND APPROXIMATIONS, . . . 236
Decimalization of Money. Possible Errors in Decimalization.

43. PRACTICE, ... 244

SECTION PAGE

44. ERRORS IN DATA, ... 254

45. APPROXIMATE OR CONTRACTED MULTIPLICATION, . 259

46. CONTRACTED DIVISION, ... 268

47. SQUARE ROOT, ... 273
General Method. Square Roots of Vulgar Fractions.

48. INTEREST, ... 278
Simple Interest.

REVISION PAPERS, ... 289

PART II

SECTION PAGE

49. MEASUREMENT OF LENGTH (*Continued*) . . . 297

50. THE USE OF THE CHAIN 303

51. APPROXIMATIONS (*Continued*) 307

Approximations in Multiplication. Approximations in Division. Approximations in the Extraction of Square Root. Surds. Approximations in Square Root (*Continued*). Square Root (*Continued*).

52. PROPORTION (*Continued*) 323

53. MEASUREMENT OF ANGLES 327

54. AREAS OF CIRCLES 331

55. THE PRISM AND CYLINDER 334

56. SIMILAR FIGURES 338

57. INVERSE PROPORTION 342

Examination of Quantities for Proportion.

58. AREAS OF RECTILINEAR FIGURES 358

59. LOGARITHMS 387

60. LOGARITHMIC TABLES AND INSTRUMENTS . . . 396

Evaluation of Formulae. The Slide Rule.

SECTION PAGE

61. COMPOUND INTEREST 414
The Compound Interest Law. Geometrical Progression.

62. THE AREA OF A TRIANGLE 432

63. BILLS OF EXCHANGE AND BANKER'S DISCOUNT . . 436

64. CALCULATION OF AREAS (*Continued*) 438
Spherical Surfaces.

65. CALCULATION OF VOLUMES (*Continued*) . . . 447
Volumes of Cones and Pyramids. Volumes of Frusta. Volumes of Spheres. Graphic Calculation of Volumes.

66. EQUATION OF PAYMENTS 478

67. FOREIGN EXCHANGES 482
Money Exchange between different Countries.

68. FOREIGN BILLS OF EXCHANGE 487

69. STOCKS AND SHARES 490
Brokerage and other Charges.

70. STATISTICS 505
Averages. Errors in Practical Work. Method of Mean Squares. Permutations and Combinations. Probabilities. Expectation of Life.

TYPICAL EXAMINATION PAPERS 555

TABLES 585

ANSWERS 599

INDEX 659

A MODERN ARITHMETIC

1. NOTATION AND NUMERATION.

Arithmetic is the science treating of numbers and calculations.

Numbers are expressed by means of certain signs or symbols. These are, in the very great majority of cases, figures; and occasionally the capital letters I, V, X, L, C, D, M are used. Calculations are always made with figures.

Figures, or digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, represent one, two, three, four, five, six, seven, eight, nine units respectively.

The fact of our figures being often called digits (Latin, *digitus*, a finger) suggests that no doubt the primitive way of expressing numbers up to ten was by means of the fingers of the two hands.

The letters which are occasionally used to represent numbers are: M, a thousand; D, five hundred; C, one hundred; L, fifty; X, ten; V, five; and I, one.

The basis of all calculation is the unit 1, one; and since there are only the above symbols to represent numbers, it is evident that in order to express numbers exceeding 9 it is necessary to give a **place value** or **local value** to the digits; that is to make the value of a digit depend upon position as well as upon the symbol.

Figures have two values—an intrinsic one, and a place value when arranged to form numbers. So, too, has the symbol I in the case of letters.

The symbol o—naught, nought, or cypher—has neither an intrinsic nor a place value, but is introduced in expressing the place value of the other digits.

Letters used to represent numbers have no place value, excepting I and X. I or X, placed to the left of a symbol, signifies that the number represented is *short* by that amount, while, if placed to the right, signifies that the number represented is *greater* by the amount. Thus IV, VI, VIII, IX, and XII represent severally 4, 6, 8, 9, 12, etc. XC is 90.

A figure in the first place of a number denotes so many units, and the same figure in the second place (proceeding from right to left) denotes 10 times as many. Hence, figures in the second place are often called **Tens**. A figure in the third place represents ten times as many as it would in the second place, or a hundred times as many as it would in the first place. These third place figures are called **Hundreds**. Similarly, figures in the fourth place are called **Thousands**; in the fifth place, **Tens of Thousands**; in the sixth place, **Hundreds of Thousands**. When we come to the seventh place a figure there represents **Millions**. The million is often taken as a new kind of unit, and a repetition of the former process then occurs. A million millions is called in Great Britain a billion, beyond which few numbers are ever taken. (In France and the United States a billion is taken to be one thousand millions.)

The notation and numeration table up to a million would then be as on the next page.

It will be noticed that a figure in occupying places successively one place to the left represents a value ten times greater than before.

Common System of Notation.

Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Numbers expressed in Words.
						9	Nine.
					1	0	Ten.
					9	9	Ninety nine.
				1	0	0	One hundred.
				9	9	9	Nine hundred and ninety nine.
			1	0	0	0	One thousand.
			9	9	9	9	Nine thousand, nine hundred and ninety nine.
		1	0	0	0	0	Ten thousand.
		9	9	9	9	9	Ninety nine thousand, nine hundred and ninety nine.
	1	0	0	0	0	0	One hundred thousand.
	9	9	9	9	9	9	Nine hundred and ninety nine thousand, nine hundred and ninety nine.
1	0	0	0	0	0	0	One million.

It is customary and advantageous to divide or separate the figures expressing a given number by a comma into periods of three, thus 3,465,908

Notation is the method of expressing a number given in words by means of figures, and **Numeration** is the method of writing a number in words when represented by figures. When any value, as tens, thousands, etc., is not given, the place is supplied by cypher 0. Thus, seven thousand and six would be written 7,006

EXERCISES I.

1. Make a table like that on page 3, and show in it the numbers (*a*) 3 million 2 hundred and 6, (*b*) 71 thousand and twenty five, (*c*) 5 hundred thousand 2 hundred, (*d*) 9 thousand and 9.

Express in numbers :

2. (*a*) Twenty, (*b*) twenty three, (*c*) fifteen, (*d*) ten, (*e*) eighteen, (*f*) sixty two.

3. (*a*) Eighty six, (*b*) one hundred, (*c*) one hundred and one, (*d*) one hundred and ten, (*e*) one hundred and eleven.

4. (*a*) One hundred and twenty eight, (*b*) one hundred and eighty two, (*c*) one hundred and eight, (*d*) one hundred and eighty.

5. (*a*) Three hundred and three, (*b*) three hundred and thirty, (*c*) three hundred and thirty three.

6. (*a*) Two thousand and six, (*b*) six thousand and two, (*c*) six thousand and twenty, (*d*) six thousand two hundred.

7. (*a*) Eight thousand, seven hundred and six, (*b*) eight thousand and seventy six, (*c*) eight hundred and seventy six.

Express in words :

8. (*a*) 3, (*b*) 8, (*c*) 18, (*d*) 81, (*e*) 25, (*f*) 52, (*g*) 520, (*h*) 300, (*i*) 3000, (*j*) 3005, (*k*) 3050, (*l*) 3500, (*m*) 5003, (*n*) 186, (*o*) 168, (*p*) 861, (*q*) 816, (*r*) 71, (*s*) 701, (*t*) 7001.

2. ADDITION.

Addition is the process of finding *one* number, or quantity, which is equivalent to two or more numbers or quantities. The simple numbers which are to be collected into one are often called *terms*, or *addends*, while the result obtained is called the **Sum**, or **Total**.

In finding the Sum of any number of terms it is usual to arrange them vertically, taking care that units figures are

vertically under one another, and so with the tens, and other digits.

EXAMPLE.— *To find the sum of* 476, 3089, *and* 940.

The numbers are arranged.

Th.	H.	T.	U.
	4	7	6
3	0	8	9
	9	4	0
4	5	0	5
1	2	1	

The units added up make fifteen (15) which is 1 ten and 5 units. The five units are placed in the units column, and the one ten carried to the tens column.

The tens increased by this 1 make twenty tens (200), which is 2 hundreds and 0 tens. The cypher 0 is put in the tens column, and the 2 hundreds carried on to the hundreds, which become fifteen hundreds, or one (1) thousand and 5 hundreds. Placing the 5 under the hundreds and carrying the 1 thousand to the 3 thousands the total becomes 4 thousands.

The sum or total of the 3 addends is 4505.

It is by no means necessary to place the terms vertically under one another. They may be arranged horizontally, and then added up. Here care has to be taken that the units are added to units, the tens to the tens, and so on.

In the process of addition the two essential requisites are speed and accuracy. To ensure speed it is a good practice to go through such exercises as the following. Start with a given number, say 7, and then continue to add say 3's, up to any given number, thus 7, 10, 13, 16, etc. The same may be done by adding on other numbers; continuing from a given start, thus, 6, and adding on 7, we obtain 6, 13, 20, 27, and so on for any number of times.

To secure accuracy, it is well to add the columns from bottom to top, and then from top to bottom; or, when placed horizontally, to add the figures from left to right and then from right to left. If the totals agree in each case, it is very likely that the answer is correct.

EXERCISES II.

Find the sums of the following, checking your results by adding from bottom to top, and then from top to bottom.

1.	2.	3.	4.
75	963	541	829
382	875	326	314
971	314	715	692
64	65	493	508
28	802	654	344
394	68	19	182
.....	.....	.....	.....

5. A woman has three baskets full of eggs; one basket contains 22 eggs, the second basket 30 eggs, and the third basket contains 19 eggs; how many eggs are there altogether?

6. In a garden there are six apple trees, one with 107 apples on it, the second with 36 apples on it, the third with 76 apples, the fourth with 52 apples, the fifth with 110 apples and the sixth with 82 apples. How many apples does the garden contain?

7. There are three bags of marbles, one containing 26 marbles, the second 57 marbles, and the third 48 marbles; how many marbles are there altogether?

Add:

8. 26, 212, 18, 77, 84.
9. 38, 146, 372, 237, 96
10. 345, 246, 348.
11. 318, 76, 208, 79.
12. 592, 685, 23, 82, 71.
13. 734, 72, 83, 174, 536.
14. 492, 218, 339, 297, 355, 210.
15. 34, 25, 36, 48, 348, 19, 7.
16. 207, 316, 450, 38, 27, 34.

Find the totals of the following, checking your results by adding from right to left and from left to right.

17. 826, 45, 167, 45, 792, 75.
18. 957, 63, 14, 770, 845, 64.
19. 145, 896, 56, 549, 14, 342.
20. 567, 265, 26, 367, 129, 36, 267, 30.
21. 589, 742, 180, 246, 352, 705.

EXERCISES III.

Express in words the numbers represented by the figures marked with an asterisk (*).

1. $\overset{*}{3}82$. 2. $\overset{*}{1}082$. 3. $21\overset{*}{6}1$. 4. $\overset{*}{8}18$. 5. $\overset{*}{7}56$.
6. $216\overset{*}{1}$. 7. $\overset{*}{1}623$. 8. $18\overset{*}{8}5$. 9. $32\overset{*}{6}$. 10. $2\overset{*}{3}6$.
11. $18\overset{*}{3}2$. 12. $183\overset{*}{2}$. 13. $17\overset{*}{6}1$. 14. $\overset{*}{3}81$. 15. $101\overset{*}{3}$.
16. $\overset{*}{1}01$. 17. $1\overset{*}{1}0$.

Write down the sum of the numbers represented by the digits marked with an asterisk.

18. $\overset{*}{8}325$, $261\overset{*}{3}$, $18\overset{*}{2}2$, $276\overset{*}{4}$.
19. $\overset{*}{8}325$, $261\overset{*}{3}$, $182\overset{*}{2}$, $276\overset{*}{4}$.
20. $\overset{*}{8}00\overset{*}{2}$, $\overset{*}{8}2$, $\overset{*}{2}008$, $\overset{*}{2}08$.
21. $73\overset{*}{6}$, $\overset{*}{8}17$, $\overset{*}{2}926$, $\overset{*}{3}184$.
22. $\overset{*}{7}63$, $\overset{*}{5}68$, $\overset{*}{9}21$, $\overset{*}{3}74$.
23. $\overset{*}{5}36$, $\overset{*}{2}30$, $\overset{*}{2}03$, $121\overset{*}{8}$.
24. $\overset{*}{5}36$, $\overset{*}{2}30$, $\overset{*}{2}03$, $\overset{**}{1}218$.
25. $\overset{*}{1}72$, $\overset{*}{1}72$, $\overset{*}{1}72$.
26. $\overset{*}{1}72$, $\overset{*}{7}12$, $\overset{*}{2}71$.
27. $\overset{*}{1}72$, $\overset{*}{7}12$, $\overset{*}{7}12$.

Use of Signs.

The symbol + (**plus**) is used to express addition. Thus, $4+5$ means 4 added to 5, and the answer is 9, and the statement that four added to five makes nine could be represented by making use of another symbol—the equality symbol =.

Thus, $4+5=9$.

The symbols $\therefore$ and $\because$ will also frequently be employed;

$\therefore$ signifies **therefore**,

$\because$,, **because**.

EXERCISES IV.

Evaluate, that is, find the value of:

1. $28+316+459+326+182$.
2. $56+89+376+83+672$.
3. $36+112+172+56+29$.
4. $42+226+672+53+726$.

Evaluate, that is, find the value of:

5. $38+176+54+365+27$.
6. $126+58+98+462+567$.
7. $316+98+327+42+389$.
8. $389+627+544+89+72$.
9. $16+54+237+84+112$.
10. $56+3276+81+8964$.
11. $12+87+543+898+276+543$.
12. $6+13+897+26+8679+345+6$.
13. $856+983+72+543+876$.
14. $32+97+856+321+765+50$.
15. $3+895+726+86+5890+32$.
16. $5432+674+52+1000+371$.
17. $296+154+172+345+89+1654$.

Addition by Steps.

The following are the results of borings for wells, the tables giving the thicknesses of the various sorts of strata met with; add up and give the depths of the different strata below the surface, also the total depth of the well, as in the example (Well at Romford).

EXAMPLE.—Well at Romford:

Strata.	Thickness in feet.	Depth.
Soil	7	7
Gravel	9	16
Blue clay	29	45
Stone	2	47
Green sand	18	65
Loamy sand	17	82
Bright green sand	20	102
Green sand	32	134
Hard blue clay	18	152

EXERCISES V.

1. Well at Hampstead :

Strata.	Thickness in feet.	Depth.
Brown clay	30	.....
Blue „	170	.....
Sandy „	84	.....
Basement bed	5	.....
Plastic clay	40	.....
Sand	49	.....
Hard chalk	40	.....
Soft „	4	.....
Hard „	28	.....

2. Well at Walham Green :

Strata.	Thickness in feet.	Depth.
Soil	6	.....
Sand and Gravel	15	.....
Blue London clay	119	.....
Mottled clay	36	.....
Sand	4	.....
Mottled clay	15	.....
Sand, stones, water	7	.....
Brown sand and clay	35	.....
Green „ „	3	.....
Flints	1	.....
Chalk, Flints, Rock chalk	170	.....

3. West Drayton :

Strata.	Thickness in feet.	Depth.
Mould	3	.....
Clay	2	.....
Gravel	11	.....
Clay	61	.....
Basement bed	2	.....
Blue clay	10	.....
Clay	11	.....
„	8	.....
Red clay	7	.....
Sand	28	.....
Clay	6	.....

4. Westminster, a boring through an old well :

Strata.	Thickness in feet.	Depth.
Old well	118	.....
Sand	5	.....
Clay	10	.....
Hard rock	2	.....
Sand and stone	2	.....
Sand and clay	10	.....
Shale	2	.....
Mottled stone	16	.....
Green sand	1	.....
Sand and pebbles	21	.....
Grey sand	24	.....
Flints	1	.....
Chalk	205	.....

5. Wandsworth :

Strata.	Thickness in feet.	Depth.
Yellow clay - -	12	.....
Blue „ - -	158	.....
Black „ - -	20	.....
Mottled „ - -	41	.....
Yellow „ - -	5	.....
Green sand - -	36	.....
Flints - - -	2	.....
Chalk - - -	60	.....

Addition of Long Columns.

In adding long sums it is frequently advisable to mark the hundreds as in the following example :

EXAMPLE.—*Add* 28, 76, 126, 92, 75, 38, 127, 36, 84, 59, 96, 74, 89, 438, 127, 399, 418, 385, 59.

	From Units Column.	Tens.	Hundreds	
28	126	112		
76	118	110		
126	112	103	28	
92	106	**101**		
75	**104**	92		
38	99	85		Add up the units in the
127	91	82	27	usual way, the totals, as the ad-
36	84	80		dition proceeds, being marked
84	78	77		on the right ; these however
59	74	69		should *not* be written down
96	65	64		until the hundred is reached,
74	59	55		when the 5 is ticked and 104
89	55	48		marked on the right.
438	46	40	26	The tens column is treated
127	38	37	22	similarly.
399	31	35	21	
418	22	26	18	
385	14	25	14	
59		17		
2826				

EXERCISES VI.

Find the sum of the following groups, marking the hundreds, etc., as in the preceding example.

1. 36, 48, 32, 70, 83, 56, 34, 92, 57, 55, 32, 29, 38, 59, 78, 98, 8, 57, 96, 88, 57, 68, 39.

2. 99, 88, 78, 79, 67, 75, 58, 78, 49, 89, 96, 87, 98, 92, 75, 56, 57, 58, 199, 77.

3. 239, 259, 39, 57, 68, 79, 38, 36, 55, 66, 74, 85, 97, 97, 38, 56, 54, 187, 88, 94, 29, 37.

4. 57, 66, 78, 87, 84, 96, 58, 58, 67, 66, 28, 27, 39, 57, 86, 78, 93, 99, 98, 64, 67, 19, 18, 17, 55.

5. 86, 88, 95, 98, 97, 67, 76, 54, 47, 48, 49, 74, 89, 97, 96, 95, 59, 76, 69, 68, 26, 35, 44, 78, 89, 199, 97, 76, 89, 98, 94, 57, 69, 78.

6. 59, 58, 67, 68, 49, 98, 95, 57, 49, 58, 57, 68, 69, 79, 87, 88, 279, 368, 599, 594, 29, 37, 48, 49, 57, 198, 197, 179, 939, 19, 39, 46, 42, 57, 68.

7. 235, 267, 319, 429, 487, 576, 387, 563, 397, 876, 875, 587, 939, 948, 952, 269, 268, 479, 89, 97, 88, 89, 99, 57, 58, 89, 76, 378, 492, 865, 296, 93, 39, 29, 36, 76, 78, 88, 89.

8. 76, 49, 48, 47, 46, 57, 59, 56, 86, 58, 89, 89, 28, 29, 37, 49, 39, 65, 66, 79, 89, 47, 93, 58, 57, 79, 76, 77, 89, 98, 99, 197, 58, 67, 69, 84, 56, 189, 95, 78, 97, 79, 89.

9. 78, 45, 69, 78, 64, 28, 39, 49, 58, 97, 98, 78, 87, 88, 96, 69, 67, 74, 76, 88, 89, 98, 276, 398, 892, 89, 74, 59, 96, 87, 88, 96, 98, 96, 88, 92, 76, 75, 89, 289, 274, 98.

10. 472, 477, 88, 99, 79, 86, 57, 58, 96, 279, 375, 49, 58, 97, 78, 88, 38, 56, 9, 8, 19, 18, 29, 36, 47, 58, 98, 98, 376, 49, 389, 98, 97, 365, 69, 37, 47, 59, 48.

3. SUBTRACTION.

Subtraction is the operation of finding how much larger or smaller one quantity is than another. This may be expressed in two ways:

(*a*) How much must be added to a given number to make another.

(*b*) By how much must a given number be diminished so as to be equal to another.

The quantity from which a second is to be subtracted is the *Subtrahend*, and the quantity which has to be taken away is the *Minuend*. The first is usually preceded by the word *From*, and the latter by *Take*.

The sign (–), called **minus**, indicates the operation of subtraction, and expresses that the quantity *following* it is to be taken from the quantity preceding it. Thus, 24 – 13 means that 13 is taken away from 24.

The operation of subtraction may be done in two ways, as will be seen from the following example:

EXAMPLE.—*Take* 945 *from* 6128.

Arrange as in addition.

	Th.	H.	T.	U.
From	6	1	2	8
Take		9	4	5
	5	1	8	3

Take 5 units from 8 units, 3 units are left; the 3 is set down in the units column. Since 4 tens cannot be taken from 2 tens; take one of the hundreds, and convert it into 10 tens; thus making 12 tens in all; subtract 4 tens from 12 tens and we have 8 tens, which is set down under the tens. Next, 9 hundreds cannot be taken from 0 hundreds; take one of the thousands and convert it into 10 hundreds, 9 hundreds from 10 hundreds is 1 hundred which is set down in the hundreds column.

Having taken 1 thousand from the 6 thousands, 5 thousands remain, which is set down.

Such is the full reasoning for the operation, but in actual working this is largely curtailed, thus—5 from 8 is 3 : 4 from 2 I cannot : 4 from 12 leaves 8 : 9 from 0 I cannot : 9 from ten is 1. Bring down the 5.

Note that practically the subtrahend has been modified thus :

	Th.	H.	T.	U.
From	5	10	12	8
Take		9	4	5
	5	1	8	3

The result or answer to a Subtraction Sum is called the **remainder** or the **difference**, and one or other of these words should in the earlier stages of work be always entered.

The result can be proved by seeing if when 5183 (the remainder) is added back again to 945 (the number taken away) the first number, 6128, is once more obtained.

Thus,

945 Number taken away. } Add
5183 Remainder. }

6128 Original number.

It can thus be seen if we truly have that number, which, when added to 945, makes 6128.

This suggests another method of doing subtraction, known as **Complementary Addition**.

EXAMPLE.—Thus, in the same example,

6128 – 945.

In the answer let the units digits be represented by U., the tens digits by T., hundreds digits by H., and thousands by Th.

Th.	H.	T.	U.	
6	1	2	8	is to be the answer when
9	4	5		are added together.
5	1	8	3	

Suppose we were adding

The Units digit + 5 would equal 8 ; ∴ U. = 3.

The Tens digit + 4 would make, clearly not 2 but 12 ; ∴ T. = 8.

The 1 would be carried, and 1 + 9 + Hundreds digit would equal 11 ; ∴ H. = 1.
We should again carry 1, 1 + Thousands digit would make 6 ; ∴ Th. = 5.
The remainder is ∴ 5183.

Expressed more briefly, the working of the sum may be as follows :

5 + a digit = 8 ; ∴ digit = 3.
4 + „ = 12 ; ∴ „ = 8.
Carry 1.
1 + 9 + „ = 11 ; ∴ „ = 1.
Carry 1.
1 + „ = 6 ; ∴ „ = 5.

EXERCISES VII. a.

Mental or Oral.

1. What must be added to 8 to make (*a*) 17, (*b*) 24, (*c*) 30, (*d*) 35?

2. What must be taken away from 16 to leave (*a*) 7, (*b*) 8, (*c*) 6, (*d*) 5?

3. One boy has 13 marbles in his pocket, a second boy has only 6 ; how many more marbles has the first boy than the second?

4. A girl has 15 pennies in her pocket, and gives her companion 6 ; how many has she left?

5. A boy has 30 yards of string, gives some away and finds he has 25 yards left ; what did he give away?

6. A rod is 26 inches long, two pieces are cut off, the first 8 inches in length, the second 5 inches in length ; what is now the length of the rod?

7. A piece of string is 12 yards long ; what length must be tied on to it, to make it 16 yards long?

8. What weight must be added to 16 lbs. to make up a quarter, or 28 lbs.?

EXERCISES VII. b.

1. Find the remainders in the following cases, and prove your answer:

(*a*) From 1435 Take 518	(*b*) From 1063 Take 107	(*c*) From 1063 Take 914 .....
(*d*) From 2150 Take 1026	(*e*) From 839 Take 76	(*f*) From 2618 Take 1926 .....
(*g*) From 1764 Take 617	(*h*) From 8820 Take 312	(*i*) From 1000 Take 998 .....

2. (*a*) 78 – 25; (*b*) 100 – 84; (*c*) 262 – 118;
(*d*) 309 – 106; (*e*) 323 – 248; (*f*) 778 – 90;
(*g*) 716 – 182; (*h*) 1086 – 689; (*i*) 2175 – 889;
(*j*) 3162 – 1469; (*k*) 5269 – 4182; (*l*) 1000 – 286;
(*m*) 1001 – 929; (*n*) 1072 – 974; (*o*) 2116 – 1119.

Supply the missing figures in the following subtraction examples:

3. From 9163 Take 4271	**4.** From 7642 Take 3421	**5.** From 1008 Take 206	**6.** From 3736 Take 3582
7. From 4168 Take 2190	**8.** From 7290 Take 6361	**9.** From 2081 Take 82	
10. From 3333 Take 1865	**11.** From 4571 Take 737	**12.** From 8088 Take 6162	

EXERCISES VII. c.

In the following tables, giving the various strata met with in making the wells specified, the depths to the bottoms of the various strata and the well bottom are given; fill up the table by giving the thicknesses of the different strata, similarly to the worked example.

EXAMPLE.—Southend Water Works:

Strata.	Depth in feet.	Thickness.	
Surface soil	3	3	
Yellow clay	33	30	... by subtracting 3 from 33
Blue „	417	384	„ „ 33 „ 417
Running sand	598	181	„ „ 417 „ 598
Chalk	900	302	„ „ 598 „ 900

1. Well at New Barnet:

Strata.	Depth in feet.	Thickness.
Yellow clay	24	.....
Blue „	111	.....
Quick sand	115	.....
Mottled clay	131	.....
Silver sand	135	.....
Pebbles in sand	136	.....
Mottled clay	137	.....
Dark sand and pebbles	144	.....
Dark sand	157	.....
Sand and flints	159	.....
Chalk	439	.....

2. Well at Elstree:

Strata.	Depth in feet.	Thickness.
Yellow clay	20	.....
Hard blue clay	170	.....
Sandy „ „	183	.....
Sand and water	184	.....
White pebbles	189	.....
Coloured sandy clay	196	.....
Clay and pebbles	202	.....

3. Well at Essendon:

Strata.	Depth in feet.	Thickness.
Soil	2	.....
Gravel and sand	26	.....
Blue clay	130	.....
Yellow clay	170	.....
Flints	172	.....
Chalk with flints	344	.....

4. Well at Eltham Park:

Strata.	Depth in feet.	Thickness.
Sand	44	.....
Yellow clay with fragments of shells	50	.....
Green sand	60	.....
Black pebble bed	70	.....
Sand	122	.....
Chalk	216	.....

5. Well at Alperton :

Strata.	Depth in feet.	Thickness
Yellow clay	25	.....
Blue „	165	.....
Mottled „	201	.....
Sand	214	.....
Pebbles	218	.....
Flints	220	.....
Chalk	400	.....

EXERCISES VII. d.

Mental or Oral.

EXAMPLE.—*Find the result of the following operations :*

8+ 7 −6+ 9 − 3 − 2 + 6 + 8 − 2 − 9.
15 9 18 15 13 19 27 25 16

Starting with the +8, the results are figured underneath, as above, the process however being mental; the answer is **16.**

Evaluate :

1. 5+6−4−2+7−9+6−5.
2. 3+9+10−5−6+8−5+4.
3. 6+7−3−7+8+6−9−5.
4. 8+5+6−3−4+6−3+6.
5. 6−4+9+2−3−6+8+2.
6. 9+2−3−4+2+4−6+8.
7. 7−3+4+6−3+8−5−7.
8. 2+4+3−5+6−7+8+2.
9. 6−5+6−3−2+6+4−6.
10. 2−1+3+6−4−2+8+2.
11. 6+4+3−4−6+6+4−6.
12. 8−6+3+6−4−2+6+3.
13. 4+6−3−2+6−4+3−4.
14. 3+2−4+5−2+3+6−5.
15. 8+7−5+6−8−4+9−8.
16. 5−4+8−5+6+3−6+8.

4. MULTIPLICATION.

Multiplication is only a shortened method of addition, where the addends are all equal. In arithmetical operations the sign for multiplication is represented by $\times$; if, for example, we wish to find the value of $4+4+4+4+4$, *i.e.* of 5 fours added together, the operation is equivalent to multiplying 4 by 5, and is represented by 5×4.

Similarly 2×8 means $8+8$.

8×2 means $2+2+2+2+2+2+2+2$.

In 2×8 the answer 16 is termed the **product**, the 2 is called the **multiplier**, and 8 the **multiplicand**, while 2 and 8 are spoken of as the **factors** of 16.

Remembering that multiplication is merely shortened addition, we can construct a set of multiplication tables.

1 two is 2

2 twos make 4, for $2+2=4$.

3	„	„	6	3 twos are 2 greater than 2 twos, *i.e.* 2 greater than 4.	
4	„	„	8	2 greater than	6,
5	„	„	10	„ „	8,
6	„	„	12	„ „	10,
7	„	„	14	„ „	12,
8	„	„	16	„ „	14,
9	„	„	18	„ „	16,
10	„	„	20	„ „	18,
11	„	„	22	„ „	20,
12	„	„	24	„ „	22.

EXERCISES VIII. a.

1. Construct a table of multiples for the numbers 3 to 12 similar to the above, thus

1 three is 3,
2 threes make ...
etc.

2. Arrange the results of all these tables as in Fig. 1 (filling in the blank spaces): where the result of multiplying any number, such as 7 by 8, is put in that square which is under the 7, and in the same line horizontally as the 8.

1	2	3	4	5	6	7	8	9	10	11	12
2											
3											
4											
5											
6											
7											
8						56					
9											
10											
11											
12											

FIG. 1.

Note the peculiarity of the table, that the rows and the columns might be interchanged, for example,

$$8 \times 7 = 56$$

and $7 \times 8 = 56$;

accordingly $8 \times 7 = 7 \times 8$,

so also $6 \times 4 = 24 = 4 \times 6$.

This fact may be illustrated graphically on 'squared' or chequered paper.

For example, to show that $4 \times 3 = 3 \times 4$.

Three squares can represent 3 units, thus (Fig. 2)

FIG. 2.

four such rectangles will represent the product (4×3) (Fig. 3).

FIG. 3.—To illustrate 4×3.

But the big rectangle representing the product can be divided up into 3 rectangles each of 4 squares, thus

and represents 3×4 (Fig. 4)

FIG. 4.—To illustrate 3×4.

EXERCISES VIII. b.

Practical.

[*Apparatus*: *Ruler and Squared Paper.*]

Represent graphically the following products, and show in each case that the multiplier and multiplicand can be interchanged:

1. 5×3. **2.** 4×5. **3.** 6×2. **4.** 8×7. **5.** 6×9.

These results could be generalized by stating that $a \times b = b \times a$, where a and b stand for any numbers.

EXERCISES VIII. c.

Mental or Oral.

EXAMPLE.—*Give factors for* 72.

Since $6 \times 12 = 72$, then 6 and 12 are said to be **factors** of 72; similarly, 8 and 9 are also factors of 72.

1. Give factors for the following numbers:

   (*a*) 48, (*b*) 28, (*c*) 35, (*d*) 90, (*e*) 88, (*f*) 32.

2. Write down the results of the following operations:

   (i) $3 \times 4 + 2$; (ii) $5 \times 6 - 8$; (iii) $12 \times 9 - 9$;
   (iv) $12 \times 9 - 12$; (v) $8 \times 11 + 8$; (vi) $8 \times 11 + 3$.

Multiplication by single digits.

The method of multiplying by numbers consisting of a single digit, is best illustrated by considering an example.

EXAMPLE.—*To multiply* 876 *by* 4.

Here we have to evaluate 4×876; or, what is the same thing, $876 + 876 + 876 + 876$.

We do this by addition first.

$$\begin{array}{r} 876 \\ 876 \\ 876 \\ 876 \\ \hline 3504 \\ \hline {\scriptstyle 3\ 2} \end{array}$$

Next by multiplication:

$$\begin{array}{r} 876 \\ 4 \\ \hline 3504 \\ \hline {\scriptstyle 3\ 2} \end{array}$$

As in addition we added 4 sixes, so we now take 4×6 making 24, setting down 4 and carrying the 2 tens. Corresponding to adding the 4 sevens, and this 2, we obtain mentally the value of $4 \times 7 + 2$ or 30, set down 0, and carry 3. Next we obtain $4 \times 8 + 3$ or 35; and the answer is **3504**.

Multiplication by factors.

EXAMPLE.—*To multiply* 586 *by* 45.

We have to find the sum of 45 numbers each equal to 586.

These numbers could be arranged in 5 groups of 9, for 45 is 5×9.

The number in each group is $9 \times 586 = 5274$,

„ „ „ the 5 groups is $5 \times 5274 = 26370$.

The result has been obtained by multiplying,

1st by one factor 9,

2nd by the second factor 5;

i.e., we multiply in succession by all the factors which make up the multiplier.

To multiply by 10, 100, or 1000 is easy, for when multiplied by 10, units become tens, tens become hundreds, and hundreds thousands.

Thus, 215 multiplied by 10 becomes $50 + 100 + 2000$, *i.e.* 2150.

Notice that the place value of each digit of the original number has been increased tenfold, but that all that has been done is to add a cypher after the unit figure.

Similarly, when we multiply by 100, units become hundreds and tens become thousands; the place value of each digit is increased one hundredfold, and 2 cyphers are added.

Thus, 18 multiplied by 100 yields 1800, and so on.

EXAMPLE.—*To multiply* 324 *by* 30.

That is, to multiply by a number the factors of which are 10 and 3.

Multiplying 324 by 10 we obtain 3240.

„ 3240 by 3 „ 9720.

The two operations are conducted simultaneously, *i.e.* we multiply by 3 and add a cypher.

General Case.

EXAMPLE.—*To multiply* 276 *by* 315.

It is required to find the number which would result, if 276 were written down 315 times, and the sum obtained; the

result must be the same as if 276 be first taken 300 times, then another 10 times and finally another 5 times. Hence, to obtain the result multiply 276 by 300 first, then multiply 276 by 10, and lastly by 5, and finally add the 3 products together.

Or, $315 \times 276 = (300 \times 276) + (10 \times 276) + (5 \times 276)$.

The working is set down usually as below

276	(i) Multiply 276 by 300, by multiplying by 3 and
315	the addition of 2 cyphers to increase the place
	value a hundredfold.
82800	(ii) Multiply by 10, by adding a cypher to increase
2760	the place value of each digit tenfold.
1380	(iii) Multiply by 5.
86940	(iv) Add the three products, and obtain 86940 as
	the product of 276 and 315.

NOTE.—It is convenient to place the numbers with their unit digits beneath one another.

Generally the work is shortened very slightly by omitting the cyphers used in increasing the place value in the partial products. Thus,

276
315
828
276
1380
86940

The gain in time is very small, and it is better to omit nothing at first.

EXERCISES IX. a.

Mental or Oral.

1. A yard (1 yd.) contains 3 feet; how many feet are there in (*a*) 4 yds., (*b*) 3 yds., (*c*) 6 yds., (*d*) 5 yds., (*e*) 8 yds.?

2. A foot (1 ft.) contains 12 inches; how many inches are there in (*a*) 8 ft., (*b*) 6 ft., (*c*) 4 ft., (*d*) 5 ft., (*e*) 9 ft.?

3. One sixpenny piece has the same value as 6 pennies ; how many pennies should be exchanged for

(*a*) 4 sixpenny pieces, (*b*) 5, (*c*) 7, (*d*) 2, (*e*) 8 ?

4. How many days are there in the following number of weeks :

(*a*) 4 weeks, (*b*) 12 weeks, (*c*) 7 weeks, (*d*) 9 weeks ?

5. A foot contains 12 inches ; how many inches are there in 8 feet ?

6. How many marbles are there in 6 bags, each containing 9 marbles ?

7. A yard contains 36 inches ; how many inches are there in a roll of paper 5 yards in length ?

8. A man gives 7 pennies to each child in a school of 160 children ; how many pennies does he give away altogether ?

Evaluate the following :

9.	25, multiplied by	(*a*) 2,	(*b*) 4,	(*c*) 6,	(*d*) 8.	
10.	25, „	(*a*) 3,	(*b*) 5,	(*c*) 7.		
11.	125, „	(*a*) 4,	(*b*) 8,	(*c*) 3,	(*d*) 5,	(*e*) 7.
12.	144, „	(*a*) 3,	(*b*) 4,	(*c*) 12,	(*d*) 5,	(*e*) 6.
13.	236, „	(*a*) 2,	(*b*) 5,	(*c*) 7,	(*d*) 8,	(*e*) 9.
14.	121, „	(*a*) 11,	(*b*) 3,	(*c*) 6,	(*d*) 5.	
15.	37, „	(*a*) 3,	(*b*) 6,	(*c*) 9,	(*d*) 4.	
16.	28, „	(*a*) 4,	(*b*) 8,	(*c*) 3.		
17.	112, „	(*a*) 5,	(*b*) 8,	(*c*) 9.		

State all the factors of

18. (*a*) 24, (*b*) 32, (*c*) 45.

EXERCISES IX. b.

1. How many names are there on 9 registers, each containing 45 names ?

2. How many panes of glass are there in 12 windows, each containing 27 panes ?

3. A man has 276 oranges in each of 5 boxes, how many oranges has he altogether ?

4. How many lines are there in a page of 9 columns, each of which contains 136 lines?

5. A book has 280 pages with 48 lines to the page; what is the total number of lines?

6. What is the weight of 28 blocks of stone, each one of which weighs 137 pounds?

7. If a mile contains 1760 yards, how many yards are there in 26 miles?

Multiply the following:

8. 76 by 35.	**9.** 89 by 47.	**10.** 57 by 23.
11. 93 by 87.	**12.** 96 by 64.	**13.** 296 by 27.
14. 305 by 76.	**15.** 1905 by 63.	**16.** 7560 by 640
17. 906 by 267.	**18.** 693 by 709.	**19.** 790 by 652.
20. 319 by 1035.	**21.** 737 by 680.	**22.** 543 by 706.
23. 318 by 723.	**24.** 6389 by 38.	

25. A store has in it 326 boxes each containing 138 apples, 326 boxes each containing 262 apples; how many apples are there altogether in the store?

26. Evaluate:

(*a*) $(24 \times 260)+(38 \times 260)+(37 \times 260)$.
(*b*) $(198 \times 325)-(68 \times 325)$.
(*c*) $(316 \times 175)-(316 \times 74)$.
(*d*) $(25 \times 140)+(65 \times 140)+(84 \times 140)-(170 \times 140)$.
(*e*) $(12 \times 36 \times 25)+(1300 \times 14)$.

27. *After working mentally, write* down the results of the following operations:

(*a*) $76 \times 5+8$.	(*b*) $37 \times 6+5$.	(*c*) $37 \times 6-8$.
(*d*) $124 \times 7+26$.	(*e*) $384 \times 9+54$.	(*f*) $256 \times 12+7$.
(*g*) $382 \times 9+123$.	(*h*) $36 \times 5-4$.	(*i*) $471 \times 11-8$.
(*j*) $374 \times 10-32$.	(*k*) $256 \times 12-24$.	(*l*) $318 \times 7-45$.
(*m*) $100-5 \times 6$.	(*n*) $180-3 \times 15$.	(*o*) $200-11 \times 17$.
(*p*) $562-7 \times 28$.	(*q*) $2362-9 \times 183$.	

28. What is the weight of 312 cart loads of bricks, each one of which weighs 1643 pounds?

29. What is the distance covered in a year (365 days) by a train making 2 daily journeys of 143 miles?

30. If a pint of a liquid weighs 23 ounces, how many ounces short of 1000 are there in a cask containing 43 pints?

31. A boy wishes to join up by a thread, two posts 4956 inches apart; he has 12 skeins of thread each 415 inches in length; how much thread has he over?

EXERCISES IX. c.

Mental or Oral.

Write down the product of the given numbers, and the numbers corresponding to the digits marked with an asterisk. Thus in **1.** (*a*) $234 \times \overset{*}{1}46$. The result would be $234 \times 40 = 9360$.

1. (*a*) 234 by $1\overset{*}{4}6$; (*b*) by $16\overset{*}{4}$; (*c*) by $\overset{*}{2}046$; (*d*) by $\overset{*}{7}13$.

2. (*a*) 893 by $36\overset{*}{3}$; (*b*) by $\overset{*}{3}63$; (*c*) by $\overset{*}{3}001$.

Write down the products of the pairs of numbers represented by digits marked by asterisks in the following:

3. (*a*) $\overset{*}{8}76$ by $32\overset{*}{5}$; (*b*) $\overset{*}{8}76$ by $\overset{*}{5}23$; (*c*) $\overset{*}{8}672$ by $\overset{*}{2}53$.

(*d*) $96\overset{*}{1}3$ by $\overset{*}{2}182$; (*e*) $\overset{*}{7}992$ by $\overset{*}{8}99$; (*f*) $\overset{*}{8}734$ by $\overset{*}{7}34$.

Squared and Cubed Numbers.

When a number is multiplied by itself it is said to be **squared**, thus

$12 \times 12 = 144$, and 144 is the **square** of 12

The operation is generally represented by the index 2 written above the number, thus

$$144 = 12 \times 12 = 12^2$$
$$16 = 4 \times 4 = 4^2$$
$$9 = 3 \times 3 = 3^2$$
$$123 \times 123 = 123^2$$
$$a \times a = a^2, \text{ where } a \text{ stands for any number.}$$

Similarly,

$a^3 = a \times a \times a$

$a^4 = a \times a \times a \times a$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$; and hence 81 is called the 4th **power** of 3.

Notice that three like numbers have to be multiplied together to obtain the third power; four to obtain the fourth power, and so on.

EXERCISES X.

1. Write down (*a*) the powers of 2 up to the 10th, (*b*) the powers of 3, also up to the 10th.

2. Write down the powers of 5 up to the 5th.

3. Name the 5 lowest powers of 2 which are also powers of (*a*) 8, and of (*b*) 16.

4. Construct a table of squares for the first 20 numbers.

5. Using your table of squares, find the numbers which, when squared, will give the numbers nearest to the following:

(*a*) 10, (*b*) 20, (*c*) 30, (*d*) 40, (*e*) 50,
(*f*) 60, (*g*) 80, (*h*) 110, (*i*) 130, (*j*) 150.

6. Construct a table of the cubes of the first 20 numbers.

7. A number is known to be (*a*) a squared number, find the possible digits in the units place; what digits are possible if the number is known to be (*b*) a cubed number?

8. Write down as concisely as possible:

(*a*) $a \times a \times a \times a \times a$, (*b*) $b \times b \times b \times b \times b \times b$, (*c*) $x \times x \times x$,
(*d*) $y \times y$, (*e*) $z \times z \times z \times z \times z \times z \times z \times z \times z$.

EXERCISES XI.

Practical.

[*Apparatus: Squared Paper.*]

1. The accompanying figure represents the square of 5. Draw similar figures representing the results of squaring the following numbers :

3, 4, 8, 11, 13, 21.

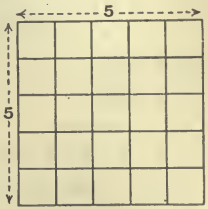


FIG. 5.—To represent the square of 5.

2. Show by drawing figures that

$3\times4+3\times5$ is the same as 3×9.
$5\times2+5\times3$ „ „ 5×5,
$11\times10+11\times1$ „ „ 11^2,
$14\times10+14\times4$ „ „ 14^2.

3. Show by drawing figures that

(*a*) 3×13 is the same as $3\times10+3^2$,
(*b*) 4×14 „ „ $4\times10+4^2$,
(*c*) 3×23 „ „ $3\times20+3^2$,
(*d*) 20×23 „ „ $20^2+20\times3$,
(*e*) 6×36 „ „ $6\times30+6^2$,
(*f*) 30×36 „ „ $30^2+6\times30$.

4. Draw a square to represent 36^2, divide it up into two rectangles one of which represents 6×36, the other 30×36.

Divide the rectangle representing 6×36 as in (*e*) in question (3).
„ „ „ 30×36 (*f*) „ (3).
Show from your divided figures that

$$36^2=30^2+6^2+\text{twice } 30\times6.$$

5. Draw figures showing that

(*a*) $13^2=10^2+3^2+$twice 3×10,
(*b*) $18^2=10^2+8^2+$ „ 8×10,
(*c*) $24^2=20^2+4^2+$ „ 4×20.

6. Making use of the method suggested by (5), *write down* the squares of the following numbers :

(*a*) 15, 25, 35, 45, 55 ; (*b*) 17, 22, 33, 91, 82 ; (*c*) 102, 203, 305.

7. Draw as in the accompanying diagram a figure to represent the square of 10; divide it up into 10 parts, as in the figure, and write down the numbers corresponding to each part; thus the shaded portion in the diagram is the 8th part and represents 15, the first represents 1, the second 3 and so on.

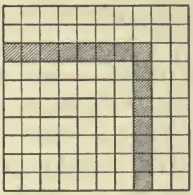


Fig. 6.

(*a*) Find the sum of the first 3 parts,
(*b*) „ „ 4 „
and so on up to (*c*) the first 10 „

8. Draw a figure to represent the following sum:

$$1+3+5+7+9+11+13+15+17+19+21+23+25$$

and find the sum.

9. Represent any odd number, say (*a*) 33, as the difference between two squared numbers; also (*b*) 29, (*c*) 31.

10. A man puts 9 pennies, resting against one another, with their centres in a straight line, against these he puts another row of 7, then another row of 5, and so on; how many rows will there be altogether, and how many pennies?

11. A man puts 24 rows of shillings against each other, 1 in the first row, 3 in the second row, 5 in the third row and so on; how many shillings has he?

5. DIVISION.

Division is the operation of ascertaining how often one quantity is contained in another, or how many times the same quantity may be taken away from another. From this second aspect of the operation, division may be looked upon as repeated subtractions where the minuend remains constant. Thus, 7 is contained in 50, 7 times with a remainder of 1; or, 7 may be subtracted 7 times from 50 and 1 remains.

Hence (1) a concrete quantity may be divided by a concrete

quantity of the same kind, as, how often is £7 contained in £21?

(2) A concrete quantity may be divided by an abstract one, as, divide 12 cwts. into 4 equal parts.

(3) An abstract quantity may be divided by another abstract quantity, as, divide 28 by 4.

The quantity to be divided is called the **Dividend**; the quantity we divide by is known as the **Divisor**, the result obtained as the **Quotient**; and when a quantity is left at the end of the operation it is called the **Remainder**. It is evident that

Divisor × Quotient + Remainder = Dividend.

The explanation of the process is seen from the following example:

EXAMPLE.—*It is required to divide* 3419 by 7.

Arrange the work thus:

7|3419
488 remdr. 3.

Since 3 is less than 7, regard 3 thousand and 4 hundred as 34 hundred; divide by 7; the result is 4 hundred as quotient and 6 hundred as remainder;

$\therefore 7 \times 4 + 6 = 34.$

Regard this 6 hundred remainder and the 1 ten as 61 tens; divide by 7; the result is 8 tens as quotient and 5 tens as remainder;

$\therefore 7 \times 8 + 5 = 61.$

Regard this 5 tens and the 9 units as 59 units; divide by 7; the result is 8 units as quotient and 3 as remainder;

$\therefore 7 \times 8 + 3 = 59.$

The answer or quotient is 488, remainder 3.

To **prove** the result, see if

Divisor × Quotient + Remainder = Dividend.

Thus $(7 \times 488) + 3 = 3416 + 3 = 3419 = \text{Dividend}.$

The operation of division is often represented by the symbol ÷, which signifies that the quantity before the sign is

to be divided by the quantity following it. It is also expressed in other ways; thus 12 divided by 4 may be written as $\frac{12}{4}$, or 12/4.

EXERCISES XII. a.

Mental or Oral.

1. How often is 8 contained in 48, 80, 144 and 192?
2. „ „ 9 „ „ 63, 117, 288, 72?
3. How often can 11 be taken away from 100, 200, 154, 362?
4. A basket containing 124 apples is shared equally among some boys, giving each one 8. How many boys will have shares? What apples, if any, remain?
5. What must 12 be multiplied by to give 84, 120, 48 and 480?

EXERCISES XII. b.

1. Divide 2375 by (*a*) 3, (*b*) 5, (*c*) 7; prove your result in each case.
2. What number when divided by 7 yields 285 as quotient and 6 as remainder?
3. How many numbers less than 1200 are multiples of 8?
4. There are 12 inches in 1 foot; how many feet are there in 3002 inches; how many inches over?
5. A flagon can hold 7 pints; how often can it be filled from a cask containing 1250 pints?
6. How many sixpences are equivalent to 3276 pennies?
7. A man has 2561 pennies; what is the least number of pennies he must add to them, so that he can exchange them for an exact number of shillings?

Division by Factors.

When a divisor is the product of 2 or more factors, the division may be performed by each of these successively. The

final result is the quotient. A little care is required in determining the remainder when the factors do not divide the quantities exactly.

EXAMPLE.—*To divide* 7563 *by* 32.

It is required to find how many lots of 32 can be taken from 7563, and how many units will then remain.

4 | 7563
8 | 1890 remdr. 3, { *i.e.* 7563 can be divided into 1890 groups of 4, and 3 units over.
236 remdr. 2, { *i.e.* 1890 groups of 4 are equivalent to 236 groups of 32, and 2 groups of 4 over.

Quotient = 236 with remdr. $2 \times 4 + 3$, or 11 units.
Hence total remainder is 2 groups of 4, + 3, or 11 units.
The same factors may have their order changed, thus:

8 | 7563
4 | 945 remdr. 3.
236 remdr. 1.

Quotient 236 with 1 group of 8 and 3 units, or 11 units, as remainder.
This method can be extended to 3 or more factors.

EXERCISES XII. c.

Perform the following operations, using factors when advisable. In each case, when factors are used, change their order and thus verify your own work.

1. 2694 ÷ (*a*) 4, (*b*) 5, (*c*) 6, (*d*) 7, (*e*) 8, (*f*) 9, (*g*) 11, (*h*) 12.
2. 28407 ÷ (*a*) 14, (*b*) 15, (*c*) 16, (*d*) 18, (*e*) 21, (*f*) 24.
3. 30269 ÷ (*a*) 27, (*b*) 32, (*c*) 35, (*d*) 36, (*e*) 40, (*f*) 44.
4. 1125 ÷ (*a*) 45, (*b*) 48, (*c*) 49, (*d*) 55, (*e*) 60, (*f*) 64.
5. 2784 ÷ (*a*) 72, (*b*) 84, (*c*) 96, (*d*) 105, (*e*) 110, (*f*) 112.
6. 3269 ÷ (*a*) 135, (*b*) 147, (*c*) 168, (*d*) 180, (*e*) 192, (*f*) 210.

EXERCISES XII. d.

Graphic.

[*Apparatus*: *Squared Paper.*]

1. Divide graphically 82 by (*a*) 7, by (*b*) 17, by (*c*) 13. (Mark off a distance representing 82, and find the number of times a distance representing 7 can be marked off on this.)

2. Mark off a distance representing 97, and see how many distances representing 5 can be marked off on this, next take these distances in groups of 3; how many groups are there, what is left over?

3. Similarly, divide 86 by (*a*) 15 (*i.e.* by 3×5) and by (*b*) 24.

4. The accompanying figure contains 166 small squares. Divide it up so as to show that when 166 is divided by 28, by the method of factors, first by 7 then by 4, the remainder is of the form $3 \times 7 + 5$, and equal to 26. What is the Quotient?

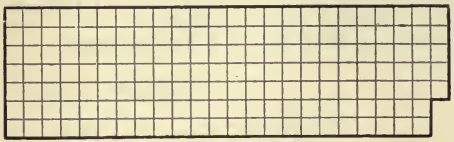


FIG. 7.

5. Draw in a similar manner to that employed in (4) a figure divided up to show the form of the remainders when 166 is divided by 35, by 42 and by 63.

6. Draw figures as in (4) and (5), showing the forms of the remainder when 154 is divided by 72, by 54 and by 45.

Division by the General Method.

EXAMPLE:—41184 ÷ 234.

```
        6
       70
      100
234 ) 41184 (
    23400 = 100 times 234
    -----
    17784
    16380 = 70 times 234
    -----
     1404
     1404 = 6 times 234
     ----
```

411 hundreds odd is greater than 100 times 234 and less than 200 times 234.

We subtract therefore 100 times 234 from 41184, and the remainder is 17784; and we set 100 above 41184 as in the example.

17784 we know is less than 100 times 234; we have to find the greatest exact *ten* multiple of 234 less than 17784; now 2 hundred and odd divides into 177 hundred odd, clearly something nearly 80 times, and by multiplication 80 times is found to be too large; 70 times 234 is however less than 17784, and we accordingly multiply 234 by 70 and subtract from 17784, leaving 1404; we set the 70 above the 100.

By inspection it is seen that 234 divides into 1404, something like 6 times; 6 is tried, and found to equal 1404 exactly, and the result of the division is 176.

The method is usually contracted as follows:

```
          176 = Quotient
      ------
234 | 41184
      234
      ----
      1778
      1638
      ----
       1404
       1404
       ----
```

To prove the result, test if

Quotient × Divisor + Remainder = Dividend.

Here $234 \times 176 + 0 = 35200 + 5280 + 704 = 41184$.

The result is therefore correct.

Italian Method.

The writing work is sometimes abbreviated by writing down the *remainders* only in the division.

EXAMPLE :—*To evaluate* $87643 \div 387$.

Worked out in full.

226 = Quotient

387 | 87643
774
1024
774
2503
2322

181 = Remainder.

Worked out by Italian method.

226 = Quotient

387 | 87643
1024
2503
181

twice 7 = 14, 14 and 2 = 16, carry 1.
„ 8 = 16, with 1 = 17, 17 and 0 = 17, carry 1.
„ 3 = 6, with 1 = 7, and 1 = 8, so obtaining the 1st partial remainder 102.

Bring down the 4.

twice 7 = 14 + 0 = 14, carry 1.
„ 8 = 16, with 1 = 17, 17 + 5 = 22, carry 2.
„ 3 = 6, with 2 = 8, 8 + 2 = 10, so obtaining the 2nd partial remainder 250.

Bring down the 3.

6 times 7 = 42, and 1 = 43, carry 4.
6 „ 8 = 48 with 4 = 52, and 8 = 60, carry 6.
6 „ 3 = 18 with 6 = 24, and 1 = 25, and the **final remainder is 181.**

Division true to the Nearest Unit.

Frequently in division we are not concerned with the remainder, and it is more convenient to state the answer true to the nearest unit (or to the nearest ten, or hundred).

EXAMPLE.—$87643 \div 387$.

The quotient is (as on p. 35) 226 true to the nearest unit, as 181 is less than one half of 387.

Again, in dividing 7836 by 9 the quotient is 870 and 6 over, but stated true to the nearest unit the quotient is 871, as the remainder 6 is greater than one half of 9, and 9×871, *i.e.* 7839 is nearer to 7836 than 9×870 or 7830.

EXERCISES XII. e.

Find the quotient and remainder when each of the following numbers

(i) 62341, (ii) 56375, (iii) 88216, (iv) 70062,
(v) 34562, (vi) 16324, (vii) 83110,

are divided by each of the following :

(*a*) 23, (*b*) 56, (*c*) 123, (*d*) 478,
(*e*) 525, (*f*) 417, (*g*) 731, (*h*) 922.

EXERCISES XII. f.

Mental or Oral.

1. What is the least number which can be taken from 60 so that the remainder may be divisible by (*a*) 8, (*b*) 7, (*c*) 9?

2. What is the least number which can be added to 73 so that the sum may be an exact multiple of (*a*) 6, (*b*) 7, (*c*) 11, (*d*) 5?

3. In division operations, supply the missing quantities in the following :

	Divisor.	Quotient.	Remainder.	Dividend.
(*a*)	8	3	4	*
(*b*)	9	*	*	232
(*c*)	7	123	5	*
(*d*)	6	182	3	*
(*e*)	*	7	4	200
(*f*)	*	12	10	430.

4. What digit must any number end in, to be exactly divisible by 5?

5. What are the numbers less than 100 exactly divisible by 25?

6. What must be the tens and units digits of any number if it is exactly divisible by 25?

7. State the remainders when the following numbers are divided by 25 :

80, 180, 237, 257, 983, 933, 1953, 201, 441.

8. Multiplying by 100 and dividing by 4 is equivalent to multiplying by 25. Making use of this, state a quick way of multiplying by 25.

9. Evaluate the following :

25×126, 25×18, 25×144, 25×48, 25×337.

10. State a method of quickly dividing by 25.

11. Evaluate

$2300 \div 25$, $375 \div 25$, $875 \div 25$, $1925 \div 25$, $4700 \div 25$.

6. FACTORS.

Factors.—Many rules can be established for finding possible factors, thus :

(*a*) **2.** Clearly all even numbers are multiples of 2.

(*b*) **4.** Since 100 is 25×4, any number which is an exact multiple of 100 is an exact multiple of 4; therefore any

number divisible by 4 must have the number represented by the tens and units digits also divisible by 4.

Thus, 1872 is divisible by 4, since 1800 is divisible by 4 and since 72 is also divisible.

To test if 2316 is divisible by 4, we merely note that 16 is divisible by 4.

(*c*) **8.** Similarly, since $1000 = 125 \times 8$, any number is divisible by 8 if the number represented by the hundreds, tens and units digits is divisible by 8.

(*d*) **9.** To test if a number is divisible by 9.

Consider any number such as 3426.

$$\begin{aligned} 3426 &= 3000 + 400 + 20 + 6 \\ &= 3 \times (1000) + 4 \times (100) + 2 \times (10) + 6 \\ &= (3 \times 999 + 3) + (4 \times 99 + 4) + (2 \times 9 + 2) + 6. \end{aligned}$$

But 999 is exactly divisible by 9,

also 99 „ „ „ „

and 9 „ „ „ „

The number 3426 can be divided by 9 if $3 + 4 + 2 + 6$, *i.e.* the sum of the digits, can be divided by 9.

And the remainder, when 3426 is divided by 9, is the same as that obtained by dividing $3 + 4 + 2 + 6$ by 9.

EXERCISES XIII.

1. Show that 1836, 2176, 3024, 51232, 6328 are exactly divisible by 4. Which are divisible by 8 also and which by 9?

2. Find the remainders when the following operations are gone through:

(*a*) The difference between 3265 and 2356, divided by 9, and also by 4.

(*b*) The sum of 3265 and 2656, divided by 9, and also by 4 and by 25.

(*c*) Twice 7361 divided by 9.

3. If a cubic foot of iron weighs 450 pounds, how many cubic feet can be obtained from 26732 pounds; what weight will be left over?

4. The distance between London and Sheffield is 162 miles; during which consecutive journey will a locomotive journeying backwards and forwards between these two places complete 10000 miles?

5. A number has two of its digits unknown, $1a85b$; they are represented by a and b. The remainder when the number is divided by 25 is known to be 6, and known to be 3 if the divisor is 9; what are the two digits?

6. In (5) find the possible numbers, if both are known to be exactly divisible by 36 (*i.e.* by 4 and by 9).

7. Supply the missing figures in the following example on division, worked by the Italian method:

```
      3** = Quotient
237 ) *****
      1251
       668
       194 = Remainder.
```

8. How many pieces of cloth 128 inches in length can be cut off from a roll 3646 inches long; what length will be left over?

9. How many numbers less than 1500 are exactly divisible by 51?

10. A number is divided by another by the method of factors, thus:

```
a ) *****
b ) ****  rem. 3
    ***   rem. 4      Total remainder = 35.
```

What is the first divisor a?

7. ARITHMETICAL AVERAGES.

Arithmetical averages.—The word **average** is used in a variety of senses; but when the sum of a number of quantities is divided by that number, the result is termed the **arithmetical average** of the given quantities.

EXAMPLE.—*Find the arithmetical average number of M.P.'s on the different sides in the Parliaments quoted below:*

Year.	Ministerial Side.	Opposition Side.
1833	480 Whigs	173 Conservatives
1835	380 Whigs	273 Conservatives
1837	352 Whigs	301 Conservatives
1841	367 Conservatives	286 Whigs
1847	327 Whigs	326 Conservatives
1852	333 Liberals	320 Conservatives
1857	366 Liberals	287 Conservatives
1859	348 Liberals	305 Conservatives

Here there are 8 ministries.

Adding the numbers on the Ministerial and Opposition Sides. } Total on Ministerial Side, 2953. Total on Opposition Side, 2271.

Dividing each total by the number of ministries the arithmetical average is obtained.

$\therefore$ $2953 \div 8 = 369$ is the average on the Ministerial Side to the nearest member,

and $2271 \div 8 = 284$ is the average on the Opposition Side to the nearest member.

EXAMPLE.—*The average number of merchant and other vessels (not including warships) launched in the United Kingdom from* 1893-1903 *inclusive is given in the table below. Find the arithmetical average of the number launched per year:*

Year.	Number of Ships.
1893	536
1894	614
1895	579
1896	696
1897	591
1898	761
1899	726
1900	692
1901	639
1902	694
1903	697

Add the number of ships and divide by the number of years (11) to obtain the arithmetical average.

Total number of ships = 7225.

Arithmetical average = 657 to the nearest exact number of ships.

EXAMPLE.—*During the four years* 1902-1905 *inclusive, the arithmetical average of the number of ships (not warships) under construction in the United Kingdom was* 412 *per year, the number during* 1902, 1903, 1904 were 388, 393, 393 *respectively. How many ships were under construction during* 1905?

Since the average number per year for 4 years = 412,
the total number during the 4 years = 1648,
the total number during 1902-04 = 388 + 393 + 393 = 1174;
∴ number during 1905 = 1648 − 1174
= 474.

EXERCISES XIV.

N.B.—In all the examples below, the average asked for is the arithmetical average, and the answer is to be found to the nearest unit.

1. There are 10 bags of oranges, the bags containing the following numbers of oranges: 8, 9, 11, 7, 6, 9, 7, 9, 8, 7. Find the average number of oranges contained in each bag.

2. There are 5 piles of coins, 10 coins in one pile, 8 in the next and 6 each in the remaining three. Find the average number of coins in each pile.

3. In a school there are 13 boys in Form I., 12 in Form II., 14 in Form III., 15 in Form IV., 10 in Form V. and 6 in Form VI. Find the average number of boys in each Form.

4. The number of depositors in the Post Office Savings Bank for the years 1894-1903 inclusive is given in the accompanying table. Find the average number of depositors (in thousands) during that time :

Year.	Number of Depositors in Post Office Savings Bank.
	In thousands.
1894	6,109
1895	6,454
1896	6,862
1897	7,240
1898	7,631
1899	8,047
1900	8,440
1901	8,788
1902	9,133
1903	9,404

5. (*a*) Find the average number of letters received per head per annum at the various Post Offices in the United Kingdom during the period as below :

Year.	Letters Per Head.
1896–1897	48
1897–1898	50
1898–1899	54
1899–1900	55
1900–1901	56
1901–1902	59
1902–1903	60
1903–1904	61

(*b*) Draw up also a table showing the yearly increase in the number of letters per head, and calculate the average yearly increase.

6. Find the average number of seamen in the British Navy during the period 1889-1890 to 1904-1905.

Year.	Seamen.
1889–1890	51,526
1890–1891	54,918
1891–1892	56,995
1892–1893	49,133
1893–1894	51,428
1894–1895	57,026
1895–1896	61,945
1896–1897	65,757
1897–1898	70,472
1898–1899	75,709
1899–1900	70,322
1900–1901	82,821
1901–1902	85,323
1902–1903	88,691
1903–1904	93,385
1904–1905	101,287

7. Find the average number of day schools inspected—(*a*) England and Wales, (*b*) Scotland, (*c*) Ireland—from 1894–1903 inclusive.

Year.	(*a*) England and Wales.	(*b*) Scotland.	(*c*) Ireland.
	No. of Schools Inspected.	No. of Schools Inspected.	No. of Schools Inspected.
1894	19,709	3,053	8,505
1895	19,739	3,034	8,557
1896	19,848	3,083	8,606
1897	19,958	3,086	8,631
1898	19,937	3,067	8,651
1899	20,064	3,062	8,670
1900	20,100	3,104	8,684
1901	20,116	3,141	8,692
1902	20,153	3,145	8,712
1903	20,173	3,149	8,720

8. The average mean temperatures at Greenwich, in degrees Fahrenheit for each month excepting November, of a certain year are: January 37°, February 39°, March 41°, April 45°, May 53°, June 59°, July 62°, August 61°, September 55°, October 50°, December 40°.

If the average temperature for the 12 months is known to be 49°, find the average temperature during November.

9. The average dates at which the following spring migrants arrive and leave the British Isles is given below:

Cuckoo	April 14	Aug. 25.
Swallow	April 10	Oct. 15.
Treepipit	April 15	Sept. 25.
Sand-martin	April 9	Oct. 4.
Martin	April 14	Oct. 12.
Nightingale	May 8	July 27.
Landrail	April 10	Oct. 5.

Find the average dates of arriving and of leaving of the birds as a class.

10. Find the average (*a*) population, (*b*) acreage, (*c*) number of emigrants from the counties of Ulster at the times specified:

Counties.	(*a*) Population.	(*b*) Extent in Acres.	(*c*) Irish Emigrants.
Ulster.	1901.	1891.	1903.
Antrim and Belfast co. boro.	461,240	711,276	3,042
Armagh - - - -	125,238	313,036	758
Cavan - - - -	97,368	467,025	978
Donegal - - - -	173,625	1,190,268	1,045
Down - - - -	289.335	611,927	1,317
Fermanagh - - -	65,243	417,665	310
Londonderry and co. boro.	144,329	513,388	855
Monaghan - - - -	74,505	318,806	403
Tyrone - - - -	150,468	778,943	962

EXERCISES XV.

Miscellaneous.

Graphic.

[*Apparatus*: *Squared Paper.*]

1. Draw a figure such as ABCD, representing the square of 17; also AEFG, representing the square of 13.

Produce GF to H, mark off CK equal HB and complete as in the figure.

Show from your figure that

$$17^2 - 13^2 = 30 \times 4.$$

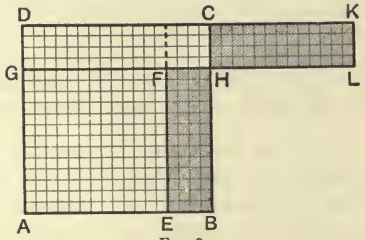


FIG. 8.

2. Draw figures showing that

$$18^2 - 7^2 = 25 \times 11.$$
$$16^2 - 10^2 = 26 \times 6.$$
$$30^2 - 29^2 = 59 \times 1.$$

State in words the general result you have obtained.

3. In the accompanying figure, note the numbers represented by the figures marked a, b, c ....

Then use your figure to find the sum of

$$1+2+3+4+5+6+7+8+9.$$

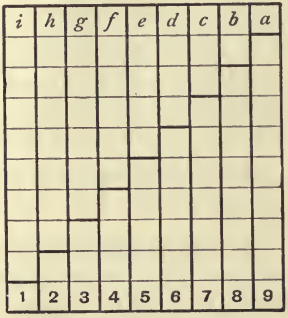


FIG. 9.

4. Draw a similar figure to find the sum of

(a) 1, 2, 3, 4, 5, 6, 7, 8.

(b) 1, 2, 3, 4, 5, 6, 7.

5. Find, by a similar method, the sum of

(a) $1+3+5+7+9+11$.

(b) $1+4+7+10+13+16+19$.

(c) $1+3+5+$etc., until there are 7 terms.

(d) $1+4+7+10+$etc., until there are 8 terms.

(e) $1+2+3+4\ldots$, until there are 12 terms.

(f) $3+5+7+\ldots$, until there are 7 terms.

EXERCISES XVI.

Miscellaneous.

1. In four villages the populations of which are 2952, 896, 1528 and 962; 3267 are males. What is the total population and how many females are there?

2. Three heaps of bricks containing 7860, 9428 and 12064 are to be cleared away in cartloads of 345 bricks each. How many loads will have to be taken and how many bricks left?

3. The population of a town on the 1st Jan., 1904, was 78,563. During the year 942 fresh people came to live in it; 329 people left it; there were 1964 births and 1428 deaths. What was the population at the end of the year?

4. A vessel has to take a cargo of 3470 tons. Of this 1874 tons are coal, 1473 tons coke, and the remainder is cement. What is the amount of cement?

5. At a football match there were 134 people on each of 13 rows in the grand stand; 7 times the number in the second seats; and in the third seats 476 more than in the other two places. What was the total number of spectators?

6. What number must be added to 7342 to make it equal to the product of 879 by 19?

7. The restoration of a church requires £6500. Towards this, there are 3 subscribers of £100, 14 of £50, 32 of £25, 8 of £20, 70 of £10, 75 of £5. What amount is unsubscribed for?

8. A house has 4 floors, with 8 windows to each floor, and each window has 12 panes. How many panes would 32 such houses have?

9. What is the least number which must be added to 718456 so as to contain 85 an exact number of times?

10. By how much must 5384 be increased so as to be equal to 10,000?

11. In a long division sum the quotient was 7054, the divisor was 572 and the remainder 342; what was the dividend?

12. To the sum of 9832 and 5716, add their difference.

13. Among how many pensioners may £2100 be divided so as to give each one £37, and what sum remains undivided?

14. Find the sum of the prime numbers between 60 and 100.

N.B.—A prime number is one which has no factor other than unity.

15. The water from a coal pit is being pumped up by an engine at the rate of 3 strokes a minute, and delivering 350 gallons at each stroke. What amount is pumped in 24 hours?

16. The product of two factors is 1580877 and one of the factors is 783. What is the other?

17. Multiply one million one thousand and one by one hundred thousand one hundred, and express the result in words.

18. The quotient in a division is 479, the dividend is 3476418 and the remainder is 794; what is the divisor?

19. What is the smallest number which, when subtracted from 99099, will make it exactly divisible by 909?

20. When 128 apples are taken away 67 times from a store there are 44 left. How many were there at first?

21. From 98,765 subtract any other smaller number consisting of the same five digits in a different order, and divide the remainder by 9. Why must the remainder be divisible by 9?

22. Multiply together 1,634,797 and 73,946, and explain any short method you know for verifying the correctness of the product. How is it adapted for the verification of a quotient?

23. The following is a summary showing the value of the exports from the United Kingdom of articles manufactured (or partly so), excluding foods, drinks, tobacco, and ships. Find the totals.

Destinations.	(*a*) 1893.	(*b*) 1894.	(*c*) 1895.	(*d*) 1896.
Certain Foreign Countries:	£	£	£	£
Germany -	14,853,000	14,550,000	17,297,000	19,175,000
Belgium -	6,429,000	6,829,000	6,516,000	6,947,000
Holland -	8,628,000	8,137,000	6,725,000	7,542,000
France - -	10,293,000	10,135,000	10,649,000	11,034,000
Russia - -	5,295,000	5,632,000	5,807,000	6,023,000
Italy - -	3,216,000	3,044,000	3,437,000	3,476,000
United States	21,087,000	16,464,000	24,985,000	18,129,000
Total of above Countries				

24. Values are given of Imports into, and Exports from, Tunis in each of the years 1892 to 1899 inclusive, distinguishing separately trade with France, Inter-Colonial trade, and trade with Foreign Countries. Add up the quantities horizontally, filling up the entries under the heads of Total.

Years.	Imports.			
	From France.	From other Colonies.	From Foreign Countries.	Total.
	1,000 Francs.	1,000 Francs.	1,000 Francs.	1,000 Francs.
1892	20,774	2,665	15,887	.....
1893	21,726	1,613	15,044	.....
1894	22,942	1,955	17,026	.....
1895	23,309	1,382	19,395	.....
1896	25,563	1,536	19,345	.....
1897	27,872	2,537	23,411	.....
1898	29,876	2,073	21,572	.....
1899	34,264	1,851	19,663	.....

Years.	Exports.			
	To France.	To other Colonies.	To Foreign Countries.	Total.
	1,000 Francs.	1,000 Francs.	1,000 Francs.	1,000 Francs.
1892	20,965	6,105	10,130	.....
1893	14,521	4,892	10,272	.....
1894	19,874	6,038	11,021	.....
1895	26,348	4,579	10,320	.....
1896	20,223	5,796	8,488	.....
1897	22,179	4,078	10,474	.....
1898	24,992	5,137	14,068	.....
1899	26,714	4,245	18,474	.....

25. Values are given of

(a) Imports into, and

(b) Exports from,

Algeria in each of the years 1892 to 1901 inclusive, distinguishing separately trade with France, Inter-Colonial trade, and trade with Foreign Countries. Add up the quantities horizontally, filling up the entries under the heads of Total.

Years.	Imports.			
	From France.	From other Colonies.	From Foreign Countries.	Total.
	1,000 Francs.	1,000 Francs.	1,000 Francs.	1,000 Francs.
1892	189,639	6,114	57,323	.....
1893	184,754	4,110	50,834	.....
1894	199,319	11,345	54,470	.....
1895	203,163	6,563	51,486	.....
1896	217,802	5,326	52,671	.....
1897	216,175	3,889	56,837	.....
1898	225,535	6,742	69,946	.....
1899	260,422	3,979	55,447	.....
1900	259,355	5,471	58,992	.....
1901	258,977	6,596	70,545	.....

Years.	Exports.			
	To France.	To other Colonies.	To Foreign Countries.	Total.
	1,000 Francs.	1,000 Francs.	1,000 Francs.	1,000 Francs.
1892	200,191	3,177	40,599	.....
1893	148,415	3,373	40,840	.....
1894	213,848	4,170	47,695	.....
1895	253,293	9,848	44,646	.....
1896	203,780	6,785	36,845	.....
1897	245,979	7,313	42,436	.....
1898	232,137	8,892	44,740	.....
1899	279,675	11,966	54,774	.....
1900	173,467	12,329	56,521	.....
1901	207,099	7,492	52,250	.....

26. Find the totals in the following table:

Statement showing the Tonnage of Foreign Sailing Vessels, distinguishing the Nationalities, entered with Cargoes at Ports of the United Kingdom from the under-mentioned British Colonial and other Possessions during the year 1902.

	(*a*) Norwegian.	(*b*) German.	(*c*) Italian.	(*d*) Russian.	(*e*) Swedish.	(*f*) Other.
Channel Islands -	654	97	...	...	...	831
Burma - - -	1,926	...	...	...	2,722	...
Australia—						
West Australia -	32,257	11,677	2,760	3,683	2,547	1,657
South Australia -	2,648	7,128	2,529	3,023	1,460	4,830
New South Wales	1,808	5,637	...	1,568	...	3,309
Victoria - -	3,318	3,284	3,780	...	1,711	3,302
Queensland -	...	1,243	...	...	...	...
New Zealand -	2,934	1,339	...	3,359	800	1,185
Canada - -	156,998	3,466	10,100	4,231	5,208	2,885
Newfoundland -	4,541	...	779	...	...	357
West Indies - -	3,468	1,437	491	870	...	3,054
British Honduras -	6,849	...	...	...	...	...
British Guiana -	2,788	...	...	1,464	...	...
All other - -	3,900	...	758	...	1,115	...
Total - -						

27. (*a*) The destination of the coke exported by the Westphalian Coke Co. has been as under:

Find the total export each year.

Destination of Coke Exports.	1900.	1901.	1902.
	Metric Tons.*	Metric Tons.	Metric Tons.
France	784,688	782,220	710,870
Belgium	188,689	66,216	153,947
Holland	40,052	41,195	47,778
Switzerland	95,055	103,030	100,305
Italy	24,805	35,338	33,687
Spain	16,255	5,750	30,327
Austria	302,634	237,422	186,405
Russia	88,158	42,793	75,612
Sweden	33,523	29,315	32,665
Norway	13,392	12,780	21,378
Denmark	21,023	13,913	15,167
England	850	1,540	2,346
Roumania	...	...	2,950
Greece	3,820	4,120	3,375
Asia Minor	...	...	1,630
Asia	4,385	6,047	5,807
Africa	502	1,300	1,990
America	39,268	71,112	183,895
Australia	2,487	5,000	1,845
Total			

* A metric ton, or *Tonneau* equals roughly 2200 lb., that is, about 40 lbs. less than the British ton.

(*b*) Find also the average yearly exports during the period 1900-1902, to

(*b*) Austria,
(*c*) Denmark,
(*d*) America,
(*e*) Asia, Africa and Australia.

VALUE OF SOME EXPORTS TO GERMANY FROM THE UNITED KINGDOM OF ARTICLES MANUFACTURED IN THE UNITED KINGDOM.

ARTICLES.	1899.	1900.	1901.
	£	£	£
Books, printed	71,701	69,971	72,599
Caoutchouc	205,934	179,792	118,023
Chemical products	289,975	306,248	253,107
Coal, products of	242,674	292,907	238,251
Cotton: Yarn	1,894,124	1,762,873	1,573,250
Manufactures	1,770,261	2,081,044	2,144,402
Cycles	32,868	18,567	20,250
Earthen and china ware	134,511	106,673	86,008
Glass	39,599	47,524	38,499
Grease, tallow	105,995	72,655	76,412
Haberdashery and millinery	42,327	39,749	35,901
Hardware and Cutlery	154,475	139,024	126,630
Hats of all sorts	51,038	40,319	45,000
Implements	79,060	52,792	48,383
Leather	409,122	413,776	389,823
Linen: Yarn	213,932	223,360	192,227
Manufactures	281,042	287,547	247,102
Machinery and Millwork:			
Sewing machines	148,774	178,803	150,526
Steam engines	324,182	314,717	213,128
All other sorts	1,715,367	1,547,277	1,174,282
Manure	329,769	256,089	386,710
Medicines, Drugs	56,459	53,044	65,523
Metals, Iron and Steel:			
Tinplates	326,910	276,710	148,930
Other iron and steel	2,427,121	2,645,283	1,100,755
Copper	491,176	350,443	435,299
All other metals	327,779	353,673	301,641
Oil and floor cloth	56,653	42,686	27,262
Oils	140,727	229,546	255,298
Painters' Colours	129,646	125,539	110,836
Paper of all sorts	58,134	58,742	52,085
Silk:			
Thrown, twist or yarn	77,594	52,476	27,260
Manufactures	84,136	77,372	79,281
Stationery, other than paper	73,453	63,411	67,412
Wool, flocks, noils, waste, and combed	1,035,673	652,102	637,605
Woollen and worsted yarn	3,159,112	2,863,939	2,141,559
Yarn, alpaca, mohair, and other sorts	1,168,625	1,022,259	1,086,721
Woollen and worsted manufactures	976,327	879,903	848,697

(Use the values of Exports given in the Table on page 52 for Exercises 28, 29 and 30.)

28. Find the total value of the following Exports in the three years 1899, 1900, 1901 :

(*a*) Cotton yarn and manufactures.
(*b*) Haberdashery and millinery and hats.
(*c*) Linen yarn and manufactures.
(*d*) Silk of all kinds.
(*e*) Wool, and woollen and worsted yarn.
(*f*) Machinery and millwork.
(*g*) Metals of all kinds.

29. Find the total value of Exports in (*a*) 1899, (*b*) 1900, (*c*) 1901.

30. Find the average value of the Exports for the three years under the heads of (*a*) cotton, (*b*) linen, (*c*) cotton and linen together, (*d*) machinery and millwork.

8. BRITISH MONEY SYSTEMS.

THE coins used in the payment of British money, with their relative values, are as follows :

4 **farthings** = 2 halfpennies = 1 **penny**.

These are the only "copper coins;" they are styled copper, although actually made of an alloy.

12 pence = 1 **shilling**.

20 shillings = 1 **sovereign** or **pound** sterling.

A number of pennies is expressed by writing a '*d*' after the number, a halfpenny by $\frac{1}{2}d.$, a farthing by $\frac{1}{4}d.$, and three farthings by $\frac{3}{4}d.$ Shillings are expressed by numbers with *s.* after them, and sovereigns by numbers with £ after or before them.

Thus, £3. 14*s.* $8\frac{3}{4}d.$ expresses three pounds, fourteen shillings, eight pence and three farthings.

In terms of the pound, shilling, pence and farthing all sums of money are expressed, but, for convenience of exchange, use is also made of the following coins:

Threepenny Piece	worth	3 pence (3*d.*).
Sixpenny Piece	„	6 „ (6*d.*).
Florin	„	2 shillings (2*s.* or 2/-).
Half Crown	„	2 shillings and 6 pence (2*s.* 6*d.* or 2/6).
Fourshilling Piece	„	4 shillings (4*s.* or 4/-).
Crown	„	5 „ (5*s.* or 5/-).
Half Sovereign	„	10 „ (10*s.* or 10/-).

A gold coin called a **Guinea**, having the value of 21*s.*, was formerly current in Great Britain, but is not now used, though payments to professional men are frequently expressed in guineas.

It is useful to be able to express sums of money either in pence, or in shillings, or in pounds, and *vice versa*.

EXAMPLE 1.—*Express* £2. 3*s.* 5*d. in pence.*

£2. 3*s.* 5*d.* = 43*s.* 5*d.* ∵ £2 are equivalent to 2 × 20 shillings;
43*s.* 5*d.* = 521*d.* ∵ 43*s.* are equivalent to 12 × 43, *i.e.* 516 pence;
or shortly, £2. 3*s.* 5*d.* = 43*s.* 5*d.* = 521*d.*

EXAMPLE 2.—*Express* 2349 *farthings in pounds, shillings and pence.*

2349 farthings = 587¼*d.* ∵ 4 farthings = 1*d.*
587¼ pence = 48*s.* 11¼*d.* Divide 587 by 12, since 12*d.* = 1*s.*
48*s.* 11¼*d.* = £2. 8*s.* 11¼*d.* Divide 48 by 20, since 20*s.* = £1,
or briefly, 2349 farthings = 587¼*d.* = 48*s.* 11¼*d.* = £2. 8*s.* 11¼*d.*

EXERCISES XVII. a.

1. Draw up a table showing the value in pence of each number of shillings from 1 to 20.

2. Draw up a table showing the value in shillings and pence of 20 pence, 30 pence, 40 pence, ... up to 100 pence.

EXERCISES XVII. b.

Mental or Oral.

1. Give the pence values of:

(*a*) 2*s.* 6*d.*, (*b*) 3*s.* 4*d.*, (*c*) 4*s.* 2*d.*, (*d*) 5*s.*, (*e*) 6*s.*, (*f*) 7*s.*, (*g*) 8*s.*, ... to 20*s.*

2. State the value in £. *s. d.* of:

(*a*) 300 pence, (*b*) 240*d.*, (*c*) 500*d.*, (*d*) 600*d.*, (*e*) 720*d.*, (*f*) 840*d.*, (*g*) 900*d.*, (*h*) 960*d.*, (*i*) 1000*d.*

EXERCISES XVII. c.

1. Express the following sums in their pence equivalents:

(*a*) £3. 14*s.* 7*d.*, (*b*) £7. 18*s.* 6*d.*, (*c*) £49. 16*s.* 2*d.*,
(*d*) £590. 14*s.* 11*d.*, (*e*) £704. 0*s.* 5*d.*, (*f*) £9674. 13*s.* 1*d.*,
(*g*) £7005. 12*s.* 4*d.*, (*h*) £17023. 18*s.* 7*d.*

2. How many sixpences are there in the following sums of money?

(*a*) £73., (*b*) £14. 7*s.* 6*d.*, (*c*) £205. 13*s.* 6*d.*,
(*d*) £7053. 11*s.* 6*d.*, (*e*) £2963. 0*s.* 6*d.*, (*f*) £1934. 2*s.* 6*d.*,
(*g*) £18347. 1*s.* 6*d.*, (*h*) £10304. 19*s.* 6*d.*

3. Reduce each of the following sums to threepences:

(*a*) £17. 5*s.* 9*d.*, (*b*) £29. 6*s.* 3*d.*, (*c*) £294. 7*s.* 6*d.*,
(*d*) £4834. 0*s.* 9*d.*, (*e*) £200. 0*s.* 3*d.*, (*f*) £965. 18*s.* 6*d.*,
(*g*) £3045. 1*s.* 9*d.*, (*h*) £597. 10*s.*

4. Convert the following sums into their equivalent number of guineas:

(*a*) £21, (*b*) £31. 10*s.*, (*c*) £47. 5*s.*, (*d*) £95. 11*s.*, (*e*) £154. 7*s.*; and the following into pounds and shillings:

(*f*) 135 guineas, (*g*) 738 guineas, (*h*) 537 half-guineas, (*i*) 3149 half-guineas.

5. Express each of the following amounts in pounds, shillings and pence:

(*a*) 759 farthings, (*b*) 3159 farthings, (*c*) 6289 half-pennies,
(*d*) 3576 pence, (*e*) 19053*d.*, (*f*) 71936 farthings,
(*g*) 793 threepenny pieces, (*h*) 9452 sixpences.

9. COMPOUND ADDITION.

EXAMPLE.—*Find the sum of* £27. 8*s.* 9*d.*, £31. 12*s.* 10½*d.*, £157. 0*s.* 5¾*d.*

The amounts are arranged in vertical columns, keeping pounds under pounds, shillings under shillings, etc.

£.	*s.*	*d.*
27	8	9
31	12	10½
157	0	5¾
£216	2	1¼

The operation is as follows:

(*a*) Obtain the total of the farthings = 5 farthings = 1¼*d.*; set down the ¼*d.* under the farthings and carry the 1*d.* to the pence column.

(*b*) Similarly, the total of the pence column with the 1*d.* added = 25*d.* = 2*s.* 1*d.*; the 1*d.* is set down under the pence column and 2*s.* carried to the shillings column.

(*c*) The total of the shillings column with the 2*s.* carried = 22*s.* = £1. 2*s.*; the 2*s.* is set down under the shillings and the £1. is carried to the pounds column.

(*d*) The pounds are summed as in simple addition, so that the total is £216. 2*s.* 1¼*d.*

The sums to be added may be arranged horizontally and then added, care being taken not to confound the figures of the different names.

Thus the sum of

£27. 8*s.* 9*d.*, £31. 12*s.* 10½*d.*, £157. 0*s.* 5¾*d.* = £216. 2*s.* 1¼*d.*

EXERCISES XVIII. a.

Mental or Oral.

Find the totals of the following:

1. (*a*) 4½*d.*, 6*d.*, 7½*d.*, 5*d.*; (*b*) 11*d.*, 9*d.*, 10*d.*, 6*d.*; (*c*) 5¼*d.*, 6¾*d.*, 9½*d.*, 2½*d.*; (*d*) 8½*d.*, 1*s.* 3½*d.*, 9*d.*, 5*d.*, 10*d.*, 3*d.*; (*e*) 1*s.* 4*d.*, 2*s.* 6*d.*, 2*d.*, 4*s.*; (*f*) 7*s.* 6*d.*, 3*s.* 4*d.*, 9*s.* 2*d.*, 18*s.* 6*d.*

2. (*a*) 13*s.* 5*d.*, 2*s.* 5*d.*, 2*d.*; (*b*) 8*s.* 3*d.*, 6*s.* 9*d.*, 2*s.* 3*d.*, 2*s.* 10*d.*; (*c*) 1*s.* 4½*d.*, 1*s.* 2½*d.*, 1*s.* 0½*d.*, 10½*d.*; (*d*) 7*s.* 6*d.*, 3*s.* 6*d.*, 1*s.* 8*d.*, 3*s.* 4*d.*; (*e*) 2*s.* 4*d.*, 4*s.* 8*d.*, 7*s.*, 6*s.*; (*f*) 10½*d.*, 1*s.* 8½*d.*, 5*d.*, 4*s.*

EXERCISES XVIII. b.

Find the sum of the following amounts:

1. £29. 16*s.* 5*d.*, £38. 15*s.* 3*d.*, £102. 0*s.* 5*d.*, £93. 18*s.* 4½*d.*, £150. 13*s.* 7¾*d.*

2. £304. 15*s.* 2*d.*, £796. 18*s.* 7½*d.*, £42. 10*s.* 5*d.*, £1903. 17*s.* 5*d.*, 14*s.* 9¾*d.*, £24. 15*s.* 8*d.*

3. £750. 18*s.* 6*d.*, £3059. 17*s.* 10¼*d.*, £9. 7*s.* 3*d.*, £845. 6*s.* 9*d.*, £7364. 11*s.* 9*d.*

4. £384. 0*s.* 8*d.*, £7132. 9*s.* 11¼*d.*, £34. 15*s.* 10*d.*, £413. 15*s.* 8¼*d.*, £70569. 14*s.* 7¾*d.*

5. £76. 13*s.* 4½d., £903. 18*s.* 9¾*d.*, £5760. 5*s.* 8¾*d.*, £9732. 14*s.* 7*d.*, £376. 19*s.* 5*d.*, £10010. 15*s.* 9*d.*

6. £3045. 17*s.* 10*d.*, £94. 3*s.* 5¼*d.*, £897. 10*s.*, £7045. 6*s.* 3½*d.*, £903. 15*s.* 8½*d.*, £6904. 13*s.* 8¾*d.*

7. £693. 4*s.* 7½*d.*, £843. 16*s.* 0½*d.*, £9763. 0*s.* 3¼*d.*, £584. 18*s.* 9*d.*, £3692. 4*s.* 3*d.*, £619. 12*s.* 7¼*d.*, £48. 7*s.* 3*d.*, £7043. 13*s.* 9*d.*, £3. 8*s.* 7*d.*, £19. 18*s.* 3¾*d.*, 12*s.* 5½*d.*, £39. 6*s.* 8*d.*

8. £7. 18*s.* 9*d.*, £5. 17*s.* 3*d.*, £409. 13*s.* 0½*d.*, £72. 13*s.* 8¾*d.*, £615. 3*s.* 7¼*d.*, £2094. 15*s.* 8*d.*, £13. 14*s.* 5¼*d.*, £783. 12*s.* 11¼*d.*, £39. 6*s.* 2*d.*, £4. 13*s.* 5½*d.*, 16*s.* 5¼*d.*, £32. 18*s.* 7*d.*

9. What is the total of the following collection: 7 sovereigns, 13 half-sovereigns, 11 crowns, 29 half-crowns, 37 florins, 93 shillings, 231 sixpences and 853 pennies?

10. Find the total value of 84,762 penny newspapers and 105,763 half-penny ones.

11. At a concert the following amounts were taken: 193 half-crowns, 213 florins, 419 shillings and 97 sixpences. What were the total takings?

12. The takings at a shop were: on Monday £17. 8*s.* 3*d.*, on Tuesday £13. 13*s.* 9*d.*, on Wednesday £1. 3*s.* 10*d.* more than on Tuesday, on Thursday £20. 11*s.* 6*d.*, on Friday £8. 13*s.* 4*d.*, on Saturday £2. 7*s.* 10½*d.* more than on Thursday and Friday together. What was the total amount?

13. What is the money value of 93 sixpenny magazines, 5093 penny papers and 7153 half-penny papers.

14. The moneys paid every fortnight as wages by a certain manufacturer in 3 months were: £79. 13*s.* 8*d.*, £68. 15*s.* 4*d.*, £82. 19*s.* 5*d.*, £76. 12*s.* 7*d.*, £77. 15*s.* 9*d.* and £81. 15*s.* 9*d.* What was the total amount, and how many sixpences would it be equal to?

15. At a certain turnstile the following payments were received: 1951 sixpences, 5035 threepences and 1751 pennies. What was the total?

16. 5 persons A, B, C, D, E form a pool: A puts in £500, B 496 shillings more than A; C puts in as much as A and B; D puts in 473 guineas and 980 crowns; E as much as C and D. What is the value of the pool?

10. COMPOUND SUBTRACTION.

The process of subtracting one amount from another of like kind may be seen by considering an example.

EXAMPLE.—*Take* £93. 14*s.* 5½*d.* *from* £200. 9*s.* 3¼*d.*

Place the larger amount first with the smaller under it.

£.	*s.*	*d.*
200	9	3¼
93	14	5½
£106	14	9¾

Proceed as follows: (*a*) ½*d.* cannot be taken from ¼*d.* Take 1*d.* from the 3*d.* in the pence column. ½*d.* from a 1*d.* is ½*d.*, which with the ¼*d.* makes ¾*d.* This is set down under the farthings.

(*b*) 5*d.* cannot be taken from 2*d.* Hence take 1*s.* from the 9*s.*; and 5*d.* from 1*s.* is 7*d.*, which with the 2*d* remaining in the pence column gives 9*d.* This is set down in the pence column.

(*c*) 14*s.* cannot be taken from 8*s.* £1 is taken from the £ column. Now 14*s.* from a £ leaves 6*s.*, which with the 8*s.* remaining gives 14s. This is set in the shillings column.

(*d*) Taking £93 from £199 leaves £106.

(*e*) The remainder is £106. 14*s.* 9¾*d.*

EXERCISES XIX. a.

Mental or Oral.

1. Take $8\frac{3}{4}d.$ (*a*) from 1*s.*, (*b*) from 1*s.* 6*d.*, (*c*) from 2*s.* 6*d.*, (*d*) from 4*s.* 2*d.*, (*e*) from 7*s.* 6*d.*, (*f*) from 10*s.* 9*d.*

2. Take $10\frac{1}{2}d.$ (*a*) from 2*s.*, (*b*) from 2*s.* 6*d.*, (*c*) from 10*s.*, (*d*) from 15*s.* 2*d.*, (*e*) from 17*s.* 6*d.*, (*f*) from 21*s.*

3. How much is 2*s.* 9*d.* (*a*) short of 4*s.*, (*b*) short of 7*s.* 6*d.*, (*c*) short of 10*s.*, (*d*) short of 15*s.* 8*d.*, (*e*) short of 20*s.*?

4. What must be added to 13*s.* 4*d.* (*a*) to make 18*s.*, (*b*) to make £1, (*c*) to make £1. 1*s.*?

5. What must be taken from 20*s.* (*a*) to leave 14*s.* $7\frac{1}{2}d.$, (*b*) to leave 11*s.* 10*d.*, (*c*) to leave 7*s.* $7\frac{1}{4}d.$, (*d*) to leave 1*s.* $1\frac{3}{4}d.$?

EXERCISES XIX. b.

Find the difference between the following amounts:

1. (*a*) £17. 5*s.* 8*d.* and £9. 3*s.* 10*d.*
(*b*) £20 and £11. 0*s.* $3\frac{1}{4}d.$
(*c*) £34. 10*s.* 9*d.* and £36. 3*s.* 4*d.*
(*d*) £51. 18*s.* $10\frac{1}{2}d.$ and £32. 16*s.* 11*d.*

2. (*a*) £2091. 13*s.* 7*d.* and £184. 0*s.* 10*d.*
(*b*) £3156. 10*s.* and £728. 13*s.* 4*d.*
(*c*) £1053. 1*s.* 6*d.* and £706. 2*s.* 4*d.*
(*d*) £59630. 18*s.* 5*d.* and £70159. 0*s.* 2*d.*

3. (*a*) £13509. 13*s.* $2\frac{1}{2}d.$ and £684. 15*s.* $1\frac{3}{4}d.$
(*b*) £10149. 5*s.* 6*d.* and £994. 17*s.* 8*d.*
(*c*) £51639. 11*s.* $7\frac{1}{2}d.$ and £50119. 12*s.* $4\frac{3}{4}d.$
(*d*) £70436. 0*s.* 3*d.* and £6945. 11*s.* 6*d.*

4. (*a*) £20563. 17*s.* $5\frac{1}{2}d.$ and £18137. 18*s.* $9\frac{3}{4}d.$
(*b*) £75632. 15*s.* 4*d.* and £7085. 3*s.* 10*d.*
(*c*) £50103. 1*s.* 5*d.* and £27639. 5*s.* $0\frac{1}{2}d.$
(*d*) £480,060. 13*s.* 4*d.* and £8119. 10*s.* 8*d.*

5. (*a*) £109,516. 0*s.* 9*d.* and £9651. 0*s.* 10½*d.*
(*b*) £311,045. 16*s.* 10*d.* and £732. 14*s.* 11½*d.*
(*c*) £94537. 0*s.* 2*d.* and £27608. 18*s.* 3½*d.*
(*d*) £200,163. 5*s.* 6*d.* and £9037. 12*s.* 8*d.*

6. How much larger is £752. 13*s.* 7*d.* than £692. 15*s.* 8*d.*?

7. In order to pay a bill of £20. 15*s.* 9*d.*, how many sixpences in addition to £18. 17*s.* 3*d.* will be required?

8. By how much must £345. 4*s.* 5*d.* be increased so as to become £400?

9. A house was sold for £920. 15*s.*, which a year previous was bought for 873 guineas. How much was the gain or loss?

10. The takings at a fête consisted of 2074 pennies, 927 threepences, 2215 sixpences, 1195 shillings, 513 half-pennies. What number of half-pennies would make the total sum taken £150?

11. A lady took £2 to market, and spent the following sums in cash purchases: meat 8*s.* 5½*d.*, butter 5*s.* 3*d.*, eggs 1*s.* 4½*d.*, bacon 2*s.* 3½*d.*, sundry vegetables 4*s.* 5½*d.* What sum remained?

12. A clerk in giving change for a £5 note gave 3 half-sovereigns, 19 half-crowns, twelve shillings, 9 sixpences, and the rest in threepenny pieces; how many of these had to be given?

13. A debt of £3500 was contracted. Two payments of £493. 18*s.* 7*d.* and £2839. 16*s.* 8*d.* respectively were made. What sum remained unpaid?

14. The receipts of a Company during the first 4 weeks of the year 1905 were £1736. 15*s.* 9*d.*, £843. 14*s.* 7*d.*, £2019. 6*s.* 4*d.* and £1875. 19*s.* 10*d.*, while the receipts in the next 4 weeks were £1839. 11*s.* 6*d.*, £951. 14*s.* 7*d.*, £2016. 5*s.* 9*d.* and £1528. 13*s.* 9*d.* Find the difference between the takings of the first month and those of the second month.

15. The correct amount of a bill to be paid was £19. 10*s.* 5½*d.*, but in a hurry it was given as £19. 5*s.* 10½*d.* What was the amount of the error, and in whose favour, the payer's or the person who received the money?

11. COMPOUND MULTIPLICATION.

The procedure may be seen by considering an example.

EXAMPLE.— *What is the value of* 9 *copper bars, each worth* £2. 15*s.* 10½*d.*

£.	*s.*	*d.*
2	15	10½
		9
25	2	10½

(*a*) 9 halfpennies = 4½*d.*, the ½*d.* is set down under the farthings column and the 4*d.* carried on to the pence column.

(*b*) 9 × 10*d.* + 4*d.* = 94*d.* = 7*s.* 10*d.*; the 10*d.* is set in the pence column and the 7*s.* carried on.

(*c*) 9 × 15*s.* + 7*s.* = 142*s.* = £7. 2*s.*; the 2*s.* is placed under the shillings column and £7 carried on.

(*d*) 9 × £2 + £7 = £25.

The result, or the amount, is £25. 2*s.* 10½*d.*

When the multiplier is a composite number, that is, one which can be split into factors, the multiplication can be performed by multiplying successively by the factors.

For instance, multiplying by 36 might be done by 6 times 6 4 times 9, or 12 times 3.

Thus, £11. 10*s.* 8¼*d.* multiplied by 36

= £11. 10*s.* 8¼*d.* × 12 × 3 = £138. 8*s.* 3*d.* × 3 = £415. 4*s.* 9*d.*

If the multiplier be a non-composite number and large, or if the factors are either unknown or inconvenient, the method of splitting up the number into a number of tens and units, or hundreds, tens and units, etc., is often adopted. A method however, of multiplying each term of the sum of money by the total multiplier and converting the products into the next equivalent or higher term may be used. Indeed it may often be expedient to adopt both, so that the pupil may check his accuracy.

Thus, let it be required to multiply £27. 13*s.* 8¼*d.* by 319.

The sum may be done by process (A) or (B).

EXAMPLE (A).—*To multiply* £27. 13*s.* 8¼*d.* by 319.

£.	*s.*	*d.*			
27	13	8¼ × 9			
		10			
276	16	10½	= 10	times	the amount.
		10			
2768	8	9	= 100	,,	,,
		3			
8305	6	3	= 300	,,	,,
276	16	10½	= 10	,,	,,
249	3	2¼	= 9	,,	,,
£8831	6	3¾	= 319	,,	,,

EXAMPLE (B).—319 *times* £27. 13*s.* 8¼*d.*

		£.	*s.*	*d.*
¼*d.* × 319 =	319*f.* = 79¾*d.* =		6	7¾
8*d.* × ,, =	2552*d.* = 212*s.* 8*d.* =	10	12	8
13*s.* × ,, =	4147*s.* = ...	207	7	0
£27 × ,, =	.....	8613		
		8831	6	3¾

Ofttimes various devices will suggest themselves to the ingenious and thoughtful pupil in dealing with multiplication exercises.

e.g. 319 times £27. 13*s.* 8¼*d.*
= 320 times £27. 13*s.* 8¼*d.* − £27. 13*s.* 8¼*d.*
= £27. 13*s.* 8¼*d.* × 4 × 8 × 10 − £27. 13*s.* 8¼*d.*

EXERCISES XX. a.

Mental or Oral.

Find the cost of

1. 7 articles at 6½*d.*, 8¼*d.*, 9¾*d.*, 1*s.* 2*d.*, 1*s.* 3½*d.*, 4*s.* 8*d.*, 5*s.* 3*d.*, 6*s.* 9*d.*, 8*s.* 11½*d.* each.

2. 8 articles at 3¾*d.*, 5½*d.*, 8¾*d.*, 9½*d.*, 1*s.* 3½*d.*, 2*s.* 5*d.*, 3*s.* 11¾*d.*, 8*s.* 1½*d.*, 9*s.* 7*d.*, 13*s.* 7½*d.* each.

3. 12 articles at $2\frac{1}{4}d.$, $3\frac{1}{2}d.$, $4\frac{3}{4}d.$, $5\frac{1}{2}d.$, $8\frac{1}{4}d.$, $9\frac{3}{4}d.$, 1*s.* $1\frac{1}{4}d.$, 2*s.* $3\frac{1}{2}d.$, 19*s.* 10*d.*, 20*s.* $4\frac{3}{4}d.$ each.

Note that in dealing with dozens each *penny* in the cost of a single article corresponds to a *shilling* in the cost of a dozen. Articles 1*d.*, 2*d.*, 3*d.* ... each are 1*s.*, 2*s.* 3*s.* a dozen, and articles $\frac{1}{4}d.$, $\frac{1}{2}d.$, $\frac{3}{4}d.$ each are 3*d.*, 6*d.*, 9*d.*, that is, quarter of a shilling, half a shilling or three quarters of a shilling a dozen.

Use this principle in the exercises marked with a star.

4. 13 articles at $2\frac{3}{4}d.$, $8\frac{1}{2}d.$, $9\frac{1}{4}d.$, $10\frac{1}{2}d.$, 1*s.* $3\frac{1}{4}d.$, 1*s.* $8\frac{1}{2}d.$, 2*s.* 3*d.*, 3*s.* 5*d.*, 3*s.* $11\frac{1}{2}d$*., 8*s.* 10*d.*, 9*s.* $11\frac{3}{4}d$*.

5. 6 articles at 4*s.* 7*d.*, 5*s.* $3\frac{1}{2}d.$, 8*s.* $7\frac{1}{4}d.$, 9*s.* 8*d.*, 9*s.* $11\frac{1}{2}d$*., 14*s.* $5\frac{1}{4}d.$, 18*s.* $7\frac{1}{2}d.$, 20*s.* 6*d.*, 24*s.* 3*d.*, 25*s.* 9*d.*, 39*s.* 6*d**.

EXERCISES XX. b.

1. Multiply £73. 9*s.* $10\frac{1}{2}d.$ by 6, 8, 10, 12.

2. „ £358. 0*s.* $11\frac{1}{4}d.$ by 7, 9, 11, 5.

3. „ £49. 16*s.* $8\frac{1}{4}d.$ by 16, 13, 19, 14.

4. „ £795. 19*s.* 10*d.* by 18, 12, 15, 17.

5. „ £843. 14*s.* $7\frac{3}{4}d.$ by 11, 10, 15, 19.

6. „ £6095. 12*s.* $6\frac{1}{2}d.$ by 24, 36, 48, 60, checking your results by using 2 sets of factors for each.

7. „ £193. 15*s.* $4\frac{1}{4}d.$ by 72, 80, 84, 128, 125.

8. „ £273. 8*s.* $9\frac{3}{4}d.$ by 192, 240, 256, 105.

9. „ £597. 15*s.* $8\frac{1}{4}d.$ by 303, 781, 634, 999.

10. „ £738. 18*s.* $9\frac{3}{4}d.$ by 792, 543, 729, 1024.

EXERCISES XX. c.

Problems.

1. What is the total cost of 39 tons of coal at 8*s.* 9*d.* a ton and 43 tons at 9*s.* 3*d.*?

2. What are the total wages of 8 men at 5*s.* $3\frac{1}{2}d.$, 23 women at 2*s.* 4*d.* and 9 boys at 1*s.* 10*d.*?

3. What will be paid for a flock of sheep numbering 583 at £1. 19*s.* 5*d.* each?

4. Find the rent to be paid for 137 acres of land at £2. 1*s.* 7*d.* per acre.

5. What will be the cost of 309 tons of ore at 7*s.* 3½*d.* per ton?

6. The cost of a ton of coal at the pit's mouth is 3*s.* 4½*d.* per ton, the railway carriage from the colliery to a ship is 3*s.* 6*d.* per ton, the dock charges and tipping 2¾*d.*, the freight 3*s.* 2*d.* and the carriage from the ship to the final destination 1*s.* 6*d.* What will be the final cost of a cargo of 1950 tons?

7. A man purchased 73 head of cattle at £3. 6*s.* 9*d.* each and their keep cost £21. 10*s.* 6*d.* What did they cost the purchaser altogether?

8. 18 bricklayers working 6 days a week, of 9 hours per day, receive 10¾*d.* each per hour. What will their wages amount to in a month?

9. Find the cost of 365 yards of steel rails weighing 66 lbs. per yard, at ¾*d.* per lb.

10. What sum is required to pay the wages of 137 men each earning £2. 14*s.* 7*d.*, 58 women each earning £1. 2*s.* 8*d.* and 35 boys at 19*s.* 4*d.* each?

11. The rent charge on a farm is the value of 246 bushels of wheat at 4*s.* 7*d.* per bushel and half the number of bushels of barley at 2*s.* 9*d.*, in addition to a fixed sum of £26. 14*s.* 3*d.* What is the yearly rental?

12. What is the value of 63 railway shares at £143. 8*s.* 9*d.* each?

13. What sum is necessary to purchase 37 houses at £285. 13*s.* 9*d.* each?

14. How much cheaper is it to buy 73 score of articles at 8*s.* 9*d.* per score, than at 5½*d.* each?

15. The amount of wages paid at a colliery every fortnight is £546. 13*s.* 8*d.* What is that per year?

16. What is the value of the yield of 3 fields with a total acreage of 138 acres, yielding on an average 2 tons of hay per acre at £4. 2*s.* 9*d.* per ton?

17. What is the cost of gas at an establishment where 54,000 c. ft. are used for lighting and 29,000 c. ft. for cooking and heating, the former being 4*s.* 7*d.* and the latter 3*s.* 5*d.* per 1000 cubic feet?

18. An artizan's weekly earnings being taken as £2. 14*s.* 9*d.* per week and his expenses as £29. 18*s.* 7*d.* per quarter, what does he save in a year?

19. What receipts would be obtained from 29 first-class passengers at £2. 5*s.* 10*d.* each, 78 second-class passengers at £1. 9*s.* 9*d.* and 135 third-class ones at 18*s.* 5*d.* each?

20. What will be the value of 18 barrels of oil, averaging 44 gallons each at $11\frac{1}{4}d.$ per gallon?

12. COMPOUND DIVISION.

The examples in division may be of two kinds: either

(1) to divide a certain sum of money into a given number of equal parts; or

(2) to find what number of sums of one value will amount to a sum of some higher value.

EXAMPLE.—*The sum of £79. 8s. $1\frac{1}{2}d.$ was divided equally among 9 persons; what did each person receive?*

Arrange either as

(A)

	£.	s.	d.
9	79	8	$1\frac{1}{2}$
	8	16	$5\frac{1}{2}$

or (B).

£79. 8*s.* $1\frac{1}{2}d. \div 9 =$ £8. 16*s.* $5\frac{1}{2}d.$

Proceed as under:

(*a*) 9 divided into £79 (as described in simple division) gives £8 with £7 over undivided: the £8 is set down under the £ in (A) or placed to right as in (B).

(*b*) The £7 over together with the $8s. = (7 \times 20s. + 8s.) = 148s.$, and 148*s.* divided by 9 gives quotient 16*s.* and 4*s.* over undivided: the 16*s.* is placed in its proper column or position.

(*c*) The 4*s.* over together with $1d. = (4 \times 12d. + 1d.) = 49d.$, and 49*d.* divided by 9 gives 5*d.* and 4*d.* over: the 5*d.* is set in its proper column or position.

(*d*) The 4*d.* over with the $\frac{1}{2}d. = (4 \times 4 + 2) = 18$ farthings, and this divided by 9 gives 2 farthings or $\frac{1}{2}d.$, which is set down at the end.

The quotient, or share of each person, is £8. 16*s.* $5\frac{1}{2}d.$

If the divisor is greater than 12, but can be split up into convenient factors, it will be frequently convenient to use them.

EXAMPLE.—*To divide* £138. 10*s.* *by* 48.

Factors of 48 are (*a*) 4 and 12, or (*b*) 6 and 8; using (*a*) we divide first by 4 and then by 12.

	£.	*s.*	*d.*
4	138	10	0
12	34	12	6
	2	17	$8\frac{1}{2}$ Ans.

The following is an example of the general process of compound long division:

EXAMPLE.— *What sum of money could be given to each of* 357 *persons from* £3742. 3*s.* 10*d.*? *What would remain over?*

£.	*s.*	*d.*	£.	*s.*	*d.*
357) 3742	3	10 (	10	9	$7\frac{3}{4}$
172					
20					
3443 shillings.					
230 „					
12					
2770 pence.					
271 „					
4					
1084 farthings.					
13 „					

(*a*) Dividing 357 into £3742, we have £10 and £172 over.

(*b*) £172 = 3440*s.*, adding on the 3*s.* we obtain 3443*s.*

(*c*) 3443*s.* divided by 357 yields 9*s.* and 230*s.* over.

(*d*) 230*s.* = 2760*d.*, adding on the 10*d.* 2770*d.* is obtained.

(*e*) 2770*d.* divided by 357 yields 7*d.* and 271*d.* over.

(*f*) 271*d.* = 1084 farthings.

1084 farthings divided by 357 yield 3 farthings and 13 farthings over.

∴ A payment of £10. 9*s.* $7\frac{3}{4}d.$ could be made to each person, and 13 farthings or $3\frac{1}{4}d.$ would remain over.

EXAMPLE.—*How many payments of £2. 6s. 9¼d. can be made with £28. 10s. 3d.?*

It is obviously necessary to reduce each quantity into its least common term, which is here farthings. The work can then be arranged as follows:

£ 2. 6s. 9¼d. = 46s. 9¼d. = 561¼d. = 2245 farthings.
£28. 11s. 3d. = 571s. 3d. = 6855d. = 27420 farthings.

By simple division 27420 ÷ 2245 = 12 and 480 over; therefore 12 payments of £2. 6s. 9¼d. can be made, and the sum of 480 farthings or 10s. will remain over.

EXERCISES XXI. a.

Mental or Oral.

1. Divide 4s. 8d. by 4, 7, 8 and 16; and 6s. 8d. by 4, 5, 8, 10, 40.
2. „ 7s. 6d. by 9, 30; and 8s. 9d. by 5, 7, 8 and 21.
3. „ 12s. 6d. by 4, 5, 6, 8, 15; and 18s. 4d. by 4, 5, 11, 22.
4. „ £2. 8s. 4d. by 5, 8, 16; and £2. 14s. 9d. by 4, 6, 9.
5. „ £13. 7s. 6d. by 4, 5, 6, 7, 8, 9, 12, and note the amount of undivided money remaining in each case.
6. How often is 3½d. contained in 2s. 4d., 2s. 11d., 8s. 2d., 11s. 1d. and 20s. 5d.?
7. How often may payments of 8d. each be made from 3s., 5s., 6s., 8s. 4d., 10s. 6d., 12s. What sums remain over or undivided in each case?
8. Divide £19. 14s. 7d. by 5, 7, 9, 11.
9. „ £216. 19s. 4d. by 6, 8, 10, 12.

EXERCISES XXI. b.

1. Divide £693. 15s. by 16, 18, 21, 24.
2. „ £7352. 9s. 8d. by 32, 45, 52, 56.
3. „ £29418. 13s. 8d. by 81, 96, 105, 112.

Note in each case the value of the undivided amount at the end of the operation.

4. Divide £2032. 13*s*. $3\frac{1}{4}d$. by 53.

5. „ £6614. 15*s*. $4\frac{1}{2}d$. by 79.

6. „ £17831. 16*s*. $11\frac{3}{4}d$. by 85.

7. „ £108163. 12*s*. $3\frac{3}{4}d$. by 247.

8. „ £346827. 16*s*. $8\frac{1}{4}d$. by 439.

9. „ £6160706. 11*s*. 10*d*. by 764.

10. „ £156143. 10*s*. by 427, and give the value of the undivided sum remaining.

11. Divide £25194. 16*s*. $2\frac{3}{4}d$. by 97, and give the value of the remainder.

12. How often must the sum of 5*s*. 9*d*. be put aside to save £19. 16*s*. 9*d*.?

13. In what time will a weekly saving of 3*s*. $7\frac{1}{2}d$. amount to £75. 8*s*.?

14. Divide £138 between 27 men and 36 boys, giving a man twice as much as a boy.

15. What is a man's average daily earnings when his yearly income is £175. 13*s*. $1\frac{1}{2}d$.?

16. Prize money to the amount of £2787. 10*s*. 6*d*. was divided among a company of sailors, giving each one £35. 14*s*. 9*d*. How many were there?

17. What is the greatest number of sums of £2. 18*s*. 4*d*. which can be formed from a donation of £100; and what amount remains?

18. The average cost per child of giving a number of children a week's stay in the country is 9*s*. $8\frac{1}{4}d$., and a sum of 50 guineas has been collected for this purpose. How many can be taken and what sum remains unused?

19. A collection containing an equal number of half-crowns, florins, shillings, sixpences and pennies amounted to £14. 5*s*. 11*d*. What was the total number of coins?

20. How many rupees, each worth 1*s*. $10\frac{1}{2}d$., would be equivalent to the sum of £286. 6*s*. 3*d*.?

21. How many quarters of wheat, each worth 53*s.* 8*d.*, together with the same number of quarters of barley each worth 32*s.* 6*d.*, would be equivalent to a rent of £249. 17*s.* 8*d.*?

22. Complete the missing terms in the given table of income or wages, stating the missing terms to the nearest farthing.

(The yearly wage is given, and from it has to be found the monthly wage, weekly wage, etc., taking 12 months to the year, 52 weeks to the year and 365 days to the year.)

	Per Year.			Per Month.			Per Week.			Per Day.		
	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
(*a*)	1	0	0	0	1	8	.....			.....		
(*b*)	5	0	0	.....			.....			.....		
(*c*)	10	0	0	0	16	8	.....			.....		
(*d*)	50	0	0	.....			.....			.....		
(*e*)	100	0	0	.....			.....			.....		
(*f*)	500	0	0	.....			.....			.....		
(*g*)	1000	0	0	.....			.....			.....		

EXERCISES XXII.

Miscellaneous Problems.

1. The daily takings at a shop in 1 week were £14. 13*s.* 3*d.*, £7. 18*s.* 6*d.*, £10. 7*s.* 9½*d.*, £18. 15*s.* 5½*d.*, £12. 4*s.* 9*d.* and £35. 9*s.* 6*d.* What was the arithmetical average daily receipt?

2. 14 barrels of oil were bought for £25. 8*s.*, 17 barrels more for £39. 8*s.* 5*d.* and 26 others for £33. 10*s.* 1*d.* If they had been bought at a uniform price per barrel, what would that price have been?

3. How many French francs, each worth 9½*d.*, would be equivalent to 95 Brazilian milreis of 2*s.* 3*d.* each?

4. What sum remains out of a yearly income of £1526, after a weekly expenditure throughout the year of £26. 9*s.* 5*d.*?

5. From the yearly receipts of a company, amounting to £79,020. 4*s.* 10*d.* the sum of £75,637. 5*s.* 4*d.* is paid for working

expenses, a sum of £2000 is set aside to form a reserve fund and the remainder divided equally among the 47 shareholders. What does each shareholder receive?

6. If I exchange 1120 lbs. of sugar at $2\frac{1}{4}d.$ a lb. for an equivalent value of rice at $3d.$ a lb., how many pounds of rice shall I receive?

7. At a certain hotel in the winter season of 16 weeks, 15 fires each burn on an average 1 ton of coal in 80 days. What is the value of the coal used at $25s.$ $9d.$ per ton?

8. A collection realized 343 silver coins: viz. crowns, double florins, half-crowns, florins, shillings, sixpences and threepenny pieces, and there was an equal number of each. What was the value of the collection?

9. A sum of £25,000 is to be raised by the issue of 650 first-class shares of £27. $12s.$ $6d.$ each and 1127 second-class shares. What is the value of a second-class share?

10. At a manufactory, a weekly levy of $2d.$ in each £ received in wages is made toward the doctor's fund. There are 128 hands employed, half earning £2. $6s.$ $3d.$ a week, and the other half £1. $12s.$ $6d.$ each. What does the levy amount to in a year?

11. The sum of £1950 ship prize money is to be shared first among 9 officers, each to have £175, and the remainder among the ship's crew, giving each £7. $16s.$ $3d.$ What was the total company of the ship?

12. I wish to convert 42 Bank shares at £33. $6s.$ $3d.$ each into 13 Railway shares at £113. $8s.$ $9d.$ each. What additional cash sum will be necessary?

13. To defray a debt of £1500, 64 sheep worth £2. $3s.$ $9d.$ a piece, 36 cows each worth £15. $16s.$ $9d.$ and 17 horses each worth £34. $12s.$ $6d.$ are sold. What sum is still owing?

14. I exchange 742 yards of silk, at $9s.$ $3\frac{1}{2}d.$ per yard, for a cash sum of £79. $16s.$ $1d.$ and a number of yards of cloth at $4s.$ $7d.$ a yard. What number of yards of cloth must be given?

15. The holiday expenses of a party of 45 were £108. $3s.$ $3d.$; 18 paid £2. $12s.$ $6d.$ each, 16 paid £2. $9s.$ $6d.$ What had each of the remainder to pay?

16. A cargo of 2344 tons of English coal is sold in San Francisco at £1. 6*s.* 9*d.* per ton What is the nearest number of dollars, each worth 4*s.* 2*d.*, that will discharge the amount?

17. Butter is bought at 6½ guineas for 112 lbs., and sold at 1*s.* 4½*d.* per lb. What quantity must be sold to realize a total profit of 10 guineas?

18. By selling 57 yards of cloth at 2*s.* 3¼*d.* a yard, a loss of £1 was made; while later in the season 73 yards of the same cloth were sold at 2*s.* 10½*d.* per yard, making a profit of £1. 14*s.* 10½*d.* What was the buying price of the whole per yard?

19. Which is cheaper, and by what amount, to buy a 9-ton truck of coal at 18*s.* 9*d.* per ton, with haulage 2*s.* 6*d.* per ton extra, or to buy the coal in sacks delivered at 2*s.* 6*d.* each, 10 sacks making up a ton?

20. A cart load of apples has 640 lbs. of apples of prime quality which are sold at 5*d.* a lb., and 760 lbs. of second quality at 3¾*d.* What difference would there be in the receipts if all were sold at 4½*d.* a lb.?

21. Find, correct to the nearest penny, the average rate of interest per cent. declared by the following Life Assurance Companies:

Name of Office.	Rate of Interest per cent.		
	£.	s.	d.
Provident	3	15	0
Provident Clerks	3	14	3
Prudential (Ordinary)	3	6	8
Refuge (Ordinary)	3	8	0
Rock	3	17	0
Royal	3	13	9
Royal Exchange	3	16	1
Sceptre	3	16	2
Scottish Accident	3	15	1
Scottish Amicable	3	17	9
Scottish Equitable	4	1	0
Scottish Imperial (Ordinary)	3	16	0

22. Find, true to the nearest penny, the average Imports and Exports per head per annum in the United Kingdom from 1892-1903 inclusive.

Year.	Imports Per Head.			Exports Per Head.		
	£.	s.	d.	£.	s.	d.
1893	10	10	1	5	13	4
1894	10	9	11	5	11	1
1895	10	12	3	5	15	2
1896	11	2	11	6	1	2
1897	11	5	4	5	17	0
1898	11	12	10	5	15	6
1899	11	17	9	6	9	7
1900	12	14	2	7	1	6
1901	12	11	3	6	14	9
1902	12	11	10	6	15	1
1903	12	16	1	6	17	3

23. What was the average price per head of cattle, of which 60 were bought for £10. 18*s.* 6*d.* each, 60 for £10. 16*s.* 9*d.* each, 50 for £10. 15*s.* 3*d.* each and 30 for £10. 12*s.* 5*d.* each?

24. Add together £175. 17*s.* 6*d.*, five hundred guineas, eighty-seven half-crowns and 1143 fourpenny pieces.

25. Find the whole cost of 20 dozen boxes of fruit at 14*s.* 7½*d.* per box; 40 dozen boxes at 13*s.* 9¼*d.* and 60 dozen boxes at 12*s.* 8*d.*

26. Two horses (Bob and Farmer) and a carriage cost £250. Bob and the carriage cost £181. 13*s.* 7*d.*, while Farmer and the carriage cost £155. 6*s.* 5*d.* What was the separate price of each of the three?

27. At a collection for a Sanatorium there were 79 sovereigns, 135 half-sovereigns, 69 crowns, twice as many half-crowns, 30 score and 6 shillings, 1145 sixpences and 593 pennies. What was the total amount collected, and what was it short of £500?

28. From 271 times £35. 4*s.* 2*d.* take £9441. 6*s.* 8*d.*, and divide the remainder by 89.

29. A boy bought a knife for 2*s.* 6*d.*, and afterwards sold it for 9*d.* and 210 marbles worth a penny a dozen. What did he lose by the sale?

30. A flock of 54 sheep is sold at £2. 5*s*. 6*d*. each and a second flock of 78 is sold at £2. 7*s*. 9*d*. each. What was the average price per sheep to the nearest penny?

31. The average cost of a sheep in a flock of 648 was £2. 14*s*. 9*d*.; 420 of the sheep were worth a total of £1000. What was the average cost of the remaining sheep to the nearest penny?

32. A farmer having 240 acres pays a rent of £2. 10*s*. per acre on 100 acres, of £3. 5*s*. per acre on 80 acres and £4. 3*s*. 4*d*. per acre for the remainder. What is the average rent per acre?

33. If 30 tons of coal at 20*s*. 6*d*. a ton be mixed with 22 tons at 24*s*. per ton, find the cost per ton of the mixture, true to the nearest penny.

34. A man borrowed £200, of which he paid at Lady Day £40. 10*s*. 9*d*., at Midsummer £68. 10*s*. 8*d*. and at Michaelmas £25. 11*s*. 1*d*. How much of the £200 does he still owe, and how many half-crowns will pay it?

35. The sum of £17 is made up of the same number of sovereigns, half-sovereigns, half-crowns, shillings and sixpences. How many are there of each?

36. How many payments of £74. 16*s*. 3*d*. can be made with £5000, and what sum is left?

37. How many payments of £86. 18*s*. 9*d*. each can be made with £1000, and what sum is left?

38. Among how many men can £5828 be divided, giving each man £1. 18*s*. 9*d*., and what money, if any, is left?

39. In a bag of 2760 coins there is an equal number of sovereigns, half-sovereigns, crowns, half-crowns, florins, shillings, sixpences and threepenny pieces. What is the number of each kind of coin, and their total value?

40. If the total revenue of a country is £107,780,179 and the population is 34,908,560, what is the average amount of revenue raised per head of the population (true to the nearest penny)?

41. A prize of £6374. 18*s*. 6*d*. was divided between 9 officers, each receiving £193. 14*s*. 6*d*., and the remainder between 732

privates. Find to the nearest penny what each private received, and the value of the remainder, if any.

42. A man buys 497 sacks of potatoes for £339. 1*s*. 11¾*d*. and sells 248 sacks at 17*s*. 9½*d*. each, and the remainder he sells at 18*s*. 2½*d*. each. Find his total gain.

43. If a person's income for the year 1880 be £37. 15*s*. per calendar month, and he spends at the rate of £1. 4*s*. 7½*d*. per day, how much will he have left at the end of the year?

44. A person bought 374 eggs at 2 a penny, and some others at 3 a penny. He paid for them in all £1. 9*s*. 11*d*. How many did he buy at 3 a penny?

13. THE MEASUREMENT OF LENGTH.

In early times lengths were measured as multiples, or as portions, of different parts of the human body, this being indicated by the names which still survive, as, for example:

The foot; the hand, a term used even now in expressing the heights of horses; the span; the pace and others.

Such measurements would be rough, and vary with the person measuring; now, when by modern instruments it is easy to copy accurately a standard length, more precise methods are used. In our country, lengths are expressed in terms of multiples, or portions, of the distance between 2 marks on a rod kept at the Standards Office and copies at other places, and called the **yard**; the multiples and portions in general use are the **inch**, the **foot** and the **mile**, connected as in the table:

British Measures of Length.

12 inches (in.)	make 1	**foot.**
3 feet (ft.)	,, 1	**yard.**
.....		
1760 yards (yds.)	,, 1	**mile.**

(A number of inches is frequently indicated by writing down the number with two accents above it—thus

3 inches by 3″—and a number of feet by the number with one accent above it—thus 2 feet by 2′; so that 2′. 4″ represent two feet and four inches.)

The system in use in most countries, and in much scientific work in England, is the

METRIC SYSTEM,

in which the unit is the **Metre**. The chief portions and multiples of the metre are as indicated in the table below:

10 **millimetres**	make	1 **centimetre.**
10 centimetres	„	1 **decimetre.**
10 decimetres	„	1 **metre.**
1000 metres	„	1 **kilometre.**

One metre is nearly 40 inches.

14. REDUCTION.

In many instances it may be found necessary to change from yards to feet and inches; from miles to smaller lengths, and *vice versa*. The following examples are typical of the methods employed;

EXAMPLE.—*Express* 4 *yds.* 2 *ft.* 8 *in. as a number of inches.*

4 yds. 2 ft. 8 in.
3
14 ft. 8 in.
12
176 in.

(*a*) Express the 4 yds. in feet, by multiplying by 3 (as each yard is the same as 3 ft.); adding on the 2 ft., it is seen that the 4 yds. 2 ft. is the same as 14 ft.

(*b*) Change from feet to inches in a similar manner, *i.e.* by multiplying the number which expresses the length in feet by 12, and adding on the number of inches (here 8).

Thus 4 yds. 2 ft. 8 in. = 176 in.

EXAMPLE.—*Express* 5 *miles* 77 *yards* 1 *foot as a number of feet.*

5 mi. 77 yds. 1 foot.
1760
8877 yds. 1 foot.
3
26632 ft.

(*a*) Multiply the 5 by 1760 and so express the 5 miles by the corresponding number of yards—add on the 77.

(*b*) Reduce the yards to feet by multiplying by 3, add on the 1.

Thus 5 miles 77 yards 1 foot = 26632 feet.

EXAMPLE.—*Express* 341 *inches in terms of yards, feet and inches.*

12 | 341 in.
3 | 28 ft. 5 in.
9 yds. 1 ft. 5 in.

(*a*) Divide the 341 by 12 to find how many feet there are in 341 inches. The result shows that there are 28 ft. and 5 inches over.

(*b*) Divide the 28 by 3 to find how many yards there are in 28 ft.

EXERCISES XXIII. a.

Mental or Oral.

1. Express in feet and inches
   (*a*) 16 inches, (*b*) 26 inches, (*c*) 36 inches,
   (*d*) 50 inches, (*e*) 100 inches, (*f*) 200 inches.

2. How many inches are there in
   (*a*) 2 ft. 5 in., (*b*) 1 yd. 2 ft. 3 in., (*c*) 10 yds.,
   (*d*) 8 yds. 2 ft., (*e*) 100 yards, (*f*) 10 yds. 4 in.?

3. How many feet are there in
   (*a*) 1 mile, (*b*) 220 yds., (*c*) a quarter of a mile,
   (*d*) a mile and a half. (*e*) one-eighth of a mile?

EXERCISES XXIII. b.

Practical.

[*Apparatus required: Foot-rule. Mathematical Instruments. Squared Paper.*]

1. Find, in inches, as accurately as your scale permits, the length, breadth and diagonal of a page of your (1) Arithmetic text-book, (2) Arithmetic exercise book.

2. Measure the width of your desk and (either individually or collectively) the length of the desk.

Make a drawing of the desk, where each foot on the desk is represented by a line, one inch, half-inch or a quarter-inch in length, as you think most suitable.

3. Set off on squared paper a line AB in one direction and a line AC at right angles to it ; make AB and AC of the lengths given below ; measure the length of BC in each case.

(*a*) AB 3 inches, AC 4 inches,
(*b*) AB 2 inches and 3 twelfths, AC 3 inches,
(*c*) AB 5 eighths of an inch, AC 1 inch and 2 twelfths,
(*d*) AB 3 inches and a half, AC 4 inches and 8 twelfths.

EXERCISES XXIII. c.

1. How many inches are there in 1 mile?

2. A train goes 60 miles an hour ; how many feet will it travel in 1 minute?

3. A train goes 44 feet in one second ; how many miles will it travel in one hour?

4. What is the length in yards, etc., of 32 steam pipes, each one being 19 ft. 9 in. long?

5. The circumference of a bicycle wheel is 13 ft. 8 in. Express in miles, yards and feet the distance covered when 2764 revolutions are recorded.

6. The drum of a winding engine is 22 ft. 6 in. in circumference. What length of rope in yards is let out by 200 revolutions?

7. A chain consists of 1742 links, each 2 inches long. What is the total length of the chain?

8. What is the length covered by 2000 steel rails, each 23 ft. 6 in. long, placed end to end, with a total space of 20 ft. between their adjacent ends, as in a railway?

9. Express in yards and feet the excess of 1 knot (6080 ft.) over a mile.

10. The highest mountain in the world is stated to be 29,002 ft. high. Express this in miles, yds. and ft.

11. A boy has used 6 balls of cord, each 145 yds. long, in flying his kite. What is the height of his kite in ft. and in. if the height is one-tenth the length of the string?

12. The driving wheel of a locomotive engine is 23 ft. 9 in. in circumference, while the small front one is 9 ft. 6 in., how many more revolutions will the small one perform than the larger one in going a distance of 76 miles 380 yds.?

13. From the ground floor of a house to the attic are 54 steps each five inches in height. What is the height of the attic above the ground floor in feet and inches?

14. The top of a tower is reached by 372 steps each having a rise of 4 in. What is the height of the top of the tower in yds., ft. and in.?

15. What distance does a soldier cover in a march of 4 hours, making an average of 110 steps each 2 ft. 3 in. per minute?

15. ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF LENGTHS.

Addition, subtraction, multiplication and division of lengths are done by following the plan employed in similar operations in money questions, thus:

EXAMPLE.—*Three rods have lengths respectively:* 3 *yards* 1 *foot* 6 *inches,* 2 *yards* 2 *feet* 8 *inches and* 8 *yards* 2 *feet* 9 *inches; what is their combined length?*

Arrange with yards under yards, feet under feet, etc.

yds.	ft.	in.
3	1	6
2	2	8
8	2	9
15	0	11
2	1	

(*a*) Adding the numbers in the inches column, we get 23 in., representing 1 ft. 11 in. We set down the 11 in the inches column and carry on the 1 to the feet column.

(*b*) Adding the numbers in the feet column, to the 1 carried, we obtain 6, representing 6 ft. or 2 yards 0 ft. Set the 0 in the feet column, carry on the 2 to the yards column. Adding this, we obtain 15.

The combined length is 15 yards 0 ft. 11 inches.

EXAMPLE.—*The length of a piece of string is* 27 *yards* 1 *foot* 8 *inches; a piece* 12 *yards* 2 *ft.* 9 *inches is cut off; what length remains?*

As in complementary addition:

yds.	ft.	in.
27	1	8
12	2	9
14	1	11
1	1	

(*a*) 11 inches added on to 9 inches give 1 ft. 8 in. We set down 11, carrying 1 on to the 2 in the feet column obtaining 3 ft.

(*b*) 1 ft. added on to 3 ft. makes 1 yd. 1 ft. Carry 1 on to the 12 in the yards column.

(*c*) 14 added to 13 gives 27. The length of string which remains is therefore 14 yds. 1 ft. 11 in.

EXAMPLE.—*What is the total length of* 15 *balls of twine each* 223 *yards* 2 *feet* 3 *inches long?*

miles.	yards.	feet.	inches.
0 .	223 .	2 .	3
			15
1 .	1596 .	0 .	9
1	11		3

(*a*) 15 times 3 inches are 45 inches or 3 ft. 9 inches. Set down the 9 inches, carry on the 3 ft.

(*b*) 15 times 2 ft. are 30 ft.; with the 3 ft. brought forward this becomes 33 ft. or 11 yds. 0 ft. Set down the 0, carry on the 11.

(*c*) 15 times 223 yards are 3345; carry on the 11, and obtain 3356 yds. or 1 mile 1596 yds.

The total length is 1 mile 1596 yds. 0 ft. 9 in.

EXAMPLE.—*A cord* 2 *miles* 30 *yards long is divided into* 24 *equal parts; what is the length of each part?*

0 miles. 147 yards. 2 ft. 9 in. *Ans.*

24 | 2 miles. 30 yards. 0 ft. 0 in.
1760

3550 yards. 0 ft. 0 in.
115
190

22 yards. 0 ft. 0 in.
3

66 ft. 0 in.

18 ft. 0 in.
12

216
...

(*a*) 2 divided by 24 gives 0 and 2 over.

(*b*) Reducing the 2 miles to yards and adding the 30, we obtain 3550.

(*c*) Dividing 3550 yds. by 24, we obtain 147 yds. and 22 yds. over.

(*d*) Reducing this 22 yards to feet we obtain 66.

(*e*) Dividing by 24, we obtain 2 ft. and 18 ft. over

(*f*) Reducing to inches, 216 in. is obtained; dividing by 24, the result is 9 in.

Each piece is therefore 147 yards 2 feet 9 inches long.

EXAMPLE.—*A board is* 8 *yds.* 2 *ft.* 7 *in. in length; how many pieces* 1 *yd.* 0 *ft.* 3 *in. long can be cut off it, and what length of board will be left over?*

Reducing both lengths to inches, we have to find out how many lengths of 39 inches can be cut from a board 26 ft. 7 in or 319 inches long.

Dividing 319 by 39, we obtain 8 and 7 over.

The board may therefore be cut into 8 pieces each 1 yd. 0 ft. 3 in. long, and a piece 7 in. long will remain.

EXERCISES XXIV.

1. The three sides of a triangle are respectively 2 yds. 2 ft. 2 in., 4 yds. 0 ft. 11 in. and 3 yds. 1 ft. 10 in. What is the length of the perimeter or distance right round the triangle?

2. In order to get from a place A to a place B, one man follows a straight road AC 1 mile 725 yards long, turns at C and follows another road 1 mile 1200 yards long. A second man takes a direct cut from A to B, and thereby saves a distance of 800 yards. What is the direct distance between A and B?

3. How many tiles each 6 in. long will be necessary to enclose a square plot of grass, each side being 23 ft. 9 in.?

4. A wire fence 212 yds. 1 ft. long is fitted with uprights 2 ft. 2 in. apart. What number of uprights will be required?

5. Each turn of a coil of wire is 1 ft. 7 in. long. What length of wire will be required to make 200 turns?

6. In paving one side of a street 872 kerb stones each of an average length of 2 ft. 4 in. are used. What is the length of the street?

7. From a roll of calico 60 yds. long, lengths of 34 yds. 8 in. and 49 ft. 9 in. are cut off. What length remains?

8. A path in a park is edged on both sides with tiles 5 in. long. What is the length of the path when 3642 tiles are required?

9. A short piece of railway measuring 3 miles 1482 yds. is to be made by 63 men. What length must each make?

10. Three roads in a parish are severally 4 miles 753 yds. 2 ft., 2 miles 1628 yds., and 6 miles 905 yds. in length, and are kept in repair by 5 men. What is each man's share?

11. Goods have to be taken a distance of 50 miles. Of this 9 mi. 1342 yds. is by canal, and 36 mi. 570 yds. by rail, and the remainder by cart. What distance is performed by cart?

12. Fill in the tables below, giving the thicknesses and depths of the strata met with in making the wells specified:

(*a*) Well at Marylebone Road:

Strata.	Thickness. ft.	in.	Depth.
Made ground	3	0	.....
Gravel and clay	8	0	.....
„ „ sand	12	8	.....
London clay	92	7	.....
Mottled „	41	7	.....
Peaty earth	0	2	.....
Clayey sand	1	8	.....
Green „	7	4	.....
Grey „	21	0	.....
Chalk	178	0	.....

(*b*) Well at Greenwich Hospital:

Strata.	Thickness. ft.	in.	Depth.
Made ground	11	0	.....
Gravel	33	0	.....
Black sand	4	10	.....
Blue clay	0	8	.....
Shelly rock	4	0	.....
Red clay	6	0	.....
White sand	4	0	.....
Green sand and pebbles	4	0	.....
Thanet sand	55	10	.....
Flint bed	1	0	.....
Chalk with flints	180	6	.....

(*c*) Well at Waltham Abbey:

Strata.	Thickness. ft.	in.	Depth.
Soil	1	0	.....
Clay	2	6	.....
Peat	4	6	.....
River gravel	10	0	.....
Blue clay	26	0	.....
Sandy clay	10	6	.....
Sand and shell	0	4	.....
Fine hard sand	1	0	.....
Pebbles	0	2	.....
Blue clay	8	0	.....
Sandy clay	4	0	.....
White sand	4	11	.....
Sandy clay	1	1	.....
Green sand	8	0	.....
Coloured sand	3	0	.....
Light „	5	0	.....
Coarse „	4	6	.....
Fine „	27	9	.....
Flints	0	9	.....
Chalk	75	0	.....

(*d*) Well at Lea Bridge:

Strata.	Thickness. ft.	in.	Depth.
Made ground	6	0	.....
Red ballast	12	0	.....
Peat	2	0	.....
Ballast	7	0	.....
Grey sand	8	0	.....
Loam	0	6	.....
Sand	1	0	.....
Dark sand	1	8	.....
Dark sand and sandstone	4	0	.....
Conglomerate	2	0	.....
Dark sand	2	0	.....
Grey „	21	0	.....
Very hard sand	6	11	.....
Soft green „	0	7	.....
Hard „ „	2	1	.....
Soft sand	2	6	.....
Flint	0	10	.....

(*e*) Well at Dartford :

Strata.	Depth. ft.	Depth. in.	Thickness. ft.	Thickness. in.
Bog	6	0	.....	
White sand	9	0	.....	
„ „	10	0	.....	
„ gravel	12	0	.....	
Marl	12	6	.....	
White sand and flints	30	0	.....	
Grey chalk	36	0	.....	
White sand	37	0	.....	
Grey chalk	60	1	.....	
Hard grey chalk	79	10	.....	
Chalk with flints	152	10	.....	
Chalk	202	7	.....	
Sandy marl	203	5	.....	
Rotten chalk	210	5	.....	
Chalk	250	5	.....	

(*f*) Well at Ealing :

Strata.	Depth. ft.	Depth. in.	Thickness. ft.	Thickness. in.
Yellow clay	35	0	.....	
Clay with clay-stones	253	6	.....	
Clay and green sand	264	0	.....	
Mottled clay	304	0	.....	
Loam	306	0	.....	
Mottled clay	307	3	.....	
Hard sand	310	3	.....	
Red clay	321	2	.....	
Green and black sand	328	9	.....	
Chalk and flints	330	3	.....	
Chalk	347	3	.....	
Chalk and flints	418	5	.....	
Hard chalk	430	11	.....	
Chalk	480	10	.....	
Grey chalk with flints	488	10	.....	
Chalk with flints	500	4	.....	
Chalk	501	4	.....	
Chalk with flints	538	4	.....	
Grey chalk with flints	601	8	.....	

EXERCISES XXV.

Practical.

[*Apparatus : Scale graduated in fractions of an inch. Paper.*]

1. Employ your scale to find ;

(*a*) The number of *eighths* in 15 twelfths ;
(*b*) The number of *eighths* in 27 twelfths ;
(*c*) The number of *twelfths* in 14 eighths ;
(*d*) The number of *quarters* in 33 twelfths.

2. Draw any four parallel straight lines (or take any four which are ruled on the paper) at equal distances apart. Draw a straight line meeting the four parallel lines, and letter the points of meeting A, B, C, D taken in order ; measure AB, BC, CD.

Repeat with five other lines like ABCD ; measure the parts cut off between the parallel lines in each case, and state what you notice about these parts.

3. Draw a line 4 inches and 7 twelfths, divide into five equal parts, measure the length of one of these parts.

4. Use your scales to find the sum of 2 inches, 3 inches and a quarter, 1 inch and 5 eighths, 2 inches and a twelfth.

5. Copy accurately the accompanying figures, and mark on them the dimensions of the various lines.

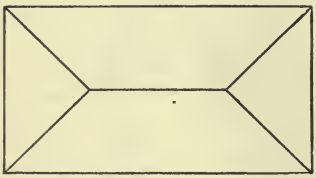


FIG. 10 (*a*).

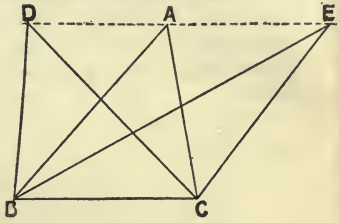


FIG. 10 (*b*).

6. The accompanying figure represents the two faces of an ordinary brick, which meet along one edge.

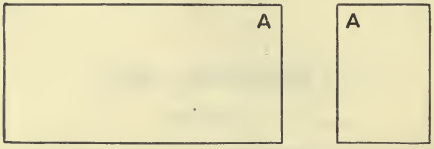


FIG. 11.

(The figure should, however, be enlarged six times to give a true representation).

Draw a figure representing the other face which meets the two given faces at the corner A, and write down the length, breadth and thickness of the brick.

16. REDUCTION IN THE METRIC OR DECIMAL SYSTEM.

In this system the process of reduction is very simple, owing to the multiples and submultiples all being powers of 10. Thus:

EXAMPLE.—*To express* 3 *metres* 4 *decimetres* 2 *centimetres* 8 *millimetres in millimetres.*

It is usual to abbreviate by representing metres by m., decimetres by dm., centimetres by cm. and millimetres by mm.

∴ 3 m. 4 dm. 2 cm. 8 mm. have to be expressed in mm.

3 m. 4 dm. 2 cm. 8 mm.
= 34 dm. 2 cm. 8 mm.
= 342 cm. 8 mm.
= 3428 mm.

Since 3 m. are 30 dm.
„ 34 dm. „ 340 cm.
„ 342 cm. „ 3420 mm.

EXAMPLE.—*To express* 7812 *mm. in metres, decimetres, etc.*

7812 mm. = 781 cm. 2 mm. = 78 dm. 1 cm. 2 mm.
= 7 m. 8 dm. 1 cm. 2 mm.

EXERCISES XXVI. a.

Practical.

[*Apparatus: Dividers. Scale on English and Metric systems. String.*]

1. Measure the length and breadth of your book with your scale, writing the measurements down:

(*a*) In inches;
(*b*) In decimetres, centimetres and millimetres;
(*c*) In centimetres and millimetres;
(*d*) In millimetres.

2. In the same way obtain:

(*a*) The width of the desk;
(*b*) The length of your pen;
(*c*) The length and breadth of your exercise book.

3. Set off on your exercise books lines of the following lengths:
(*a*) 8 cm. 3 mm., (*b*) 57 mm., (*c*) 1 dm. 2 cm. 5 mm.,
(*d*) 1 dm. 0 cm. 5 mm., (*e*) 2 dm. 3 cm. 8 mm.

4. Use your scale to find:
(*a*) The difference between 1 dm. and 6 cm. 5 mm.
(*b*) The sum of 8 cm. 4 mm. and 4 cm. 8 mm.

5. Cut pieces of string or tape of the following lengths:
(*a*) 1 metre, (*b*) 1 metre 2 dm., (*c*) 2 metres 3 dm. 4 cm.

6. Find the number of inches in your string of 1 metre length.

7. Cut off a piece of string or tape of length equal to the difference between a metre and a yard; express this difference:
(*a*) In inches; (*b*) In cm. and mm.; (*c*) In millimetres.

8. Find from your scale the number of dm., cm. and mm. in one foot.

EXERCISES XXVI. b.

Mental or Oral.

(A) Express in millimetres:

1. 6 cm.	2. 6 dm.	3. 6 m.
4. 6 m. 6 cm.	5. 6 m. 6 dm.	6. 6 m. 6 dm. 6 cm.
7. 7 m. 3 dm.	8. 7 m. 3 mm.	9. 3 dm. 3 mm.
10. 5 cm. 2 mm.	11. 18 m.	12. 1 m. 1 cm.
13. 1 m. 1 dm. 2 cm.	14. 6 m. 2 cm.	15. 6 m. 2 mm.
16. 4 m. 2 dm.	17. 4 m. 3 dm. 2 cm.	18. 6 m. 6 mm.
19. 7 m. 8 cm.	20. 8 m. 2 cm.	21. 8 m. 3 cm. 2 mm.
22. 9 m. 9 cm. 1 mm.	23. 2 m. 2 cm. 9 mm.	24. 2 dm. 3 cm. 8 mm.
25. 16 m. 8 dm. 3 mm.	26. 5 m. 3 dm. 2 mm.	27. 9 m. 2 cm. 7 mm.

(B) Express in metres, decimetres, etc.:

1. 76 mm.	2. 760 mm.	3. 706 mm.	4. 8260 mm.
5. 98 cm.	6. 5309 mm.	7. 234 dm.	8. 9102 mm.
9. 912 mm.	10. 9012 mm.	11. 8716 mm.	12. 876 cm.
13. 87 dm.	14. 962 mm.	15. 8345 cm.	16. 8345 mm.
17. 8046 mm.	18. 7623 mm.	19. 926 cm.	20. 1763 mm.

(c) State in metres, decimetres, etc., the lengths indicated by the figures marked with an asterisk (*)

1. $\overset{*}{3}428$ mm.	2. $34\overset{*}{2}8$ mm.	3. $\overset{*}{3}428$ mm.	4. $\overset{*}{1}0862$ mm.
5. $\overset{*}{7}21$ cm.	6. $61\overset{*}{0}3$ cm.	7. $\overset{*}{7}123$ cm.	8. $\overset{*}{8}35$ mm.
9. $\overset{*}{7}273$ mm.	10. $\overset{*}{8}342$ dm.	11. $\overset{*}{8}312$ mm.	12. $83\overset{*}{0}1$ mm.
13. $\overset{*}{7}56$ dm.	14. $\overset{*}{3}602$ mm.	15. $\overset{*}{7}432$ cm.	16. $81\overset{*}{5}5$ mm.
17. $\overset{*}{3}137$ cm.	18. $\overset{*}{8}106$ dm.	19. $\overset{*}{3}134$ mm.	20. $\overset{*}{8}1752$ dm.
21. $\overset{*}{8}1752$ cm.	22. $\overset{*}{2}321$ mm.		

In the metric system, it will be noticed that multiplying or dividing lengths by 10 is equivalent to changing the denominations in terms of which the lengths are expressed.

EXAMPLE (1) 3 cm. × 10 = 3 dm.
(2) 3 cm. ÷ 10 = 3 mm.

Again,

	m.	dm.	cm.	mm.			m.	dm.	cm.	mm.
(1)	0	3	4	2	× 10	=	3	4	2	0
(2)	5	2	5	0	÷ 10	=	0	5	2	5

EXERCISES XXVII.

Write down the following lengths, perform the multiplications indicated and arrange as above :

	m.	dm.	cm.	mm.			m.	dm.	cm.	mm.	
1.	0	0	0	2	× 10 =	2.	0	0	0	2	× 100 =
3.	0	0	0	2	× 1000 =	4.	0	0	1	2	× 100 =
5.	0	2	3	0	× 10 =	6.	0	2	0	5	× 100 =
7.	0	0	4	4	× 1000 =	8.	0	4	4	0	× 100 =
9.	0	4	0	4	× 10 =	10.	2	3	2	0	× 10 =
11.	0	0	1	7	× 100 =	12.	3	4	5	0	× 10 =
13.	13	1	3	0	× 10 =						

Divide the following :

	m.	dm	cm.	mm.				m.	dm.	cm.	mm.		
14.	1	. 0	. 0	. 0	÷	10 =	15.	1	. 0	. 0	. 0	÷	100 =
16.	1	. 0	. 0	. 0	÷	1000 =	17.	3	. 2	. 0	. 0	÷	100 =
18.	2	. 3	. 3	. 0	÷	10 =	19.	2	. 3	. 0	. 0	÷	10 =
20.	2	. 0	. 3	. 0	÷	10 =	21.	23	. 1	. 0	. 0	÷	100 =
22.	106	. 1	. 2	. 0	÷	10 =	23.	106	. 7	. 0	. 0	÷	100 =
24.	24	. 1	. 3	. 0	÷	10 =	25.	57	. 4	. 0	. 0	÷	10 =
26.	88	. 8	. 4	. 0	÷	10 =	27.	7	. 0	. 6	. 0	÷	10 =

17. INTRODUCTION OF DECIMAL NOTATION.

We have seen that a length like 2 m. 3 dm. 4 cm. 2 mm., can be written also 234 cm. 2 mm.

Here, considering the figures representing the number of centimetres, the 3 in the tens place represents 3 decimetres or 30 centimetres, *i.e.* its value is 10 times as great as it would be if it had been located in the units place. Similarly, the 2 in the hundreds place has a value 100 times as much as if it were written in the units place, and ten times as much as if it had been written in the tens column.

The length represented by any of the figures depends upon the place of the particular figure. Further, the 2 in the mm. column represents a length one-tenth of what it would be were the 2 placed in the units place in the centimetre column ; and the length 234 cm. 2 mm. may be expressed in terms of *cm.* by adopting some device to indicate the unit figure. This is done by placing a **decimal** point immediately after the units figure so that 234·2 cm. means

2 hundreds of centimetres
3 tens ,, ,,
4 *units* ,, ,,

and the 2 following the decimal point has one-tenth the value it would have if placed immediately to the left of the decimal point, *i.e.* 2 millimetres or 2 tenths of a centimetre.

The notation is extended still further to the right of the decimal point, and thus:

34·24 dm. is the same as 3 metres 4 dm. 2 cm. 4 mm.

Although the examples given are all connected with lengths expressed in the metric system, the decimal notation can be applied to all kinds of quantities.

Thus, 34·24 ft. means 34 feet 2 tenths of a foot and 4 hundredths of a foot.

Again in 36·63 lb., the 6 after the decimal point represents just one-tenth the mass that the 6 before the decimal point represents, while the first 3 refers to a mass one-thousand times the 3 appearing in the 2nd decimal place.

Moreover, as in the case of ordinary numbers, we can make use of decimals in the abstract without reference to any concrete quantities at all.

Thus 0·3, 0·5, 0·6;

and 0·3 is one tenth of 3,
5 is ten times 0·5,
6 is ten times 0·6.

EXERCISES XXVIII.

(*These exercises may be worked through orally first, and then again considered as written work.*)

Express:

1. 3 m. 2 dm. 8 cm. as metres and decimals of a metre.

2. 0·416 metre as (*a*) decimetres, (*b*) centimetres and (*c*) millimetres.

3. 7 m. 2 cm. 5 mm. as (*a*) decimetres, (*b*) centimetres and (*c*) millimetres.

4. 14 m. 6 cm. 9 mm. as metres and decimals of a metre.

5. 29 cm. 6 mm. as metres and decimals of a metre.

6. 132 dm. 3 cm. 6 mm. as metres and decimals of a metre.

7. 84·750 metres as (*a*) decimetres, (*b*) centimetres and (*c*) millimetres.

8. 0·718 m. as cm.
9. 0·071 m. as cm.
10. 0·007 m. as cm.
11. 3·17 dm. as cm.
12. 3 m. 2 cm. as metres.
13. 6 m. 2 dm. as metres.
14. 3 m. 2 dm. 2 mm. as metres.
15. 4 m. 3 mm. as metres.
16. 3 m. 7 dm. 3 mm. as dm.
17. 2 m. 2 mm. as dm.
18. 3 m. 4 mm. as cm.
19. 2 m. 3 dm. 2 mm. as cm.
20. 0·36 dm. as cm.
21. 0·412 m. as mm.
22. 0·018 m. as cm.
23. 4·16 m. as mm.
24. 6 m. 2 cm. as mm.
25. 6 m. 2 cm. as dm.
26. 6124 cm. as m.
27. 0·012 m. as cm.
28. 4·12 dm. as mm.
29. 3 m. 2 cm. 4 mm. as metres.
30. 1·260 m. as mm.

Express, as asked for, the lengths indicated by the figures marked with an asterisk (*) in the following :

31. $\overset{*}{3}4{\cdot}24$ cm. in cm.
32. $\overset{*}{3}4{\cdot}24$ cm. in mm.
33. $\overset{*}{5}3{\cdot}24$ m. in cm.
34. $\overset{*}{5}3{\cdot}24$ m. in mm.
35. $53{\cdot}\overset{*}{2}4$ m. in cm.
36. $6{\cdot}\overset{*}{2}5$ metres in mm.
37. $17{\cdot}\overset{*}{3}2$ metres in mm.
38. $80{\cdot}\overset{*}{0}1$ cm. in mm.
39. $800{\cdot}\overset{*}{1}$ cm. in metres.
40. $6{\cdot}\overset{*}{2}3$ dm. in cm.

How many times greater is the quantity represented by the figure marked *a* than the length represented by the figure marked *b*?

41. $\overset{a}{3}{\cdot}4\overset{b}{2}4$ metres.
42. $\overset{a}{2}4{\cdot}\overset{b}{2}4$ metres.
43. $\overset{a}{2}4{\cdot}\overset{b}{2}4$ miles.
44. $\overset{a}{6}0{\cdot}\overset{b}{0}6$ miles.

45. $\overset{a}{60}\cdot\overset{b}{06}$ £.

46. $0\cdot\overset{ab}{863}$ £.

47. $\overset{a}{36}\cdot\overset{b}{63}$ £.

48. $\overset{a}{1}\cdot\overset{b}{234}$ yards.

49. $\overset{a}{4}\cdot\overset{b}{321}$ pounds.

50. $\overset{a}{1}\cdot\overset{b}{065}$ gallons.

18. ADDITION AND SUBTRACTION OF METRIC LENGTHS.

EXAMPLE.—*One string is* 4 *metres* 2 *decimetres* 3 *centimetres* 8 *millimetres in length, another string is* 6 *metres* 8 *decimetres* 2 *centimetres* 4 *millimetres long, and a third is* 8 *metres* 7 *decimetres* 7 *centimetres* 8 *millimetres; what is the combined length?*

The example may be set down as in the addition of length in yards, feet and inches, thus:

m.	dm.	cm.	mm.	
4	2	3	8	(8+4+8) mm. are 20 mm., *i.e.* 2 cm. 0 dm.
6	8	2	4	(2+7+2+3) cm. are 14 cm., *i.e.* 1 dm. 4 cm.
8	7	7	8	(1+7+8+2) dm. are 18 dm., *i.e.* 1 m. 8 dm.
19	8	4	0	(1+8+6+4) m. are 19 m.
1	1	2		

The combined length is therefore 19 m. 8 dm. 4 cm. 0 mm.

The example may be worked more conveniently in terms of one denomination and decimals, thus:

EXAMPLE.—

(1) *Expressing in metres.*	(2) *Expressing in dm.*	(3) *Expressing in cm.*	(4) *Expressing in mm.*
4·238	42·38	423·8	4238
6·824	68·24	682·4	6824
8·778	87·78	877·8	8778
19·840 metres.	198·40 dm.	1984·0 cm.	19840 mm.

Care must be taken that the *decimal points* are all under one another, in order that units are under units, tens under tens, etc.

EXAMPLE.—*A rod is* 3 *m.* 0 *dm.* 2 *cm.* 8 *mm. long; it is cut into two pieces, one of which is* 1 *m.* 2 *dm.* 0 *cm.* 9 *mm. long. What is the length of the other piece?*

Here it is required to subtract 1 m. 2 dm. 0 cm. 9 mm. from 3 m. 0 dm. 2 cm. 8 mm.

m.	dm.	cm.	mm.
3	0	2	8
1	2	0	9
1	8	1	9

The remaining piece is therefore 1 m. 8 dm. 1 cm. 9 mm.

Or, working in decimals of a metre we subtract 1·209 from 3·028, setting decimal points underneath one another for the same reason as in addition.

3·028
1·209
1·819

The remaining length is seen to be 1·819 metres.

EXERCISES XXIX.

1. Add

m.	dm.	cm.	mm.
3	2	0	6
0	0	8	2
1	0	7	6
2	3	0	1

Set down the lengths as decimals of a metre also, and add up again, checking the two results by one another.

Find the following sums, verifying as in the Example above.

2. 3 m. 3 dm. 3 cm. 3 mm., 4 m. 2 dm. 2 cm. 6 mm., 4 m. 2 dm., 4 m. 2 cm.

3. 15 m. 2 cm. 3 mm., 2 cm. 3 mm., 6 m. 3 dm. 8 cm., 8 m. 2 cm.

4. 6 m. 6 cm. 6 mm., 2 mm., 8 m. 8 cm. 8 mm., 9 m. 4 cm. 5 mm., 6 cm. 4 mm.

5. 23 m. 4 dm. 5 cm. 6 mm., 4 dm. 5 mm., 9 cm. 8 mm., 7 m. 4 cm. 3 mm., 7 m. 2 dm. 5 cm. 4 mm.

6. 8 m. 3 cm. 3 mm., 3 m. 8 cm. 8 mm., 6 m. 5 cm., 8 m. 9 dm. 7 cm. 7 mm.

7. 100 m. 2 cm. 6 mm., 8 m. 9 dm. 7 cm., 24 m. 8 dm., 3 m. 2 dm. 3 cm.

8. 7 m. 6 dm. 2 mm., 18 m. 9 dm. 6 cm. 5 mm., 13 m. 4 cm. 5 mm., 12 m. 2 dm. 9 mm.

9. 11 m. 8 mm., 1 m. 1 dm. 8 cm., 11 m. 8 cm., 11 m. 8 dm., 1 dm. 1 cm. 8 mm.

10. 9 m. 5 dm. 3 mm., 3 m. 5 cm. 9 mm., 13 m. 8 dm. 7 mm., 13 m. 7 cm., 7 dm. 8 mm.

11. 26 m. 2 mm., 26 m. 2 cm., 26 m. 2 dm., 2 m. 6 dm. 2 mm., 2 dm. 6 cm. 2 mm.

12. 19 m. 8 dm. 6 mm., 19 m. 2 mm., 37 m. 8 dm., 3 m. 8 dm. 9 cm., 3 m. 8 dm. 9 mm.

13. 49 m. 5 mm., 6 dm. 7 mm., 8 m. 7 dm., 15 m. 8 dm. 7 cm. 2 mm., 15 m. 8 cm. 7 mm.

14. 5 m. 8 dm. 6 cm., 6 m. 8 dm. 5 mm., 6 dm. 8 cm. 5 mm., 6 m. 8 dm. 5 mm.

In Exercises 15-34 perform the subtraction operations indicated, verifying by working also in decimals of a metre:

	m.	dm.	cm.	mm.		m.	dm.	cm.	mm.
15. From	12	2	3	4	take	4	3	2	6
16. „	8	8	0	8	„	8	0	8	8
17. „	9	0	0	0	„	7	7	6	3
18. „	26	1	8	3	„	21	8	1	3
19. „	103	4	6	7	„	26	8	3	8
20. „	100	2	1	2	„	3	8	9	9

21. Subtract 3 m. 2 dm. 2 mm. from 6 m. 2 mm.

22. „ 4 m. 2 dm. 3 cm. 5 mm. „ 10 m.

23. „ 18 m. 3 cm. 3 mm. „ 38 m. 1 dm. 1 mm.

24. „ 7 m. 2 cm. 1 mm. „ 10 m. 8 dm.

25. „ 6 m. 6 dm. 6 cm. 6 mm. „ 8 m. 2 dm. 3 mm.

26. „ 15 m. 6 dm. 3 cm. 2 mm. „ 18 m. 3 cm. 3 mm.

27. „ 19 m. 3 cm. „ 19 m. 3 dm.

28. „ 7 m. 4 dm. 2 cm. 2 mm. „ 12 m. 2 mm.

29. Subtract 4 dm. 3 cm. 2 mm. from 11 m.

30. „ 3 dm. 8 mm. „ 4 m. 8 cm.

31. „ 2 m. 7 cm. 6 mm. „ 3 m. 2 cm.

32. „ 5 m. 6 dm. 1 mm. „ 6 m. 5 dm. 1 cm.

33. „ 7 m. 6 dm. 7 cm. 7 mm. „ 12 m. 8 dm. 8 cm. 8 mm.

34. „ 23 m. 7 dm. 1 cm. 2 mm. „ 30 m. 3 dm. 1 mm.

35. Find the sum of 0·896, 2·175, 32·01 and 6·08.

36. The weights of coal in four consecutive trucks in a train are respectively 8·27 tons, 10·08 tons, 12·31 tons, 7·89 tons; what total weight is contained by the four trucks?

37. Five rods are placed in line with one another, their respective lengths being 14·26 ft., 13·89 ft., 15·72 ft., 13·72 ft. and 16·03 ft.; what is their combined length?

38. A piece 3·26 ft. is sawn off a rod 12·37 ft. long; what is the length of the remainder?

39. A vessel contains 36 gallons of water, 7·83 gallons of which are run out; what does the vessel now contain?

40. Evaluate:

(i) $6·32+7·16+0·87+0·26+7·814$.

(ii) $216·1+3·814+0·072+283·1+3·141+0·082$.

(iii) $79·3+91·6+90·07+0·92+167·3+0·89$.

(iv) $7·36+18·02+106·35+0·816+29·3+30·005$.

(v) $236·2+0·89+107·341+8·36+0·97+13·776$.

(vi) $378·342-186·593$. (vii) $612·7-89·821$.

(viii) $106·82-101·61$. (ix) $81·932-11·75$.

(x) $119·171-71·91$. (xi) $379·23-193·45$.

(xii) $100-0·09$.

(xiii) $16·342+17·95-18·082-0·93$.

(xiv) $102·61+92·83-10·98-83·62$.

(xv) $37·86-79·21+64·18$.

(xvi) $116·375-182·02+217·362$.

(xvii) $89·106+3·782-0·983+2·17$.

(xviii) $108·94-893·06+0·89+972·5$.

(xix) $383·16+82·76-500+142·803$.

19. MULTIPLICATION AND DIVISION OF DECIMALS BY A SINGLE FIGURE.

EXAMPLE.—*Six rods each* 4 *metres* 3 *decimetres* 0 *centimetres* 2 *millimetres in length are placed end to end. What is the combined length?*

m.	dm.	cm.	mm.
4	3	0	2
			6
25	8	1	2

The combined length is therefore 25 metres 8 decimetres 1 centimetre 2 millimetres.

Working in decimetres and decimals of a decimetre, we proceed thus:

dm.	
43·02	
6	
258·12	The combined length is 258·12 decimetres.

EXAMPLE.—*What is the combined weight of* 7 *blocks, each one of which weighs* 14·038 *lbs.?*

By addition.	By multiplication.
14·038	14·038
14·038	7
14·038	98·266
14·038	
14·038	
14·038	
14·038	
98·266	

The procedure is as in ordinary multiplication; only, as we pass the decimal point, it is set down in the answer.

The total weight is 98·266 lbs.

EXAMPLE.—*A rope* 192 *metres* 7 *decimetres* 8 *centimetres* 6 *millimetres long is divided into six equal parts. What is the length of each part?*

	m.	dm.	cm.	mm.
6	192	7	8	6
	32	1	3	1

Or, working with lengths expressed in metres and decimals of a metre,

	m.
6	192·786
	32·131

The length of each of the portions is 32·131 metres.

EXAMPLE.—*If* 272·365 *gallons be divided into five equal parts, what will be the volume of one of these parts?*

5	272·365
	54·473

Each part therefore has a volume of 54·473 gallons.

EXERCISES XXX.

1. Multiply 8 metres 2 decimetres 3 centimetres 4 millimetres by 8; also express the length in metres and decimals of a metre and multiply the resulting number by 8.

2. Multiply as in (1):

	m.	dm.	cm.	mm.			m.	dm.	cm.	mm.	
(i)	7	4	3	2	by 7.	(ii)	12	3	2	5	by 9.
(iii)	8	2	3	6	„ 11.	(iv)	14	3	9	7	„ 6.
(v)	0	3	7	6	„ 4.	(vi)	8	6	8	1	„ 3.
(vii)	13	7	5	2	„ 7.	(viii)	8	6	2	5	„ 8.
(ix)	9	1	2	5	„ 8.	(x)	11	7	7	5	„ 4.
(xi)	8	3	7	5	„ 4.	(xii)	10	1	2	8	„ 12.

3. Divide 37 m. 2 dm. 3 cm. 6 mm. by 6; also express the length as a decimal of a metre, and divide this number by 6.

4. Divide as in (3):

	m.	dm.	cm.	mm.			m.	dm.	cm.	mm.	
(i)	2	6	3	5	by 5.	(ii)	6	3	8	0	by 4.
(iii)	3	1	4	1	„ 9.	(iv)	5	3	7	6	„ 6.
(v)	9	0	2	0	„ 11.	(vi)	7	1	3	1	„ 3.
(vii)	4	9	6	5	„ 5.	(viii)	8	3	2	3	„ 7.
(ix)	2	6	2	4	„ 8.	(x)	36	5	1	3	„ 9.
(xi)	6	0	0	0	„ 8.	(xii)	21	2	3	4	„ 3.
(xiii)	16	5	6	4	„ 4.	(xiv)	2	0	7	5	„ 5.
(xv)	8	9	7	6	„ 12.						

5. In lifting a weight by a block of pulleys, the weight only rises one-fifth of the distance the rope is pulled down by the man lifting the weight. What distance will the weight be lifted when the man pulls in 6·75 ft. of rope? State also the length of rope pulled in by the man when the weight is lifted through a distance of 3·28 ft.

6. Multiply:

(i) 8·961 by (*a*) 3, (*b*) 5, (*c*) 7, (*d*) 9.
(ii) 0·0625 by (*a*) 2, (*b*) 4, (*c*) 6, (*d*) 8.
(iii) 14·016 by (*a*) 7, (*b*) 9, (*c*) 8.
(iv) 3·162 by (*a*) 4, (*b*) 3, (*c*) 7, (*d*) 9.

7. Divide:

(i) 7·6 by (*a*) 2, (*b*) 4, (*c*) 8.
(ii) 0·0621 by (*a*) 3, (*b*) 9.
(iii) 13·629 by (*a*) 3, (*b*) 7.
(iv) 624·375 by *a* (5), *b* (3), *c* (9).
(v) 1·0 by (*a*) 2, (*b*) 4, (*c*) 8.
(vi) 3·0 by (*a*) 4, (*b*) 8.
(vii) 411·264 by (*a*) 4, (*b*) 3, (*c*) 12.
(viii) 0·0024 by (*a*) 3, (*b*) 4, (*c*) 6, (*d*) 8, (*e*) 12.
(ix) 0·61782 by (*a*) 3, (*b*) 6.
(x) 36·0024 by (*a*) 2, (*b*) 4, (*c*) 8.
(xi) 7·2 by (*a*) 5, (*b*) 12, (*c*) 8, (*d*) 9.

8. What is the sixth part of (*a*) £4·7268, (*b*) 18·324 lbs., (*c*) 170·8146 ft.?

9. Subtract the seventh part of £8·743 from the twelfth part of £36·324.

10. Add the eleventh of 89·32 lbs. to the fifth part of 9·3 lbs.

20. MULTIPLICATION OF DECIMALS.

General Case.

When a number, say 328·25, is multiplied by 10, the effect, as has been seen, is that

where we had 3 hundreds we now have 3 thousands,
„ „ 2 tens „ „ 2 hundreds,
„ „ 8 units „ „ 8 tens,
„ „ 2 tenths „ „ 2 units,
„ „ 5 hundredths „ „ 5 tenths.

The place value of each digit is raised tenfold; the result is thus represented by 3282·5, and each digit is moved one place to the *left* with reference to the decimal point.

If we multiply by 100, the units become hundreds, the tens become thousands, tenths become tens, and hundredths become units; the place value of each digit is raised a hundredfold; the result 32825 is obtained by moving each digit two places to the left.

When, in a similar way, we multiply by 0·1, each unit becomes a tenth, and following the analogy each ten becomes a unit; so that in the product of 328·25 by 0·1,

instead of 3 hundreds we have 3 tens,
„ 2 tens „ 2 units,
„ 8 units „ 8 tenths,
„ 2 tenths „ 2 hundredths,
„ 5 hundredths „ 5 thousandths.

The place value of each digit is reduced to one-tenth of its value, so that 328·25 when multiplied by 0·1 becomes 32·825; and we notice that all the digits are moved one place to the *right* relatively to the decimal point.

Similarly, multiplying by 0·01, the place value will be further reduced, so that thousands digits become tens digits, hundreds digits become units, tens digits become tenths digits and so on; and $328{\cdot}25 \times 0{\cdot}01$ becomes 3·2825; each figure being moved two places to the right.

EXAMPLE.—*To multiply* 328·25 *by* 0·4.

The result is evidently 4 times that of multiplying by 0·1, reducing the place value of each digit one-tenth and multiplying by 4; in other words, multiply by 4 and move each digit *one* place to the right.

EXAMPLE.—*To multiply* 328·32 *by* 0·04.

The place value of each figure is reduced to a hundredth of the former value, and the altered number then multiplied by 4.

Thus, $328{\cdot}32 \times 0{\cdot}04$ becomes $3{\cdot}2832 \times 4 = 13{\cdot}1328$.

EXERCISES XXXI. a.

Mental or Oral.

1. (*a*) Multiply 1000 by 4, 0·04, 0·4.
   (*b*) „ 300 by 30, 0·03, 0·003, 0·5, 0·06.
   (*c*) „ 250 by 0·4, 0·2, 0·02, 0·008, 12, 1·2.
   (*d*) „ 800 by 0·05, 0·005, 5, 0·07, 0·9.
   (*e*) „ 0·05 by 10, 100, 300, 0·01.
   (*f*) „ 37·2 by 0·1, 0·001, 0·01.
   (*g*) „ 1·6 by 0·3, 0·9, 9, 0·09.

2. One line is 3 m. 2 dm. 1 cm. long, a second line is 0·4 times as much; what is its length?

3. What is the length of a line 0·07 times that of a line 83 m. 2 dm. long?

4. 0·4 of a street 378 m. 4 dm. long has to be asphalted and the remainder to be paved. What lengths have to be treated in each way?

EXAMPLE.—*To multiply* 26·3 *by* 24·52.

Following the reasoning employed in ordinary multiplication, we may say that we have to find

$$(26{\cdot}3 \times 20) + (26{\cdot}3 \times 4) + (26{\cdot}3 \times 0{\cdot}5) + (26{\cdot}3 \times 0{\cdot}02).$$

The arrangement is as follows:

```
 26·3
 24·52
------
 526·
 105·2
  13·15
   ·526
------
644·876
```

(*a*) Multiply by 20 by first increasing the place value by 10 (moving figures one place to left), and then multiply the result by 2, obtaining 526.

(*b*) Multiply by 4 and obtain 105·2.

(*c*) Multiply by 0·5, *i.e.* move the figures one place to right relatively to the decimal point, and multiply by 5, obtaining 13·15.

(*d*) Multiply by 0·02, by moving the figures two places to the right relatively to the decimal point, and multiplying by 2, obtaining 0·526.

(*e*) Add up 526, 105·2, 13·15, 0·526, obtaining as final result 644·876.

We can conveniently arrange the work as below, where it will be noticed that

```
 26·3
  2 4·52
------
 526·
 105·2
  13·1 5
   ·5 26
------
644·8 76
```

(*a*) The units digit of the multiplier is placed under the last figure of the multiplicand.

NOTE—When the multiplier is less than unity, say 0·24, this cypher, which should always precede the decimal point, must take the place of the units digit, and be placed under the last figure of the multiplicand.

(*b*) When we multiply by any digit, the partial product is set down with its figure of lowest place value immediately under that digit, *i.e.* in multiplying by 4, the 2 is set underneath the 4; in multiplying by 0·02 in the last partial product, the 6 is underneath the 2 and so on.

(*c*) The decimal point in the answer is now underneath the decimal point of the multiplicand before the multiplication took place.

(*d*) There are as many decimal figures in the product as there are altogether in the two numbers multiplied; thus in the example we have two decimal figures and one decimal figure respectively in the two numbers whose product is found, and three, *i.e.* 2 + 1 decimal figures in the answer.

EXERCISES XXXI. b.

1. If one cubic foot of fresh water weighs 62·5 lbs., what will be the weight of 11·11 cubic feet?

2. If a gallon equals 0·16 of a cubic foot, what will be the weight of a gallon of water, assuming one cubic foot of water to weigh 62·5 lbs.?

3. If sea water be 1·027 times as heavy as fresh water, and one cubic foot of fresh water weighs 62·5 lbs., how much will a cubic foot of sea water weigh?

4. If iron be 7·72 times as heavy as fresh water, what is the weight of a cubic foot of iron?

5. An acreage of 27·75 acres is rented at £12·8 per acre. What is the rent?

6. An engine draws up from a pit 376 trams of coal each averaging 0·645 tons. What is the total amount raised?

7. A liner completes a voyage in 6·4 days, and makes an average run of 18·25 knots per hour. What distance has been run, assuming a knot to be 1·15 miles?

8. What is the value to the nearest franc of 79·24 metres of silk at 8·75 francs per metre?

9. The wheat yield of a field of 19·6 acres is at the rate of 27·45 bushels per acre. What is its value at £0·325 per bushel?

10. If 0·917 of a sovereign be pure gold, how much pure gold is there in 23·27 lbs. of standard gold? (Standard gold is the gold from which sovereigns are made.)

11. If the weight of a sovereign when turned out from the mint be 123·27 grains, what is the weight of pure gold that a sovereign contains? (Refer to previous question.)

12. If £1 be equivalent to 25·25 francs, what sum in francs would be equivalent to (*a*) £3·5, (*b*) £13·42?

13. If one foot of lead piping weighs 1·82 lbs., what will be the weight of a piece of such piping 22·3 feet in length?

14. If 1 lb. of water has a volume of 0·016 cubic feet, what will be the volume of an amount of water which weighs 26·2 lbs.?

15. When a body floats it displaces its own weight of water. What volume of water will be displaced by a floating body whose weight is 7·26 lbs.? (1 lb. of water occupies a volume equal to 0·016 cub. ft.)

16. What will be the volume of water displaced by a boat weighing 420 lbs.? (Refer to previous question.)

17. Assuming 1 metre = 39·37 inches, find the English equivalents in feet and inches of the following lengths:

(*a*) 5 m.	(*b*) 3 m. 2 cm.	(*c*) 3 m. 2 dm.
(*d*) 3 m. 2 mm.	(*e*) 36 m. 3 mm.	(*f*) 78·32 cm.
(*g*) 91·82 cm.	(*h*) 101·63 mm.	(*i*) 238 cm.
(*j*) 2·707 dm.	(*k*) 0·5885 m.	(*l*) 101·2 mm.

NOTE.—1 metre is not *exactly* 39·37 inches, so that the results obtained in this exercise are not quite in accordance with the truth.

EXERCISES XXXI. c.

1. Multiply 763 by (*a*) 7·26, (*b*) 0·726.

2. „ 893·21 by (*a*) 0·0076, (*b*) 76, (*c*) 0·076, (*d*) 2·82, (*e*) 0·282, (*f*) 317, (*g*) 3·17.

3. Multiply 0·0935 by (*a*) 87·64, (*b*) 876·4, (*c*) 8764, (*d*) 590, (*e*) 5900.

4. „ 100·82 by (*a*) 375, (*b*) 3·75, (*c*) 0·0375, (*d*) 13·86, (*e*) 1·386.

5. „ 97·875 by (*a*) 424, (*b*) 4·24, (*c*) 42·4, (*d*) 800, (*e*) 80, (*f*) 0·008.

6. „ 68·125 by (*a*) 128, (*b*) 12·8, (*c*) 0·0128, (*d*) 1728, (*e*) 17·28, (*f*) 172·8.

7. „ 89·926 by (*a*) 212·5, (*b*) 21·25, (*c*) 2·125, (*d*) 0·02125, (*e*) 18, (*f*) 0·18.

8. Evaluate (*a*) $(76·32)^2$, (*b*) $(8·4)^3$, (*c*) $(0·85)^3$, (*d*) $(1·75)^3$.

9. „ (*a*) $(7·6)^2 \times (2·1)^2$, (*b*) $(98)^2 \times (0·025)^2$, (*c*) $(1·2)^3 \times (0·04)^2$.

10. „ (*a*) $(10·2)^2$, (*b*) $(10·3)^3$, (*c*) $(100·5)^2$, (*d*) $(500·1)^2$.

11. „ (*a*) $(896·2)^2$, (*b*) $(86·21) \times (0·762)$, (*c*) $(98·2) \times (9·1)$.

12. „ (*a*) $(1001·2) \times (998·8)$, (*b*) $(100·25) \times (99·75)$, (*c*) $(80·1) \times (79·9)$.

EXERCISES XXXI. d.

Mental or Oral.

Write down the product of the numbers represented by the digits marked with an asterisk.

1. $\overset{*}{4}32·12$ and $0·0\overset{*}{6}2$.
2. $59·\overset{*}{8}2$ and $2·3\overset{*}{2}$.
3. $\overset{*}{5}·86$ and $\overset{*}{1}02$.
4. $37·40\overset{*}{2}$ and $0·0\overset{*}{3}72$.
5. $\overset{*}{3}74·02$ and $0·\overset{*}{3}72$.
6. $0·7\overset{*}{1}82$ and $\overset{*}{7}18·2$.
7. $0·\overset{*}{5}72$ and $0·5\overset{*}{7}2$.
8. $\overset{*}{8}88·2$ and $2·\overset{*}{8}8$.
9. $\overset{*}{1}003·5$ and $0·00\overset{*}{1}035$.

EXERCISES XXXI. e.

Practical.

[*Apparatus: Calipers, blocks, coins, tubes, spheres and scales.*]

1. Employ your calipers, or other instruments, to measure:

(*a*) The diameter of a halfpenny.

(*b*) The diameter of a penny.

(*c*) The dimensions of the given block.

(*d*) The inside diameter of the given tube.

Express your measurements in inches, and also in centimetres and millimetres.

2. Measure the diameter of the given sphere.

To measure the diameter of a ball an indirect method must be used, because of the difficulty of measuring the diameter of a sphere directly with a scale.

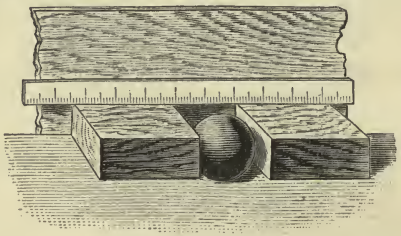


FIG. 12.—Measurement of the diameter of a sphere.

Procure two rectangular wooden blocks larger across than the sphere. Push the two blocks against a third, or against any upright surface, so as to keep them parallel, and then place the wooden sphere between them and measure its diameter, as in Fig. 12. The scale is there shown placed on the blocks "edge on," which is the correct way to use a wooden scale for such measurements as this. A thin metal scale, however, may be laid flat on the blocks. Make measures with the sphere in different positions, and find the mean or average result.

Measure the diameter of the sphere also by means of calipers.

3. (*a*) Draw 9 parallel lines at a distance apart of one inch, and number them 0, 1, 2, ... 8. Take any point O on the '0' line and draw a straight line OA to the '8' line; measure along the '8' line, a distance AB of one inch, and join OB. Mark the intercepts on your lines 1, 2, etc., by OA and OB, (i), (ii), (iii), (iv), etc., respectively; measure the intercepts, and test them by the scale of eighths.

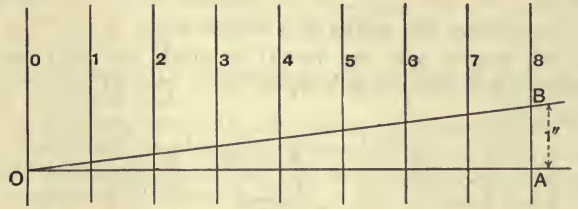


FIG. 13.

(*b*) Draw a new set of 9 parallel lines, at any distance apart, provided the lines are equidistant, and repeat the procedure in 3 (*a*).

[The results of the class may be compared on the board.]

(*c*) Repeat the procedure in 3 (*a*), using however 11 straight lines, and again test the lengths of the intercepts.

The gradual increase of length of the intercepts suggests a means of measuring small distances by means of a wedge.

4. Take a strip of squared paper, divided into centimetres, and cut from it a right-angled triangle with a base 10 cm. long and the perpendicular 1 cm. long.

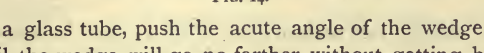


FIG. 14.

Using a glass tube, push the acute angle of the wedge into the tube until the wedge will go no farther without getting bent, then read off the diameter of the tube.

5. Construct a similar wedge, but where the height is one inch and the length 10 inches.

(*a*) Measure the diameter of the tube to the hundredth part of an inch.

(*b*) Express in cm.:
(i) 0·1 in., (ii) 0·16 in., (iii) 0·2 in., (iv) 0·3 in., (v) 0·36 in.

(*c*) Express in inches:
(i) 1 cm., (ii) 1·8 cm., (iii) 0·9 cm., (iv) 1·5 cm. (v) 2·1 cm.

6. Repeat, if necessary, Exercise 3 (*a*) and 3 (*b*), and notice that, if in each case the intercepts be produced a further distance of one inch,

(i) all the ends will still lie on a straight line;

(ii) the lengths will now be (*a*) one inch, one inch and an eighth, one inch and two eighths, etc.; (*b*) 1 inch, 1·1 inch, 1·2 inch, 1.3 inch, etc.

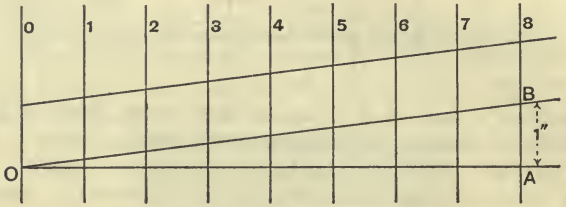


FIG. 15.

Study a diagonal scale; notice that you can read off distances as small as 0·01 inch and rising by steps of 0·01 inch to 5 inches.

7. Use a diagonal scale to draw lines of the following lengths:
0·82 in., 0·28 in., 1·28 in., 1·82 in., 4·82 in.
4·28 in., 2·56 in., 2·65 in., 3·65 in., 3·56 in.

8. Calculate in each case the length in inches equivalent to
1 cm., 4 cm., 6 cm., 8 cm., 5 cm., 3 cm.

9. If a straight line measured from left to right be accounted + and „ „ „ „ „ right to left „ „ −, find graphically the result of the following operations, and verify by calculation:

(*a*) $2\cdot36+3\cdot01-2\cdot63+1\cdot06-3$. (*b*) $4\cdot62-1\cdot68+0\cdot91-3\cdot42+2$.
(*c*) $4\cdot62-1\cdot68+0\cdot91-3\cdot42-0\cdot3$. (*d*) $1\cdot87+0\cdot53-0\cdot35-1\cdot78$.
(*e*) $1\cdot87+0\cdot53-0\cdot36-0\cdot78$.

21. DIVISION OF DECIMALS.

General Case.

EXAMPLE.—*How many lengths each* 23·28 *cm. can be cut off a rod* 345·681 *cm. long? What also will be left over?*

The rod has to be divided, and the problem is the same as if we were told that

each length was 2·328 dm. and the rod 34·5681 dm.

or that each length was 232·8 mm. and the rod 3456·81 mm.

That is to say, the decimal point in the divisor can be moved any number of places to right or to left, provided that in the dividend the decimal point be moved the *same* number of places to the right or to the left as the case may be.

It will be most convenient to move the decimal point until, in the *divisor*, the *digit of highest place value* is the *units* digit.

Thus $345{\cdot}681 \div 23{\cdot}28 = 34{\cdot}5681 \div 2{\cdot}328$, where the decimal point has been moved one place to the left in divisor and dividend, so that in the divisor 2·328 the figure of highest place value, *i.e.* the first 2, is in the units place.

Remembering that the divisor is between 2 and 3, proceed as in *short* division until the first *significant* figure is obtained; here it is easily seen to be 1, its position is over the 3; and *the first part* of the *quotient* is 10.

4· = 2nd part of Quotient.
10· = 1st part of Quotient.

2·328	34·5681	
	23·28	Proceed now as in ordinary division and obtain the
	11·2881	... *first partial remainder, i.e.* 11·2881.
	9·312	Divide again; the 2nd part of the quotient is 4, its position is over the 4; and the
	1·9761	... *remainder* is 1·9761 *dm.*

Hence (10 + 4), *i.e.* 14 lengths, can be cut off, and a piece 1·9761 dm. or 19·761 cm. will be left over.

The work may be shortened by writing remainders only, and then appears thus:

14 = Quotient

2·328 | 34·5681
11·2881
1·9761 = Remainder in *dm.*

The problem of division may be expressed in another way, thus:

What number must 23·23 be multiplied by so as to yield 30·299?

In problems of this kind the divisor may be greater than the dividend, and the quotient will be a decimal less than unity.

EXAMPLE. — *To find the value of* 0·323 ÷ 62·5.

As in the last example, the decimal point is moved in divisor and dividend, until in the divisor the digit of highest place value is in the units place; 0·0323 ÷ 6·25 is thus obtained.

Remembering that in any decimal, cyphers may be added to the right of the last decimal figure, without altering the value of the decimal, the procedure is as in the last example.

0·000008 .....4th part of the quotient.
0·00006 .....3rd „ „ „ „
0·0001 .....2nd „ „ „ „
0·005 .....1st „ „ „ „

6·25 | 0·03230000
0·03125

0·001050 .....1st remainder.
0·000625

0·0004250 .....2nd remainder.
0·0003750

0·00005000 .....3rd remainder.
0·00005000

The complete quotient is 0·005168.

Many of the cyphers are often omitted in actual work, thus:

0·005168 = Quotient.

6·25 | 0·03230000
0·03125
1050
625
4250
3750
5000
5000

or writing remainders only:

0·005168 = Quotient.

6·25 | 0·03230000
1050
4250
5000
....

It will be noticed that when the necessary cyphers are added to the dividend, the *number* of decimal figures in the dividend is *equal* to the *sum of the number in the divisor and the quotient.*

EXERCISES XXXII. a.

Mental or Oral.

1. Divide 4 by (*a*) 0·4, (*b*) 5, (*c*) 0·8, (*d*) 20.
2. „ 16 by (*a*) 5, (*b*) 50, (*c*) 0·8, (*d*) 0·08.
3. „ 24 by (*a*) 8, (*b*) 0·6, (*c*) 60, (*d*) 0·3, (*e*) 300, (*f*) 40.
4. „ 240 by (*a*) 6, (*b*) 60, (*c*) 600, (*d*) 0·6, (*e*) 0·12, (*f*) 800.
5. „ 30 by (*a*) 4, (*b*) 40, (*c*) 75, (*d*) 5, (*e*) 0·5, (*f*) 0·6, (*g*) 600.
6. „ 48 by (*a*) 8, (*b*) 80, (*c*) 60, (*d*) 12, (*e*) 0·12, (*f*) 2·4, (*g*) 24, (*h*) 240.
7. „ 4·8 by (*a*) 0·3, (*b*) 0·48, (*c*) 0·6, (*d*) 8, (*e*) 0·8, (*f*) 16.

8. Divide 7·2 by (*a*) 2·4, (*b*) 72, (*c*) 0·9, (*d*) 0·72, (*e*) 0·8, (*f*) 0·24, (*g*) 36, (*h*) 3·6.

9. „ 0·64, by (*a*) 0·4, (*b*) 6·4, (*c*) 8, (*d*) 0·8, (*e*) 32, (*f*) 0·32, (*g*) 1·6.

10. „ 9·6 by (*a*) 12, (*b*) 0·8, (*c*) 0·16, (*d*) 3·2, (*e*) 48, (*f*) 480, (*g*) 0·6, (*h*) 0·06.

11. How often is 0·25 of a yard contained in (*a*) 5 yds., (*b*) 50 yds., (*c*) 4 yds.?

12. How many times is 0·8 of a lb. contained in (*a*) 5 lb., (*b*) 7·2 lb., (*c*) 16 lb.?

13. Divide 7·5*d.* into (*a*) 5*s.*, (*b*) 2*s.* 6*d.*, (*c*) 10*s.* and (*d*) £1.

14. By what must 3·2 be multiplied to give (*a*) 1·6, (*b*) 48 and (*c*) 80?

22. DIVISION TRUE TO A CERTAIN NUMBER OF DECIMAL PLACES.

Just as in earlier problems (p. 36), results were required to the nearest unit or to the nearest hundred, etc., so frequently in the division of decimals the process need not be carried on very far; it is enough to obtain the result true to a certain number of decimal figures.

Thus, in dividing 0·0423 by 7, we obtain

0·006,04285....

The result of the division is 0·006 true to three decimal places, 0·006043 true to the nearest figure in the sixth place, and so on.

EXERCISES XXXII. b.

1. The weight of one cubic foot of iron being 485 lb., and the weight of one cubic foot of water being 62·5 lb., how many times heavier is iron than water? (In other words, find the *specific gravity* of iron.)

2. What is the volume in cubic inches (true to 3 decimal places) of a 7 lb. cast-iron weight when a cubic inch of it weighs 0·26 lb.

3. Given that a metre is equal to 39·37 in., what multipliers must be used to convert metres into yards and to convert yards into metres (true to four places of decimals)?

4. A litre is equivalent to 1·7617 pints; knowing that 8 pints make 1 gallon, express 36 gallons in litres (true to two places of decimals).

5. How many times can a vessel holding 3·45 pints be filled from one holding 18·732 gallons, and what amount remains? (1 gallon equals 8 pints.)

6. A metre being known to be equal to 39·37 inches; how many metre rods could be cut off from a rod 10 yards long? What would be the length of the piece left over?

7. How many metres could be cut off from a rod 7 ft. long? State the length of the piece remaining.

8. If one cubic foot of brass weigh 551·25 lb., how many cubic feet will there be in a brass block weighing 1290·95 lb.?

9. When a certain screw is turned round through one complete revolution, its head goes forward by 0·132 of an inch. How many turns must you give such a screw to drive it into a piece of wood to a depth of 1·716 inches?

10. The pitch of a screw being 0·166 inch, how many turns must you give it to move it forward through a distance of 1·328 inches? (The *pitch* of a screw is the distance the screw moves forward when turned through one complete turn or revolution.)

11. Divide (*a*) 108·1 by 2·3, then write down the quotients of (*b*) 10·81 by 0·23, (*c*) 1·081 by 230, (*d*) 0·01081 by 23, (*e*) 1·081 by 0·0023.

12. Divide (*a*) 2·511 by 0·27, then write down the quotients of (*b*) 251·1 by 27, (*c*) 0·002511 by 0·027, (*d*) 0·2511 by 2700, (*e*) 2511 by 0·0027.

13. Divide (*a*) 3024 by 3·6, then write the quotients of (*b*) 3·024 by 3·6, (*c*) 30·24 by 0·036, (*d*) 0·03024 by 360, (*e*) 0·003024 by 0·0036.

14. Divide (*a*) 336·42 by 0·54, then write the quotients of (*b*) 0·33642 by 540, (*c*) 0·033642 by 5·4, (*d*) 3·3642 by 0·0054, (*e*) 3·3642 by 54.

15. Divide (*a*) 500·208 by 0·68, then write the quotients of (*b*) 5002·08 by 0·68, (*c*) 5002080 by 0·068, (*d*) 0·00500208 by 680, (*e*) 0·500208 by 6800.

16. Divide (*a*) 5726·885 by 60·283, then write the quotients of (*b*) 57·26885 by 602·83, (*c*) 572·6885 by 6028·3, (*d*) 0·5726885 by 0·060283, (*e*) 0·05726885 by 60·283.

17. Divide (*a*) 29019·69 by 36·5, then give the quotients of (*b*) 290·1969 by 0·365, (*c*) 0·02901969 by 365, (*d*) 2901·969 by 3650.

18. Divide (*a*) 74·820025 by 1867, then give the quotients of (*b*) 0·74820025 by 18·67, (*c*) 7482·0025 by 18670, (*d*) 748200·25 by 186·7, (*e*) 7·4820025 by 0·01867.

19. Divide (*a*) 5942·6842 by 73·4, then give the quotients of (*b*) 59·426842 by 734, (*c*) 59426·842 by 7340, (*d*) 0·59426842 by 0·0734, (*e*) 594·26842 by 0·00734.

20. Divide (*a*) 281·37084 by 57·9, then give the quotients of (*b*) 28·137084 by 579, (*c*) 281370·84 by 57900, (*d*) 0·28137084 by 5·79, (*e*) 2813·7084 by 0·0579.

21. Divide (*a*) 117·1408 by 2091·8, then give the quotients of (*b*) 117140·8 by 20·918, (*c*) 1171·408 by 20918, (*d*) 1·171408 by 0·0020918.

22. Divide (*a*) 2791·490 by 3·94, then give the quotients of (*b*) 27·9149 by 394, (*c*) 0·279149 by 39·4, (*d*) 27914·9 by 0·00394.

23. Divide (*a*) 71656·228 by 95·06, then give the quotients of (*b*) 71·656228 by 950·6, (*c*) 716562·28 by 9506, (*d*) 7165·6228 by 0·9506, (*e*) 716·56228 by 0·009506.

24. Divide (*a*) 240493·344 by 62·4, then give the quotients of (*b*) 2404·93344 by 6240, (*c*) 24·0493344 by 0·624, (*d*) 0·240493344 by 6·24, (*e*) 240493344 by 62400.

25. Divide (*a*) 882·54019 by 9070·3, then give the quotients of (*b*) 88254·019 by 90·703, (*c*) 8825·4019 by 9·0703, (*d*) 88254·019 by 907·03, (*e*) 88254019 by 9070300.

23. BRITISH MEASURES OF LENGTH (*continued*).

Other British systems for measuring length exist, in addition to those mentioned already (p. 74). Thus, in surveying, we have

100 **links** = 1 **chain,**
25 links = 1 **pole,**
10 chains = 1 **furlong,**
8 furlongs = 1 **mile.**

Some other measures used in special branches of work are the following:

1 **Geographical mile** or Nautical mile = 1·15 ordinary or Statute mile,
1 League = 3 miles,
1 Cable = about 100 fathoms, but a varying length,
1 **Fathom** = 6 feet,
1 Pace [Military] = 2 ft. 6 in.,
1 Pace [Geometrical] = 5 ft.,
1 Ell = 1·25 yards,
1 Cubit = 18 inches,
1 Quarter or Span = 9 inches,
1 **Hand** = 4 inches,
1 Palm = 3 inches,
1 Nail = 2·25 inches,
12 Lines = 72 points = 1 inch.

EXAMPLE.—*To express* 2 *yards* 2 *feet* 9 *inches in feet and decimals of a foot.*

Since 12 inches make 1 foot, divide 9 by 12 to change the inches to feet,

∴ 2 yds. 2 ft. 9 in. = 2 yds. 2·75 ft. = 8·75 ft.

EXAMPLE.—*To express* 3 *miles* 4 *fur.* 3 *ch.* 5 *lks. as a decimal of a mile.*

3 miles 4 fur. 3 ch. 5 lks.

= 3 miles 4 fur. 3·05 ch. — Since 100 lks. = 1 ch., 1 lk. = 0·01 ch.

= 3 miles 4·305 fur. — Since 10 ch. = 1 furlong, divide the number of chains by 10 to express in furlongs.

= 3·538125 miles. — Divide the furlongs by 8 to express in miles.

EXAMPLE.—*To express* 5·1627 *miles in miles, furlongs, chains and links.*

5·1627 miles — Multiply the *decimal* part by 8 to express in furlongs.

= 5 miles 1·3016 furlongs

= 5 miles 1 fur. 3·016 chains — Multiply ·3016 by 10 to express in chains.

= 5 miles 1 fur. 3 ch. 1·6 lks. — Multiply ·016 by 100 to express in lks.

EXERCISES XXXIII

Express as decimals of a mile:

	fur.	ch.	lks.		fur.	ch.	lks.
1.	2	3	2.	**2.**	3	8	32.
3.	7	6	8.	**4.**	4	2	64.
5.	3	3	3.	**6.**	6	9	72.
7.	3	2	6.	**8.**	6	7	76.

Express in furlongs, chains and links:

9.	3·26 of a mile.		**10.**	0·3284 of a mile.
11.	1·062	„	**12.**	0·089 „
13.	2·163	„	**14.**	4·361 „
15.	7·125	„	**16.**	0·375 „
17.	0·002	„	**18.**	6·81 „

19. A ship sails at the rate of 12 knots per hour; how long will it take to complete a voyage of 1700 statute miles? (1 knot or

nautical mile = 1·15 statute miles.) State the answer true to the nearest hour.

NOTE.—Much confusion exists relatively to the use of the term 'knot.' Some authorities treat it as in Ex. 19, *i.e.* as a distance. Other authorities use it as a speed, *i.e.* one knot is regarded as one geographical mile *per hour*.

20. A ship covers a distance of 288 miles in a day (24 hours). What is its speed in miles per hour?

21. An express train ran a distance of 457·86 miles in 7·8 hrs. What was the average speed in miles per hour?

22. Express in yards and decimals of a yard a distance of 7 ch. 29 lks.

23. The distance between the wickets on a cricket pitch is 1 chain. Express in miles the distance a man runs in scoring 120 runs, 64 of which were run out, assuming the distance covered for each run to be the same as the distance between the wickets?

24. Express in feet and inches the heights of the horses mentioned in the following extracts of advertisements:

(1) Good Brougham Horse, 16·1 hands high.
(2) Carriage Gelding, 15·2 „ „
(3) Black Brown Gelding, 16 „ „
(4) Bay Cob Gelding, 15 „ „
(5) Bay Gelding, 15·8 „ „
(6) High-stepping Bay, 14·3 „ „
(7) Cleveland Bay, 15·3 „ „

25. In the given table of velocities, some of which are only approximate, fill in the last column:

	Centimetres per sec.	Feet per sec.	Kilometres per sec.	Miles per sec.	Miles per hour.
(i) Light	3×10^6	984×10^6	300000	186420	...
(ii) Submarine Cable Signal	4×10^8	$31{\cdot}1 \times 10^6$	4000	2486	...
(iii) Earth in its orbit	30×10^5	$98{\cdot}4 \times 10^3$	30	18·6	...
(iv) Point on the Earth's Equator	465×10^2	1525·2	0·465	0·289	...
(v) Sound in the atmosphere under average conditions	$34{\cdot}96 \times 10^3$	1147	0·3496	0·2225	...

26. Using the table of velocities in Exercise 25, find how long light takes in coming from the sun to the earth, an average distance of $93{\cdot}3 \times 10^6$ miles. (State your answer true to the nearest second.)

27. The flash of a gun is noted by an observer, and 21·1 seconds afterwards he hears the report; how far off is the gun from the observer? State your answer true to the nearest decametre. (Use the table of velocities in Exercise 25.)

24. AREA.

The area of a square, of which the side is one foot long, is called one **square foot** (generally abbreviated into 1 sq. ft.). If the side of the square is an inch, the area is a **square inch**; if the side is a centimetre, the area is called a **square centimetre**, and so on.

EXERCISES XXXIV.

Graphic.

[*Apparatus*: *Squared Paper. Dividers. Scales.*]

Construct on squared paper the following rectangles and find their area:

1. Length 4 in., breadth 1 in.
2. Length 8 cm., breadth 1 cm.
3. Length 3 in., breadth 2 in.
4. Length 82 mm., breadth 22 mm.
5. Length 3 cm., breadth 4 cm.
6. Length 5 in., breadth 3 in.
7. Length 6·2 cm., breadth 5 cm.
8. Length 12 cm., breadth 10 cm.
9. Length 6·3 cm., breadth 4·4 cm.
10. If the length be a cm. and the breadth represented by b cm., how would you express the area?

Having seen that the *number* of *units of area* contained in a rectangle is equal to the *product of the number of units of length* and the *number of units of breadth*, the 'tables' of area can be easily understood.

25. BRITISH MEASURES OF AREA.

(12 × 12), *i.e.* 144 **square inches** make 1 **square foot**,
(3 × 3), *i.e.* 9 square feet „ 1 **square yard**.

In surveying, in the measurement of fields, and for similar purposes, the following system is in use:

(100 × 100), *i.e.* (10,000) square links make 1 square chain,
(4 × 4), *i.e.* 16 square poles „ 1 square chain,
(22 × 22), *i.e.* 484 square yards „ 1 square chain,
160 sq. po. or 10 square chains „ an area known as 1 **acre** (represented generally as 1 ac.).

The fourth part of an acre is known as the **rood** (abbreviated into *ro.*). Accordingly,

1 *ac.* = 4 *ro.* = 160 *sq. po.* = 10 *sq. ch.*
= 4840 sq. yards = 100000 sq. links.

Since 1 *mile* contains 80 chains,
1 sq. mile = 6400 sq. ch. = 640 *ac.*

NOTE ON TABLE.—The acre was the amount of land which could, on a rough estimate, be ploughed by a horse in a day. It was generally regarded as a space 220 yds. long (one furlong, *i.e.* one furrow long); the breadth was a chain, it was divided into 72 furrows.

Units of Area.

144 sq. in. = 1 sq. ft.,
9 sq. ft. = 1 sq. yard,
10000 sq. links = 484 sq. yds. = 16 sq. po. = 1 sq. chain,
4840 sq. yds. = 100000 sq. links = 160 sq. po.
= 10 sq. ch. = 4 ro. = 1 acre,
640 acres = 1 sq. mile.

26. METRIC MEASURES OF AREA.

(10 × 10), *i.e.* 100 sq. mm. = 1 sq. cm.,
(10 × 10), *i.e.* 100 sq. cm. = 1 sq. dm.,
(100 × 100), *i.e.* 10000 sq. cm. = 100 sq. dm. = 1 sq. metre
= 1 *centiare*,

100 sq. metres = 1 sq. decametre = 1 **are**,
100 sq. decametres = 1 sq. hectometre = 1 *Hectare*,
100 sq. hectometres = 1 sq. kilometre.

The terms *centiare*, *are*, *hectare* are used principally in dealing with area of fields, etc., and in surveying generally.

EXAMPLE.—*Find the area of a rectangular field, length* 8 *ch.* 3 *lks., breadth* 4 *ch.* 25 *lks., expressing the answer in acres, roods, sq. po., etc.*

It is convenient to express both the length and the breadth in chains and decimals of a chain, thus :

Length = 8·03 ch.
Breadth = 4·25 ch.
Area = 8·03 × 4·25 sq. ch.

```
   8·03
   4·25
  ------
  32·12
   1·60 6
    ·40 15
  ------
  34·12 75
```

= 34·1275 sq. ch.

= 3·41275 ac. ..... Divide by 10 to reduce to acres.

= 3 ac. 1·651 ro. ..... Multiply the *decimal* part by 4 to reduce to roods.

= 3 ac. 1 ro. 26·04 sq. po. ..... Multiply the decimal part of the rood by 40 to reduce to sq. poles.

∴ the area of the field is 3 acres 1 rood 26·04 square poles.

EXAMPLE.—*What is the length of a rectangular field, the area of which is* 74 *acres* 6·806 *sq. chains and the breadth of which is* 20 *ch.* 63 *links?*

The number of sq. ch. in the area
= (number of ch. in length) × (number of ch. in breadth).
∴ number of chains in the length = (number of sq. ch. in area) ÷ (number of ch. in breadth).

Area = 74 ac. 6·806 sq. ch. = 746·806 sq. ch.
Breadth = 20 ch. 63 links = 20·63 ch.
Length = 746·806 ÷ 20·63
= 36·2 chains
= 36 ch. 20 lks.

```
                36·2
          ------------
2·063 | 74·6806
        61 89
        ------
        12 790
        12 378
        ------
          4126
          4126
          ----
```

EXAMPLE.—*A room is* 12 *ft.* 8 *in. long,* 11 *ft.* 4 *in. broad. How many blocks of wood (parquets) each* 8 *inches square will be required to floor it?*

Area = 152 × 136 sq. inches
= 20672 sq. inches.

```
  152
  136
 -----
  152
  456
  912
 -----
20672
```

Area covered by 1 parquet = 8 × 8 = 64 sq. in.
Number of parquets required = 20672 ÷ 64 = 323.

EXAMPLE.—*A field is* 150·3 *metres long,* 89·7 *metres broad. Find its area, and express in ares.*

Area of field = 150·3 × 89·7 sq. metres
= 13481·91 sq. metres
= 134 ares 81·91 sq. m. (dividing by 100)

```
  150·3
   89·7
 -------
 12024.
  1352·7
   105·21
 -------
 13481·91
```

EXAMPLE.—*How many acres, roods, etc., are there in a field having an area 0·32625 of a sq. mile?*

0·32625 sq. mile
= 0·32625 × 640 ac.
= 208·8 ac.
= 208 ac. 3·2 ro.
= 208 ac. 3 ro. 8 sq. po.

0·32625
640
195·750
13·0500
208·8

EXERCISES XXXV. a.

1. The floor of a room is 32 sq. yds. 4 sq. ft. 36 sq. in.; part of it is covered by a 'square' of carpet 20 sq. yds. 8 sq. ft. 100 sq. in., find the area of the portion of the floor not covered by the carpet.

2. The floor of a room is partly covered by a square of carpet 14 ft. long 12 ft. broad, the area of the remaining portion is 8 sq. yds. 3 sq. ft. What is the total area of the room?

3. A block of wood is 8·34 cm. by 6·21 cm., in section; through it is bored a hole 20·3 sq. cm. in extent. What does the area of the section now become?

4. The section of a girder is in the form of a capital H, where the two vertical strokes are 14″ by 2·2″, the cross portion 8″ by 1·2″. What is the total area of the section?

5. Three rooms communicate by folding doors. One room is 16 ft. long 12 ft. broad, the second is 13 ft. long 12 ft. broad and the last is 16 ft. long 15 ft. broad. Express the area of each room in sq. yds. and sq. ft., and find the area of the carpet necessary to cover the three.

6. Find the sum of the following areas:

sq. yds.	sq. ft.	sq. in.
13	3	68
14	8	75
9	7	100
1	6	37

7. The tiled hearth of a fireplace is covered with 98 tiles, each tile being square—side 2 inches. What is the area of the hearth? Express in sq. ft. and sq. in.

8. What is the area of a floor covered with 1200 parquets each 9 in. long 4 in. broad? Express your answer in sq. ft.

9. What is the area of a yard paved with 84 flagstones each 2 ft. 4 in. long, and 1 ft. 10 in. broad?

10. The top floor of a house contains the following rooms:

Bedroom	-	17 ft. by 13 ft.
„	-	14 ft. „ 9 ft.
„	-	12 ft. „ 8 ft.
Box room	-	13 ft. „ 6 ft.

If the inside dimensions of the house on that floor be 23 ft. by 27 ft., what area is used for passages, landings and staircases?

11. A square field is 70 yards long; find its area in acres and decimals of an acre (true to 4 significant figures).

[*N.B.*—Make a mental note from this, as to the approximate extent of an acre.]

EXERCISES XXXV. b.

Mental or Oral.

State *roughly* the number of acres in the following fields, assuming an area of 70 yd. × 70 yd. to be approximately an acre.

(i) 140 yd. by 70 yd.	(ii) 280 yd. by 210 yd.
(iii) 140 yd. „ 35 yd.	(iv) 350 yd. square.
(v) 630 yd. „ 280 yd.	(vi) 105 yd. by 140 yd.
(vii) 700 yd. „ 350 yd.	(viii) 560 yd. „ 210 yd.
(ix) 700 yd. „ 280 yd.	(x) 245 yd. „ 210 yd.

The fields, the areas of which are given below, may be taken as approximately rectangular. One side of the field is given; find *approximately* in yards the length of the other

1.	Area =	5 ac.,	one side	70 yd.
2.	„ =	40 ac.,	„	280 yd.
3.	„ =	36 ac.,	„	630 yd.
4.	„ =	560 ac.,	„	560 yd.
5.	„ =	24 ac.,	„	210 yd.
6.	„ =	100 ac.,	„	1400 yd.
7.	„ =	35 ac.,	„	490 yd.
8.	„ =	8 ac.,	„	140 yd.
9.	„ =	12 ac.,	„	140 yd.
10.	„ =	6 ac.,	„	21 yd.
11.	„ =	84 ac.,	„	49 yd.
12.	„ =	15 ac.,	„	350 yd.

EXERCISES XXXV. c.

Practical.

[*Apparatus: Squared Paper. Dividers. Scales. Rulers.*]

1. Draw the plan of a room 15 ft. long, 13 ft. broad, representing each foot by a distance of 1 cm.; divide it up into strips 2 ft. 6 in. wide; show that this can be done exactly if the length of the strips runs in the direction of the breadth of the room.

If you wished to cover the floor of the room with carpet, 30 inches wide, find the length of carpet required, in order that none be wasted. Show that if the carpet be arranged so that the width be in the direction of the width of the room, some carpet will be wasted; find how many sq. ft. will be wasted, also the length of carpet required in this case.

2. Draw the plan of a room 18 ft. by 16 ft. (scale 1 cm. to the foot). Find the length of carpet required to cover its floor with carpet 2 ft. 3 in. wide arranged so that the wastage is least.

3. Draw the plan of a passage 20 ft. long, 6 ft. broad (scale 4 ft. represented by 1 inch). Show it covered with flagstones 2 ft. long, 2 ft. wide. How many are required?

4. Draw a plan of a room 15 ft. by 13 ft. Find the amount of timber required to floor it, the timber being boards 9 in. wide, and arranged with their length in the direction of the width of the room (scale 1 foot represented by 1 cm.).

5. Draw a representation of a roof, 20 ft. by 15 ft. (scale 4 ft. represented by 1 inch). Show it covered with slates, each slate being 1 ft. long, and 9 in. wide. How many slates will be required, supposing no overlapping of one slate over another?

6. Draw a representation of a roof 24 ft. long by 15 ft. broad, covered with slates 1 ft. by 9 in., each slate lapping over the one underneath it to a distance of 4·5 in., and the length of each slate parallel to the length of the roof. How many slates will be required? (Represent a distance of 1·5 in. by 1 mm.)

7. Draw the roof in the preceding question, where the lap is 3 in. only, and find the number of slates required in this case.

8. Draw a plan of a room 16 ft. long, 12 ft. wide. Show it carpeted with carpet 2 ft. wide, the width of the carpet being parallel to the width of the room.

Suppose the carpet has a pattern which recurs every 5 ft.; mark the points of recurrence by small arrowheads, and find the amount of carpet wasted if, in the different strips, corresponding points have to be alongside of one another.

9. A room is 14 ft. long, 12 ft. broad, 11 ft. high. Cut out a piece of paper which, when folded round, would represent the four walls of the room, and find the total area of the wall (scale 4 ft. represented by 1 cm.).

10. Draw the following rectangles ; show by cutting them into two, along the diagonal, superposing one portion on the other, that the diagonal divides the rectangle into two equal portions :

(*a*) 12 cm. by 8 cm., (*b*) 3·62 in. by 2·41 in.,
(*c*) 5 in. by 3 in., (*d*) 15 cm. by 6 cm.

11. Draw a circle 3 in. radius ; take any point O within it, at a distance of 1 in. from the centre ; draw any three chords AOB, COD, EOF. Find the areas of the rectangles the sides of which are (*a*) OA and OB, (*b*) OC and OD, (*c*) OE and OF.

12. Draw a rectangle 62 mm. long, 12 mm. broad ; fold it so as to represent the four walls of a room 18 ft. long, 13 ft. broad and 12 ft. high (scale 1 ft. represented by 1 mm.). State the area of the four walls. Find the cost of painting at 6*d.* per sq. ft.

13. Draw a rectangle and fold it so as to represent the four walls of a room 15 ft. by 12 ft. and 10 ft. high, representing 1 ft. by 1 mm. What would be the cost of painting its four walls at 6*d.* per sq. ft.?

14. Draw, cut out and fold a representation of the sides and base of a box 1 ft. long, 1 ft. broad and 1 ft. high (scale 1 ft. represented by 1 in.). What is the area of the inside surface?

15. Draw, cut out and fold a representation of a box (sides and base) 2 ft. long, 6 in. broad and 1 ft. high ; find the area of its inner surface and the cost of lining it with sheet lead at 9*d.* per sq. ft. (Represent a distance of 1 ft. by a line 1 in. long.)

16. Draw, cut out and fold paper so as to form a triangular prism (Fig. 16) 4 inches long, and the ends equilateral triangles of one inch side ; find its surface exclusive of the triangular ends.

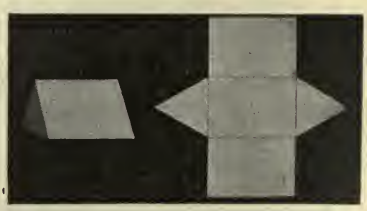


Fig. 16.—A Triangular Prism.

17. Draw a representation of the four walls of a room 16 ft. long, 14 ft. broad, 10 ft. high ; mark on it a frieze of 1 ft. (that is, a band

1 ft. wide from the ceiling round the room), also a window in the centre of one of the long side walls, a door at the end of that wall, and a fireplace in the opposite wall, the dimensions being :

Window, 8 ft. high and 6 ft. broad.
Door, 8 ft. high and 4 ft. broad.
Fireplace, 5 ft. high and 3 ft. broad.

(Represent 1 ft. by 1 mm., and calculate the remaining area of the walls.)

18. In Ex. 17 mark off paper strips 2 ft. wide suitable for papering. What length of paper would be required?

19. Draw, cut out and fold paper to represent a rectangular prism (Fig. 17) 2 ft. long, 1·5 ft. broad, and 1 ft. thick. (Represent 1 ft. by 1 inch.) If gilded at 3*s.* per sq. ft., what would the cost amount to?

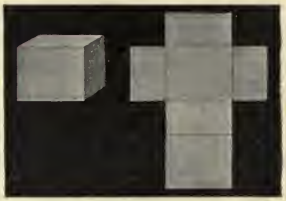


FIG. 17.—A Rectangular Prism.

EXERCISES XXXV. d.

1. Find the total surface of a brick 9 in. long, 4·5 in. broad, 3 in. thick.

2. What is the total surface of a square prism 10 in. long with an end of 2 in. edge?

3. Find the inner surface of a box 4 ft. long, 3 ft. broad, 2 ft. deep.

4. What is the cost of painting the walls of a room 16 ft. long, 15 ft. broad, 11 ft. high, at 1*s.* 3*d.* per sq. ft., making no allowance for windows, doors, etc.

5. What length of paper 21 in. in width will be required to paper a room—length 20 ft., breadth 15 ft., height 12 ft.?

6. If 1 metre is equal to 39·37 in., find (true to 4 figures) how many (*a*) sq. in., (*b*) sq. ft., (*c*) sq. yd. there are in a sq. metre.

7. Express a chain in metres, etc., to the nearest centimetre; hence, find out how many square metres there are in an acre. State also the number of *ares*, etc.

8. A man buys a plot of ground of 3 ro. 28 sq. po.; on it he intends building a house with a frontage of 50 ft. and a depth of 60 ft. What area will he have left for his garden?

9. A yard 60 yds. square is surrounded by a paved walk 3 ft. in width. If the walk be paved with flags 18 in. square, how many flags are there?

10. A room is 19 ft. long, 17 ft. broad. If the floor is to be carpeted so as to leave a margin round the room 15 in. wide, what area of carpet will be required, and what will be the extent of the surrounding margin?

11. A room has an area of 21·84 sq. metres, and its breadth is 4·16 metres; what is its length?

12. A ribbon has a surface of 47·25 sq. dm., and the width is 1·5 cm.; what is the length of the ribbon?

13. How many sq. ft. could be covered by a piece of carpet 27 in. wide and 25 yds. long?

14. A table is 5 ft. long and 2 ft. 6 in. broad; the table cloth is 8 ft. long and 5 ft. broad. What area of cloth hangs over the side of the table?

15. Find what length of plain paper, 21 inches wide, will be required to paper the rooms described below. Make no allowance for doors, windows, and so on.

(*a*)	Length,	20 ft.,	breadth,	14 ft. 6 in.,	height,	10 ft. 6 in.
(*b*)	„	18 ft. 9 in.,	„	12 ft. 9 in.,	„	9 ft. 6 in.
(*c*)	„	16 ft. 6 in.,	„	11 ft. 6 in.,	„	9 ft. 6 in.

16. The four walls of a room have a total area of 660 sq. ft.; the area of the floor is 270 sq. ft.; the width of the floor is 15 ft. What is the height of the room?

Measurement of Irregular Areas.

Irregular areas can be calculated by representing the areas to scale on squared paper and counting the number of squares covered by the figure.

EXAMPLE.—*To determine the area of the county of Norfolk.*

Each small square, in Fig. 18, represents an area of 9 sq. miles.

In estimating the portions of squares, parts less than half a square are neglected, and portions greater than half a square are counted as whole squares.

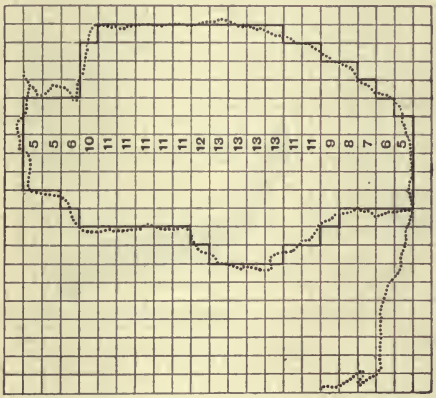


FIG. 18.—Map of Norfolk. (Scale 30 miles to the inch.)

Thus, in estimating the area, we practically work as if the area was bounded by straight lines such as those shown in Fig. 18. On counting then the number of squares included in the county, we get

$$5+5+6+10+11+11+11+11+11+12+13+13+13+13+11+11+9+8+7+6+5=202,$$

and the area of Norfolk is thus found to be

$$9 \times 202 \text{ or } 1818 \text{ sq. miles.}$$

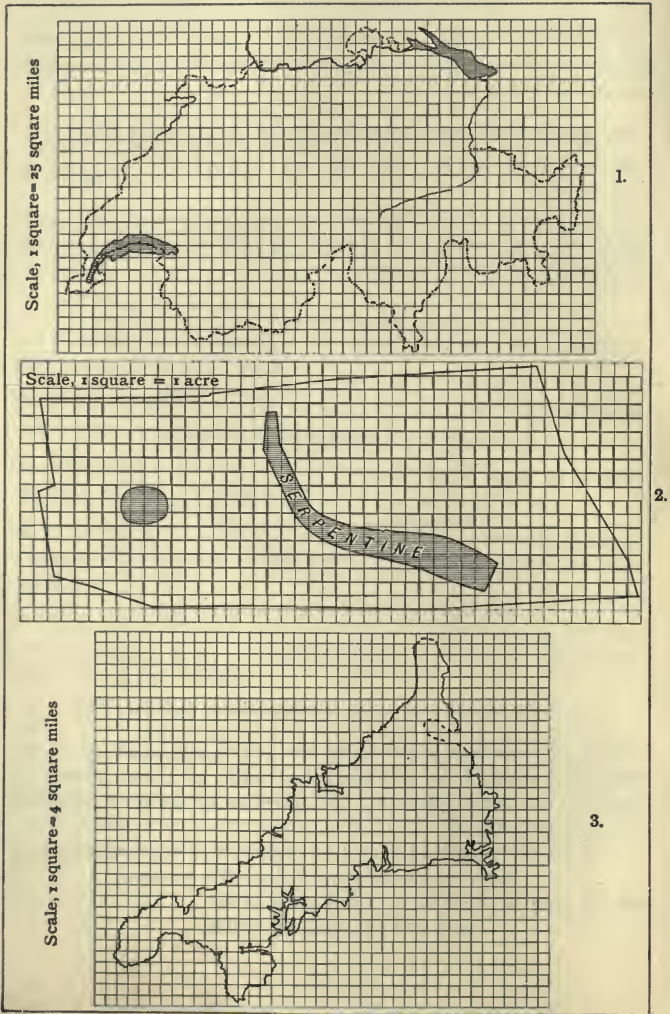
EXERCISES XXXVI.

Practical.

Apparatus: Squared paper, tracing paper.

Find the following areas:

1. Switzerland. **2.** Hyde Park. **3.** Cornwall.



4. Isle of Man.
5. Isle of Wight.
6. Wigtown.
7. Anglesea.
8. Arran.

Scale, 1 square represents 1 square mile

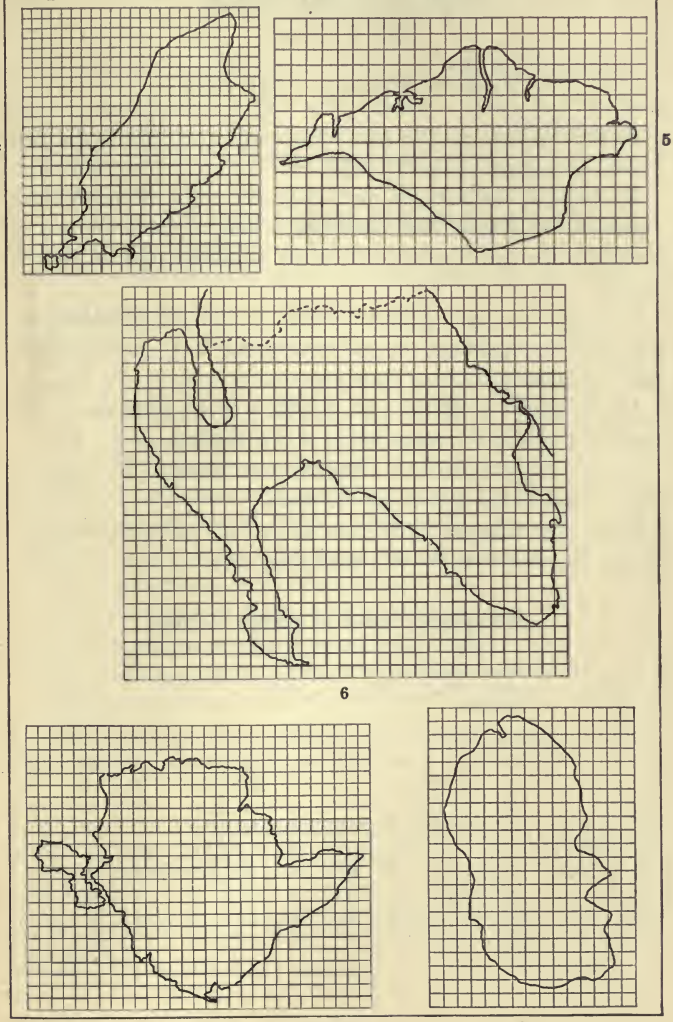


FIG 20

27. DECIMAL COINAGE.

In the majority of the more important countries a decimal system of coinage is used. Thus:

In France, Switzerland and Belgium, sums of money are expressed in **francs** and **centimes**, where 1 franc (fr.) = 100 centimes, the franc being equivalent to about $9\frac{1}{2}d.$ of British money.

In Italy francs are replaced by **lires**, centimes by **centesimos**; and, in Greece, **drachmas** and **leptas** take the place of francs and centimes.

In Germany 1 **krone** = 10 **marks** = 1000 **pfennigs**. The mark is of nearly the same value as our shilling.

In the United States 1 **dollar** = 100 **cents**. The cent is about equal to the British $\frac{1}{2}d.$ in value.

Some approximate equivalents of foreign coinage in British money are shown in the accompanying table:

Approximate Values of Foreign Coinage.

England.			France, Switzerland, Belgium.		Italy.		Greece.		United States.		Germany.	
£.	s.	d.	Frs.	Cts.	Lrs.	Cts.	Drs.	Lpts.	Dols.	Cts.	Mks.	Pfgs.
0	0	1	0	10	0	10	0	10	0	2	0	8
0	0	3	0	31	0	31	0	31	0	6	0	25
0	0	6	0	62	0	62	0	62	0	12	0	50
0	0	$9\frac{1}{2}$	1	0	1	0	1	0	0	19	0	79
0	1	0	1	25	1	25	1	25	0	25	1	0
0	4	0	5	0	5	0	5	0	1	0	4	0
0	10	0	12	50	12	50	12	50	2	50	10	0
1	0	0	25	0	25	0	25	0	5	0	20	0

EXERCISES XXXVII. a.

Mental or Oral.

1. Express as francs and centimes the following amounts:

(*a*) 2320 centimes. (*b*) 19360 centimes. (*c*) 8370 centimes.
(*d*) 9·36 francs. (*e*) 1945·3 francs. (*f*) 876·2 francs.
(*g*) 897·60 francs. (*h*) 1·7623 francs.

2. Express

(*a*) 6·3 lires in terms of lires and centesimos.
(*b*) 783 centesimos „ „ „
(*c*) 9·3 dollars in terms of dollars and cents.
(*d*) 3·756 dollars „ „ „
(*e*) 821 cents „ „ „
(*f*) 35680 leptas in terms of drachmas and leptas.
(*g*) 93213 pfennigs in terms of marks and pfennigs.
(*h*) 1·32 krones „ „ „

3. Find the cost of

(*a*) (i) 10 articles at 15 cents each.
(ii) 20 articles at 13 cents each.
(iii) 100 articles at 1 dollar 3 cents each.

(*b*) (i) 100 articles at 1 franc 50 centimes each.
(ii) 400 articles at 30 centimes each.
(iii) 1000 articles at 10 centimes each.

(*c*) (i) 8 articles at 1 drachma 40 leptas each.

(*d*) (i) 120 articles at 15 lires 20 centesimos each.
(ii) 100 at 3 lires 20 centesimos each.

(*e*) (i) 500 articles at 1 mark 3 pfennigs each.
(ii) 25 at 4 marks 6 pfennigs each.

EXERCISES XXXVII. b.

1. If 35 metres of cloth cost 26 fr. 60 c., what is the cost per metre?

2. If 89 metres of cloth cost 418 fr. 30 c., what is the cost per metre?

3. What is the cost of 23 metres of silk at 5 fr. 70 c. per metre?

4. What is the cost of 31 metres of cloth at 2 lires 90 centesimos per metre?

5. Find the cost of 320 lb. of metal at 96 cents per lb.

6. Find the cost of 81 kilo. of a substance costing 29 marks 6 pf. per kilo.

7. 3·05 kilo. of a substance cost 231 marks 8 pf.; what is the cost per kilo.?

8. Find the cost of 63 yd. of cloth worth 19 doll. 5 c. per yd.

9. Find the cost of 7560 kilo. of a substance at 6 fr. 40 c. per kilo.

10. 693 lb. of ore are worth 9826 doll. 74 c.; what is the value per lb.?

11. If 5150 fr. 80 c. be the cost of 15·8 kilo. of a substance, what is the cost per kilo.?

12. Find the cost of 1·912 kilo. of a substance at 53 lires 4 c. per gram.

13. Find the cost of 319 yds. of cloth at 10 doll. 35 c. per yd.?

14. 3194·5 metres of silk cost 24278 fr. 20 c.; what is the cost per metre?

15. 2299 krones 1 mark 40 pfennigs is the cost of 1446 kilo. of an ore; what is the cost per kilo.?

16. Find the value of 14·12 kilo. of copper at 5·43 krones per kilo.

17. Find the value per metre of cloth, when a piece 737 metres long is worth 5011 drachmas 60 leptas.

28. VOLUMES.

The volume of a block, each edge of which is one inch long, is defined as a **cubic inch**; similarly the volume of a block, each edge of which is one centimetre long, is defined as a **cubic centimetre** (c.c.).

EXERCISES XXXVIII. a.

Mental or Oral.

1. On each inch square, into which a rectangle $6'' \times 4''$ can be divided, a cube of one inch side is placed. What is the volume in cubic inches of the solid formed by the cubes? What also are the dimensions—length, breadth and thickness—of the solid?

2. Give the answers to Exercise 1 if the rectangle be replaced by a rectangle of sides:
(*a*) $4'' \times 3''$, (*b*) $3'' \times 2''$, (*c*) $8'' \times 5''$, (*d*) $36'' \times 12''$, (*e*) $12'' \times 12''$.

3. If on each cube in Exercises 1 and 2 another cube be placed, write down the dimensions and volume of the new solid formed.

4. If in Exercise 2 there be seven of the one cubic inch blocks, one above the other, placed on each square inch of the rectangle, what will be the dimensions and volume of the solid formed?

5. Write down the volumes of the following blocks:

(*a*)	Length	8 in.,	breadth	6 in.,	thickness	4 in.
(*b*)	„	6 in.,	„	4 in.,	„	3 in.
(*c*)	„	12 metres,	„	8 metres,	„	6 metres.
(*d*)	„	15 ft.,	„	8 ft.,	„	3 ft.
(*e*)	„	12·5 in.,	„	7·5 in.,	„	4 in.
(*f*)	„	25 cm.,	„	18 cm.,	„	14 cm.
(*g*)	„	x cm.,	„	y cm.,	„	z cm.
(*h*)	„	a inch,	„	b inch,	„	c inch.

6. Find the number of cubic inches in a cubic foot, *i.e.* in a cube of 12 inches edge; also the number of cubic feet in a cubic yard, *i.e.* in a cube of 3 feet edge.

7. Find the number of cubic centimetres in
(*a*) a cubic decimetre, *i.e.* a volume equal to that of a cube of 10 cm. edge;
(*b*) a cubic metre.

Tables of Volume or Cubic Measure.

British.

$12^3 = 1728$ **cubic inches** make 1 **cubic foot.**
$3^3 = 27$ cubic feet „ 1 **cubic yard.**

Metric.

$10^3 = 1000$ **cubic millimetres** make 1 **cubic centimetre,**
$10^3 = 1000$ cubic centimetres „ 1 **cubic decimetre,**
$10^3 = 1000$ cubic decimetres „ 1 **cubic metre.**

A cubic decimetre is known as a **Litre.**
A cubic metre is known as a **Stere.**

EXERCISES XXXVIII. b.

1. A brick wall 54 ft. long, 10 ft. high and 9 in. thick is built of bricks the dimensions of which (inclusive of mortar) are—length 9 in., breadth 4·5 in., thickness 3 in. Find the volume of each brick, also the number of bricks used in the construction of the wall.

2. A room is 18 ft. long, 10 ft. 6 in. high and 15 ft. broad; what is its cubic content?

3. A board is 4 m. long, 22 cm. broad and 2 cm. thick; what is the volume of the board?

4. A box has the following for its inside and outside dimensions (exclusive of cover):

Inside—Length 3 ft. 3 in., *Outside*—Length 3 ft. 5 in.,
breadth 2 ft. 4 in., breadth 2 ft. 6 in.,
depth 2 ft. depth 2 ft. 1 in.

What is the volume of the wood forming the box?

5. What is the thickness of a block of wood, the length of which is 4 ft., the breadth 3 ft. 6 in. and the cubic content 18 cub. ft. 1152 cub. in.?

6. A sheet of iron is 3·24 metres long, 2·58 metres broad, and its volume is 2089·8 cubic centimetres. What is the thickness of the sheet?

7. A wooden box with its cover on, is an exact cube internally of 1 ft. edge; what volume of lead 0·25″ thick is required to line it?

8. A room has a capacity of 3108 cubic feet; its length is 18 feet 6 inches, its height is 12 feet; what is its breadth?

9. A coverless box the internal dimensions of which are 4 ft. in length, 7 in. in breadth and 9 in. in depth, is made from 9 in. planks 1 in. in thickness; what length of plank will be required, also what will be the volume of the planking?

10. A swimming bath is 100 ft. long and 36 ft. broad; when a number of men dive into the bath, the height of the water rises by half an inch. If the average amount of water displaced by one of the men is 2 cub. ft., how many men are there in the bath?

11. A biscuit tin has internal dimensions as follows:

Length 30 cm., breadth 30 cm., height 25 cm.

It is partially filled with water and the depth of the water noted; a glass jar is then pressed into the tin, until the water is just level with the top of the jar; the rise of water in the tin is then found to be 3·2 cm.; on pressing the jar further down, so that the jar becomes full and totally immersed, the level falls by 2 cm. Find the capacity of the jar, also the volume of the glass.

12. A plasterer has 1·26 cubic feet of cement in his bucket; how many square yards of wall can be plastered to a thickness of 0·5 inch with it?

13. The sluice supplying water from a mill pond to a mill wheel is 4 ft. 4 in. broad and 8 in. high. If the water rushes through it at the rate of 5·5 ft. per second, how many cubic feet pass through in one hour?

14. A pontoon is 100 ft. long, 60 ft. broad and 30 ft. deep; it is floated into a convenient position and filled with water; how much water does it contain? If each cubic ft. of water weighs 62·5 lbs., how much lighter does the pontoon become when the water is pumped out of it?

15. A cistern is 5′ long by 4′ broad by 2′ high, and is full of water. What will be the fall in the level of the water in the cistern if 1 quart is run out of it, assuming that 25 quarts make approximately 1 cubic foot?

16. A room is 15 ft. long, 12 ft. broad and 10 ft. 6 in. high. What is its content in cubic yards?

17. A portion of a seam of coal is 240 yards long, 126 yds. broad and 4 ft. 6 in. in thickness. What weight of coal could be obtained from it, assuming that, roughly, each cubic yd. in the seam yields 1 ton at the surface?

18. A stone cross is made from a slab of stone of which the section is a square of 6 in. side, the height of the cross is 5 ft., the breadth or span 3 ft. What is the total volume of the cross?

Measurements of Capacity.

Many systems for measuring liquids exist—for example: In measuring ordinary liquids,*

4 **gills** = 1 **pint,**
2 pints = 1 **quart,**
4 quarts = 1 **gallon.**

In medicine,

60 minims (m.) = 1 drachm [f ℥],
8 drachms = 1 fluid ounce [f ℥],
20 fluid ounces = 1 pint.

* More complete tables are given at p. 138.

EXERCISES* XXXIX. a.

Practical.

[*Apparatus: Burette. Medicine bottle. Various rectangular tin boxes. Glass tumbler. Metal blocks. Scales. Medicine measure graduated. Pint mug.*]

1. Measure in inches the length, breadth and depth of the given tin box; calculate its cubic content in cubic inches (allowing for the thickness of the material of which the box is made); find by a burette the number of cubic centimetres the box holds. Hence calculate the number of cubic centimetres equivalent to 1 cubic inch, also the value of 1 c.c. as a decimal of a cubic inch.

2. Repeat Experiment 1 with other tin boxes.

3. Measure in inches the length, breadth and thickness of a given metal block and calculate its volume. Tie a fine piece of strong thread to the block and hang it in a glass tumbler, pour water into the tumbler up to a convenient height (this must be sufficient to cover the block). Withdraw the block, removing as little water as possible. Fill the glass to the same height as before by the burette, noting the volume in c.c. of the water run into the tumbler. You now have the volume of the block in cubic inches and in cubic centimetres. Calculate the number of cubic centimetres in a cubic inch, also the number of cubic inches in a cubic centimetre.

4. Measure the dimensions of a given block in cm. and in inches, calculate the volume in c.c. and in cubic inches, and find the number of cubic centimetres in one cubic inch.

5. Repeat Experiment 3, using however a graduated medicine measure instead of a burette graduated in c.c. Hence find the number of fluid ounces in a cubic inch, also the number of cubic inches in a fluid ounce.

*NOTE.—It may be convenient for many of these experiments to be performed by the teacher only.

6. How many fluid ozs. will fill a
(i) tea spoon, (ii) dessert spoon, (iii) table spoon?

7. Find the number of cubic centimetres in the given half-pint measure.

The table stated on page 136 may be extended. Some of the more common measures are the following :

LIQUID MEASURES.

4 Gills = 1 Pint (Pt.).
2 Pints = 1 Quart (Qt.).
4 Quarts = 1 Gallon (Gal.).
$4\frac{1}{2}$ Gallons = 1 Pin.
9 Gallons = 1 Firkin.
36 Gallons = 1 Barrel.
54 Gallons = 1 Hogshead.

The gill, pint, quart and gallon are the only measures which can be regarded as standard, the others varying in different cases according to the custom of the particular trade using the measure.

Thus 1 Hogshead of Ale is 54 gallons.
1 ,, Claret ,, 46 ,,
1 ,, Port ,, 57 ,,

DRY MEASURE.

Employed in measuring grain and corn.

2 Pints = 1 Quart.
2 Quarts = 1 Pottle.
4 Quarts = 1 Gallon.
2 Gallons = 1 Peck.
4 Pecks = 1 Bushel.
8 Bushels = 1 Quarter
5 Quarters = 1 Load.
2 Loads = 1 Last.

Many kinds of cereals are however usually sold by weight, though nominally in bushels.

29. PROBLEMS ON MEASURES OF CAPACITY.

EXAMPLE.—*One vessel contains* 3 *gallons* 1 *quart* 1 *pint; another vessel contains* 4 *gallons* 3 *quarts* 1 *pint. What is the total content of the two vessels?*

The process of addition is exactly similar to that employed in the addition of money and addition of length.

gal.	qt.	pt.
3	1	1
4	3	1
8	1	0
	1	1

Adding the pints, we obtain 2 pts. or 1 quart.
Adding the quarts, we obtain $(1+3+1)$ qt., *i.e.* 5 qts. or 1 gal and 1 qt.
Adding the gallons, we obtain 8 gal.

The content is 8 gal. 1 qt.

EXAMPLE.—*Express* 16 *gal.* 3 *qt.* 1 *pt. as gallons and decimals of a gallon.*

16 gal. 3 qt. 1 pt.
= 16 gal. 3·5 qt.
= 4·875 gal.

Divide the number of pints by 2, to express them in quarts.
Divide the number of quarts by 4, to express them in gallons.

EXAMPLE.—*A cask contains* 53 *gal.* 1 *qt.* 1 *pt. of ale. If this be divided equally among seven people, what does each person receive?*

Here we have only to divide 53 gal. 1 qt. 1 pt. by 7. The method again is similar to that used in Division of Money.

	gal.	qt.	pt.
7	53	1	1
	7	2	1 Ans.

53 gal divided by 7 yields 7 gal. and 4 gal. over.
4 gal. and 1 qt. = 17 qt.; dividing by 7, we get 2 qt. and 3 qt. over.
3 qts and 1 pt. = 7 pt.; dividing by 7, we obtain 1 pt. exactly.

EXERCISES XXXIX. b.

1. A gallon of wine will fill six ordinary wine bottles; how many bottles of wine could be filled from a vat of wine containing 6 hogsheads 4 gal. 2 qts.? What would be left over, if any? (The hogshead of wine is equal to 57 gallons.)

2. Four and a half quarts of ale are run out of a nine-gallon cask; how much ale remains in the cask?

3. If a gill contains 8·665 cubic inches, how many cubic feet and cubic inches are there in 6·25 gallons?

N.B.—It will be seen from this exercise that 25 gallons are very nearly the same as 4 cubic feet.

4. A cistern is 3 ft. long, 2 ft. broad and contains 125 gallons of water; what is the depth of the water within the cistern? (Assume 1 cubic foot = 6·25 gallons approximately.)

5. A decanter for holding spirits has for its inside section a square of 3 inch side; a bottleful of whisky is poured into it. What is the height of the surface of the whisky above the bottom of the decanter? (Six bottles to the gallon.)

6. If on a certain day the rainfall is 0·8 of an inch, how many gallons of water will fall on a field of 4 acres? (Assume 1 cub. ft. = 6·25 galls.)

7. What is the volume of a cube of 3·9 inches edge?

Since a litre or cubic decimetre is the volume of a cube of 3·937 inches, the answer will be a little less than a litre.

8. Assuming that 1 litre = 61 cubic inches, how many cubic inches will there be in a rectangle 4 cm. long, 5·6 cm. broad, 3·7 cm. deep?

N.B.—More accurately, 1 litre = 61·027 cubic inches.

9. If a litre is 61·02 cubic inches and 1 cubic foot is 6·25 gallons, how many pint mugs could be filled from a vessel containing 4 litres? How much would remain over?

Note therefore that approximately 7 pints = 4 litres.

10. In making up a medicine, 10 fluid ounces (f℥) 4 drachms (fℨ) 24 minims (m.) of one liquid are added to 15 fluid ounces (f℥) 7 drachms (fℨ) 49 minims (m.) of a second liquid. How much liquid is there altogether?

11. Express 0·8645 quart in pints, fluid ounces, drachms and minims.

12. Add together the following:

7 f℥ 3 fℨ 30 m.; 8 f℥ 4 fℨ 50 m.; 12 f℥ 3 fℨ 25 m.

13. A medicine is to be made up of

14 f℥ 6 fℨ of liquid A,
8 f℥ 3 fℨ of liquid B,
12 f℥ 4 fℨ of liquid C.

What would be the equivalent amounts to be taken by a dispenser who only had measures graduated according to the metric system?

14. The internal section of a medicine bottle is a rectangle 2 inches long and 1 inch broad; what is the distance in inches between the ounce marks on the bottle?

30. MEASUREMENT OF WEIGHT.

In England many systems of measurement of weight are in use. **Avoirdupois** weight is the most general. **Troy** weight is used in dealing with gold and silver. **Apothecaries'** weight is used in dispensing medicines.

On the Continent the system is that in which the unit is the weight of 1 cubic centimetre of water at a temperature of 4° C. and is called a **gram**; the system is a decimal one.

The weights employed in the different systems are as follows:

AVOIRDUPOIS WEIGHT.

16 **drams** = 1 **ounce** (oz.).
16 oz. = 1 **pound** (lb.).
14 lb. = 1 **stone** (st.).
28 lb. = 2 st. = 1 **quarter** (qr.).
112 lb. = 4 qr. = 1 **hundredweight** (cwt.).
2240 lb. = 20 cwt. = 1 **ton**.

TROY WEIGHT.

24 grains = 1 pennyweight (**dwt.**).
480 gr. = 20 dwt. = 1 ounce (oz.).
5760 gr. = 12 oz. = 1 pound.
100 lb. = 1 hundredweight.
(7000 grains troy = 1 lb. avoirdupois.)

APOTHECARIES' WEIGHT.

20 grains. = 1 scruple (℈).
3 ℈ = 1 drachm (ʒ).
8 ʒ = 1 ounce (ʒ). } These are practically
5760 gr. = 12 ʒ = 1 pound (lb.). } obsolete.

N.B.—The grain is the same weight in all three systems.

METRIC SYSTEM.

1000 milligrams = 100 centigrams = 1 decigram = 1 **gram** (gr.).
1000 grams = 1 **kilogram** (kilo.).

The gram is the weight of one cubic centimetre of water at 4° C.

In the Addition, Subtraction, Multiplication, Division and Reduction of weights the procedure is exactly similar to that employed in dealing with questions on money, length, area and volume.

EXAMPLE.—*How many pennies, three of which weigh* 1 *oz. avoir., can be made from a mass of* 2 *tons* 3 *cwt.* 1 *qr.* 4 *lb. of Bronze?*

tons.	cwt.	qr.	st.	lb.	oz.	
2	3	1	0	4	0	
20						Reduce the 2 tons to cwt. by multiplying by 20; add on the 3 cwt.
43						
4						Multiply by 4 to reduce from cwt. to qr. and add the 1 qr.
173						
2						Multiply by 2 to reduce from qr. to st.
346						
14						Multiply by 14 to reduce from st. to lb., and add the 4 lb.
346						
1384						
4						
4848						
16						Multiply by 16 to reduce from lb. to oz.
4848						
29088						
77568						

77568 oz. = 232704 pennies. Multiply by 3, since 3 pennies weigh 1 oz.

EXERCISES XL. a.

Practical.

[*Apparatus: Balance, pennies, halfpennies, tumblers, quart mugs, medicine bottles, tin biscuit boxes, pieces of wire of various lengths, pieces of sheet metal, cubes of metal of one inch side.*]

1. Find the weight in ounces, of 1 penny, 4 pennies, 4 halfpennies, 8 pennies.

2. Weigh the given objects.

3. Weigh the given half-pint glass, first when empty, secondly when full of water.

Calculate from your results the weight of a gallon of water.

4. As in (3), find the weight of water in the given quart mug, and again calculate the weight of a gallon of water.

5. Weigh the given bottle and calculate its capacity, assuming from the result of (3) and (4) the weight of a gallon of water.

6. Find the weight of the given medicine bottle (i) empty; (ii) containing (*a*) 2 f℥, (*b*) 4 f℥, (*c*) 6 f℥ of water.

7. Find the internal volumes in cubic inches of the given rectangular tin boxes, also the weights of water they contain when full. Calculate from your results the weight of a cubic foot of water.

8. Assuming the weight of a cubic foot of water, find the capacity of each of the given bottles.

9. Find the number of ounces avoirdupois in the given pound weight troy.

10. Find the weight of a litre of water in (*a*) grams, (*b*) ounces.

11. Express the weight of 1 lb. avoirdupois in the decimal system.

12. Find by means of the balance the volumes in c.c. of the given vessel.

13. Express in ounces avoirdupois the given kilogram weight.

14. You are given a coil of wire, also one foot length of it; weigh the two and calculate the length of the wire in the given coil.

15. You are given a rectangular sheet of metal one square inch in area, also an irregularly shaped piece of the same material and thickness; find the weight of both and calculate the area of the given irregularly shaped piece.

16. Weigh the given solid, also the given inch cube of the same material; from your results calculate the volume of the given solid.

EXERCISES XL. b.

1. A train consists of eight trucks of coal, the net weights being respectively: 8 ton 3 cwt. 2 qr., 9 ton 1 qr., 7 ton 15 cwt. 3 qr., 8 ton 10 cwt. 1 qr., 8 ton 11 cwt. 2 qr., 8 ton 4 cwt., 9 ton 1 cwt. 3 qr., 7 ton 19 cwt. 2 qr. What is the total net weight of the train?

2. The gross weight of a truck and load is 12 ton 3 cwt. 2 qr.; the truck itself weighs 4 ton 10 cwt. 3 qr. What is the weight of the load the truck carries?

3. A barrel of fish contains 3 qr. 15 lb. 6 oz.; if the barrel itself weighs 5 lb. 12 oz., what is the weight of the fish?

4. Reduce to lb.:

(*a*) 4 cwt. 34 lb., (*b*) 17 cwt. 3 qr., (*c*) 19 cwt. 45 lb.,
(*d*) 14 ton 2 qr. 17 lb., (*e*) 17 qr. 3 lb.,
(*f*) 793 ton 14 cwt. 2 qr., (*g*) 19 ton 1041 lb.

Find also the sum of these weights.

5. Convert (*a*) 1345 oz. into lb. (avoirdupois), (*b*) 7056 lb. into cwt., (*c*) 19043 lb. into tons, etc., (*d*) 1593 oz. into qr., (*e*) 793628 drams into cwt., (*f*) 6932 qr. into tons.

6. If 1 cubic foot of sea water weigh 64 lb., what will be the weight in tons, cwt., qr., of 35 cubic feet?

7. What volume of sea water has a weight of 3000 tons 4 cwt.? (1 cubic foot of sea water weighs 64 lb.)

8. When a body floats in water, it displaces a volume of water equal to its own weight; what will be the volume of sea water displaced by a ship of 2850 tons?

9. What would be the volume of water displaced by the ship in Ex. 8, if it were in fresh water, and if 1 cubic foot of fresh water were supposed to weigh 1000 oz.?

10. If a cubic foot of common brick work weigh 1 cwt., what is the weight of a brick wall 60 ft. long, 8 ft. 6 in. high and 9 in. thick?

11. If a Welsh fire brick crushes when it is pressed with a force per square inch equal to the weight of 7 cwt. 1 qr., what is the greatest height a brick column could be built, without crushing under the action of its own weight? (Assume each cubic inch of brick weighs 1·375 oz. and state the answer true to 0·01 ft.

12. If 1 cubic foot of fresh water weigh 1000 oz., what will be the weight in tons, etc., of the water which fills a cistern 18 ft. long, 7 ft. 6 in. deep, 15 ft. 3 in. broad inside?

13. A plate of iron is 64 cm. long, 57 cm. broad and weighs 11031·552 grams. If each cubic centimetre of iron weighs 7·2 grams, what is the thickness of the plate?

14. How many kilogram weights could be made from 2 qr. 3 lb. 4 oz. of brass; what weight of brass will remain over? (Assume 1 kilo. = 2·205 lb.)

15. A litre of water weighs 1 kilogram and a gallon of water weighs 10 lb.; how many litre measures could be run out of a 9-gallon cask, and what will then be left in the cask?

16. The wire on a reel weighs 13 cwt. 22 lb. What is its length if the weight of a foot is known to be 5·6252 oz.?

17. Assuming that approximately 1 kilo. is equal to 2·2 lb., express in French and English measures the following:

(*a*) 6 tons 12 cwt., (*b*) 1000 kilo., (*c*) 4 cwt. 3 qr. 7 lb.,
(*d*) 173 kilo. 500 grams, (*e*) 1·54 kilo.

18. A cart contains 15 sacks of flour each containing 2 cwt. 2 qr. 3 lb. 2 oz.; how much flour is there altogether in the cart?

19. Evaluate the following: 1 ton 14 cwt. 3 qr. 10 lb. 4 oz. multiplied by (*a*) 5, (*b*) 8, (*c*) 16, (*d*) 28, (*e*) 50, (*f*) 210, (*g*) 325.

20. A mass of iron weighing 37 tons 4 cwt. 3 qr. is divided into 24 equal parts; what will each of these parts weigh?

21. What is the weight per foot length of brass tubing such that a piece 124 ft. long weighs 5 cwt. 2 qr. 11 lb. 12 oz.?

22. Divide 706 tons 3 cwt. 2 qr. 10 lb. 4 oz. by (*a*) 8, (*b*) 11, (*c*) 35, (*d*) 89, (*e*) 206, (*f*) 370.

23. Express the following weights (avoirdupois) in Troy units, remembering that in 1 lb. Troy there are 5760 grains, and 7000 grains in 1 lb. avoirdupois :

(*a*) 3 lb. 2 oz., (*b*) 4 lb. 6 oz. (*c*) 1 qr. 0 lb. 8 oz.,
(*d*) 8 lb. 4 oz., (*e*) 1 stone 10 oz., (*f*) 10 lb. 12 oz.

24. Express the following weights (Troy) in avoirdupois measure. Express your answer in lbs., oz., drs. and decimals of a dram (true to 2nd decimal place) :

(*a*) 16 dwt. 1 gr., (*b*) 1 lb. 1 oz. 14 dwt. 4 gr.,
(*c*) 19 lb. 7 oz. 1 dwt. 16 gr., (*d*) 59 lb. 7 oz. 5 dwt. 23 gr.

31. MEASUREMENT OF TIME.

The chief divisions of time are :

60 **seconds** (60″) = 1 **minute**,
60 minutes (60′) = 1 **hour**,
24 hours = 1 **day**,
7 days = 1 **week**,
28 days or 4 weeks = 1 **lunar month**,
365 days = 1 ordinary or civil year,
366 days = 1 leap year.

EXERCISES XLI.

1. The earth makes a complete spin on its axis in 23 hr. 56 min. 4 sec. Find (*a*) the difference between this and the ordinary day, (*b*) what the difference amounts to in 365 days.

2. The time the earth takes in moving in its orbit once round the sun is 1 tropical year or 365·2422164 days. Express this in days, hours, minutes and seconds.

3. Express as decimals of an ordinary year the following times :

Spring - 92 days 20 hours 59 minutes.
Summer - 93 „ 14 „ 13 „
Autumn - 89 „ 18 „ 35 „
Winter - 89 „ 0 „ 2 „

State the answer to 4 decimal places.

4. Find the difference in hours and minutes between 3 ordinary years of 365 days together with 1 leap year; and 4 tropical years.

5. What is the difference between 25 groups, each consisting of 3 ordinary and 1 leap year; and 100 tropical years?

6. If a lunation be 29·5306 days and a tropical year 365·242216 days, find the difference between 19 tropical years and 235 lunations.

7. Express a lunation in days, hours and minutes.

8. The following table gives (*a*) the periods of some of the planets about the sun, also (*b*) their distances from the sun in terms of the earth's distance:

Planet.	Mean distance from the Sun in terms of the Earth's distance.	Period in Days.
(*a*) Mercury -	0·3871	87·969
(*b*) Venus -	0·7233	224·700
(*c*) **Earth** -	1·0000	365·256
(*d*) Mars -	1·5237	686·980
(*e*) Jupiter -	5·2028	4332·585
(*f*) Saturn -	9·5389	10759·220
(*g*) Neptune -	30·0370	60126·720

Express the periods in sidereal years true to 4 significant figures, and show that in these units the squares of the periods are the same as the cubes of the distances.

(A sidereal year is 365·256 days.)

9. From the following table express the duration of the longest days in terms of the duration of the corresponding shortest day. State the answer true to 3 significant figures.

	Latitude.	Longest Day.		Shortest Day.	
		hrs.	min.	hrs.	min.
(*a*)	0°	12	0	12	0
(*b*)	10°	12	35	11	25
(*c*)	20°	13	13	10	47
(*d*)	30°	13	56	10	4
(*e*)	40°	14	51	9	9
(*f*)	50°	16	9	7	51
(*g*)	60°	18	30	5	30

EXERCISES XLII.

Miscellaneous.

1. Write in figures two and seven tenths, 70 and nine hundredths, 50 and forty-six thousandths, and 123 millionths. Find the sum of the four quantities and subtract it from 200.

2. Four different bottles contain respectively 2·7, 1·65, 5·13 and 0·5 litres of wine. How many litres are there altogether?

3. Nine clocks of equal value are sold for £7. 17*s.* 6*d.* What is the cost of one?

4. How many grams are there in 2 kilograms 50 grams?

5. How many gallons are there in 71 pints and how many quarts and pints over?

6. From a caddy containing 7·25 oz. of tea 1·5 oz. are taken. How many ounces are left in the caddy?

7. If the weight of a packet of cartridges is 1 lb. 7 oz., what is the weight of 7 packets?

8. A room is 10 ft. high, 18 ft. 6 in. long and 12 ft. wide; how many cubic feet of air will it contain?

9. Find the area (in square inches) of a sheet of paper 9·2 inches long and 6·5 inches wide.

10. How many copies of a book costing £0·25 can be bought with £8?

---

11. Add together 0·375 of 13*s.* 4*d.* and 0·07 of £2. 10*s.*, and subtract the result from 0·45 of £1.

12. Find the rent of 42·75 acres of land at £1·255 per acre. Give the answer to the nearest penny.

13. Multiply 52·08631 by 0·38056. Give the product in full, and then give it true to the nearest thousandth.

14. How many times is 0·006552 contained in 155806·56? Give your answer in words.

15. Find the cost of carpeting a room 20 ft. 3 in. long and 13 ft. 4 in. wide, with a carpet 2 feet 3 inches wide at 5*s.* a yard.

16. If 3·2 pounds of sugar are taken from a bag containing 7 pounds, how much is left in the bag?

17. Ten trucks of coal have a gross weight of 168 tons 15 cwt., and the trucks themselves weigh 39 tons 18 cwt. 2 qrs. How many sacks of coal, each containing 1 cwt. 1 qr., can be filled from the trucks, and what amount of coal remains?

18. From the given tables find the average (*a*) rent and (*b*) population per acre in the following Welsh counties—rent to nearest 1*d*., population to four significant figures:

(N.B.—Take the average rent in this case as the total rent divided by the total extent in acres, and similarly for the population.)

Counties.	No. of Acres.	Population.	Gross Rental.
Anglesey -	120,417	34,808	£161,356
Brecon - -	473,087	53,951	365,137
Cardigan - -	595,285	82,707	414,856
Carmarthen -	464,587	123,570	698,284
Carnarvon -	322,742	137,236	809,253
Denbigh - -	378,309	126,458	773,656
Flint - -	106,878	60,536	377,113
Glamorgan -	576,537	866,250	7,190,934
Merioneth -	523,708	64,248	351,205
Montgomery -	591,973	63,994	514,202
Pembroke -	357,118	82,424	457,830
Radnor - -	238,660	20,241	143,561

19. Add together 0·023 of a £, 0·946 of a shilling and 3·48 pence, and subtract the sum from 0·26 of a guinea. Give the answer in pence and the decimal of a penny.

20. (*a*) Find the value of 97·96 tons of copper at £73·75 per ton. Also (*b*) find in lbs. and decimals of a lb. the weight of 1 bushel of corn when 32 bushels weigh 11 cwt. 2 qr. 16 lb.

21. The edges of a carpet in a room 25 ft. 6 in. long and 18 ft. 4 in. broad are 1 ft. 3 in. from the walls. Find the area of the carpet, in sq. yards, feet and inches.

22. How many times can a jug holding 0·078125 gal. be filled from a vessel containing 786·375 gal.? What part of a pint remains over?

23. A cistern is 6 ft. long and 5 ft. broad, and it is of such a depth that it would take 1440 bricks, each 9 inches long, 3 inches deep and 4 inches broad, to fill it: how many gallons would the cistern hold, if a pint of water contains 72 cubic inches?

24. Complete the following table by evaluating the average return for each county of the returns under each of the given heads.

Most Criminal Counties.

Counties.	Summary Jurisdiction.			Ignorance per cent. signing with marks.	Height of Adult persons. Inches.
	Committals per 1000.	Lawlessness per 1000.	Drunkenness per 1000.		
	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Hereford	0·97	21·78	5·00	14·9	66·45
Middlesex	0·93	20·72	6·76	10·0	66·69
Lancaster	0·86	27·03	13·29	20·7	67·49
Warwick	0·68	21·72	4·88	19·7	67·12
Worcester	0·65	17·89	5·26	18·8	67·22
Bedford	0·63	11·32	2·57	25·8	67·07
Monmouth	0·61	21·70	6·60	30·1	66·45
Cumberland	0·53	16·08	9·33	16·3	68·48
Gloucester	0·53	18·00	3·97	14·0	66·31
Norfolk	0·53	11·03	2·37	17·9	68·34
Northampton	0·56	11·80	2·90	14·5	67·26
Stafford	0·56	25·35	6·47	28·0	67·82
Average =	.....	.....	.....	.....	.....

Least Criminal Counties.

	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
Cornwall	0·21	6·37	1·63	19·7	67·94
Huntington	0·26	13·19	1·72	16·3	66·78
Durham	0·31	25·10	3·50	22·5	67·70
Cambridge	0·31	11·53	1·47	17·3	66·78
Devon	0·27	11·65	3·53	10·6	67·28
Derby	0·32	19·23	6·26	25·8	67·80
Nottingham	0·33	22·42	5·50	18·0	67·38
Rutland	0·33	11·85	2·19	9·4	67·29
Suffolk	0·35	2·91	0·43	18·9	67·95
Westmoreland	0·33	15·97	5·34	8·0	68·00
Northumberland	0·34	22·70	16·20	14·5	68·59
Essex	0·36	6·10	1·61	13·2	68·00
Average =	.....	.....	.....	.....	.....

25. If the water in a tank, 8 ft. long 7 ft. wide, is 4 ft. deep, and a cubic foot of water weigh 1000 oz.; what is the weight in tons of the water in the cistern?

---

26. Multiply 0·02475 by 0·64, and divide the product by 0·000125.

27. Multiply £15. 8*s.* 4*d.* by 7·8498, giving the answer in pounds, etc, to the nearest penny.

28. Add together 0·0021 of a cwt., 0·045 of a quarter, 0·37 of a lb., and subtract the sum from 35·263 ounces. Give the answer in ounces and the decimal of an ounce.

29. Below is a table of the speeds (some only approximate) of various animals and agencies:

(*a*) Fill in the missing data in (iii).
(*b*) „ „ (vi).
(*c*) „ „ (ix).
(*d*) „ „ (xi).

(*e*) Find how long the trotting horse in (iv) would take to describe a distance of 14·55 miles.

(*f*) How many times faster the average Atlantic liner is than the average racing yacht? (Assume 1 metre = 3·28 ft., and state the answer to three significant figures.)

	Centimetres per second.	Feet per second.	Kilometres per hour.	Miles per hour.
(i) Snail	0·16	0·005249	0·005759	0·00358
(ii) Hunted fly	970	31·83	34·92	21·7
(iii) Carrier pigeon	1800	...	...	...
(iv) Horse trotting	1301	...	...	...
(v) Race horse	1670	54·79	60·12	37·36
(vi) Man running	...	...	...	14·25
(vii) Racing yacht	...	...	27·80	...
(viii) Atlantic liner	...	...	...	24·41
(ix) Rifle bullet	...	1900	...	...
(x) Cannon ball	91440	3000	3292	2046
(xi) Rivers	...	...	10·80	...
(xii) Average wind	550	18·04	19·80	12·30

30. (*a*) What is the quotient when 61·061 is divided by 0·305? (*b*) How many pieces each 1·19 of an inch can be cut off from a rod which is a yard long, and what is the length of the piece remaining?

31. Find the Barometric Pressure at York on Oct. 5, from the given data: knowing that the average pressure for the 7 days is 29·88.

Barometric Pressure at York, Oct. 1-7 inclusive.

Oct. 1.	Oct. 2.	Oct. 3.	Oct. 4.	Oct. 5.	Oct. 6.	Oct. 7.
29·45	29·60	29·78	29·79	...	30·19	30·20 inches of mercury.

32. In a room 16 ft. 6 in. long by 12 ft. 4 in. broad, a carpet is laid down 10 ft. 8 in. long by 8 ft. 3 in. broad: how many sq. feet of floor are left uncovered?

33. Find, in cubic feet, the content of a room 16 feet long, 10 ft. 6 in. broad and 8 ft. 9 in. high.

---

34. How many lengths of 0·0375 ft. are contained in 31·7296875 ft.? Express the remainder as the decimal of a foot.

35. Find the dividend on £3275. 10*s.* at 14*s.* 3½*d.* in the £.

36. Divide 3·64353 by 0·0671. Explain the working.

37. Find the average monthly rainfall at the two Irish stations below, from the data given:

	Jan.	Feb.	Mar.	Apr.	May.	June.	
(*a*) Galway	5·92	2·39	1·62	1·53	3·02	3·14	inches.
(*b*) Cork	6·09	0·78	0·91	1·34	3·15	4·87	„
	July.	Aug.	Sept.	Oct.	Nov.	Dec.	
(*a*) Galway	0·95	2·60	8·30	5·88	4·88	2·73	inches.
(*b*) Cork	2·28	2·42	5·52	3·79	3·67	3·20	„

38. Find the average temperature during July 1903 at Hull and Oxford, given the average monthly temperatures as below:

1903.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	
Hull	...	58°·0	55°·3	50°·4	43°·3	37°·9	
Oxford	...	58°·7	56°·8	52°·2	44°·1	38°·6	
1904.	Jan.	Feb.	March.	April.	May.	June.	Average for 12 months.
Hull	39°·1	38°·1	40°·2	47°·8	51°·3	55°·4	48°·06
Oxford	39°·4	39°·1	41°·0	49°·3	52°·7	57°·2	49°·2

39. In the accompanying table the rainfalls in inches, for the different months of the year 1874, are stated for different parts of England.

Fill in the missing parts of the table.

	(*a*) Middlesex.	(*b*) Norfolk.	(*c*) Cornwall.	(*d*) Gloucester.	(*e*) Lancashire.	(*f*) Yorkshire, E. Riding.
Jan. -	1·34	0·89	5·62	3·93	3·53	1·05
Feb. -	1·10	0·97	3·86	2·40	1·59	1·33
March -	0·62	0·93	1·31	2·14	3·37	1·18
April -	1·33	1·03	2·34	1·99	1·00	1·01
May -	1·08	1·23	0·80	0·66	1·39	1·56
June -	2·11	2·91	1·80	1·06	1·00	0·57
July -	1·11	1·52	1·53	1·56	1·77	1·48
Aug. -	1·77	1·73	4·08	4·64	4·34	1·98
Sept. -	3·02	...	7·15	7·07	3·86	1·80
Oct. -	3·64	1·30	6·11	3·82	3·76	1·77
Nov. -	2·24	2·35	4·59	2·36	4·85	3·71
Dec. -	1·65	2·46	8·14	3·62	3·64	2·75
Monthly Average	.....	1·75	.....	.....	.....	.....

40. A ship, steaming towards a port in a fog, fires a signal gun which is answered from the port, as soon as heard, by another gun; the report of the latter reaches the ship 25·5 seconds after the first gun was fired; the ship fires again immediately, is answered as before, and this time the reply is heard in 24·5 seconds. If the sound travels 1100 feet per second, at what rate must the ship be steaming?

---

41. A rectangular piece of paper is measured in inches and in centimetres, with the following results:

Length, 8·73 inches; 22·17 centimetres.

Breadth, 5·57 inches; 14·15 centimetres.

Find the area of the paper in square inches and in square centimetres, and hence find the number of square centimetres in a square inch, true to 0·01 sq. in.

42. What is the average depth of a rainfall in which 7 million litres of water fall on a square kilometre?

43. If a cubic foot = 0·028 cubic metre, and if 1 kilogram = 2·2 lb., find the number of ounces in 1 cubic foot of water. (The kilogram is the weight of a litre of water.)

44. A wheel makes 1028 revolutions in passing over 2 miles 4 fur. 9 pls. 5 yds. 6 ins. What is the circumference of the wheel?

45. A gravel walk 6 feet wide runs round a grass-plot 60 feet long and 40 feet wide. If gravel is 3*s.* per cubic yard, find the cost of a layer of gravel on the path 3 inches deep.

46. The table below gives the average daily duration of daylight at Latitude 52° N. for each of the twelve months, with the exception of July.

The mean or average duration for the months January to July inclusive is 12 hrs. 39 min.

(*a*) What is the duration during July? (*b*) What the average duration from July to December inclusive?

DURATION OF DAYLIGHT.

Date.	Latitude 52° N.	Date.	Latitude 52° N.
	hrs. min.		hrs. min.
January -	7 50	August -	15 31
February -	9 5	September	13 40
March -	10 50	October -	11 41
April -	12 54	November	9 40
May -	14 51	December	8 8
June -	16 23		

---

47. When 14 cwt. 1 qr. 18 lb. of sugar are taken away from a ton weight of it, what decimal of a ton remains? (State your answer true to the sixth decimal place.)

48. Find the area of a room 19 ft. 4 in. long and 14 ft. 3 in. broad. What is the cost of a carpet for it, 2 ft. 9 in. wide, at 4*s.* 1½*d.* per yard?

49. Reduce 8 metres 7 decimetres 5 centimetres to yards, feet and inches. [1 metre = 1·0936 yards nearly.]

50. An open tank, measuring on the outside 6 yd. 2 ft. in length, 3 yd. 2 ft. in width and 7 ft. in depth, is made with sides and floor of brickwork 1 ft. thick, and is filled with water. Find the weight of the tank and its contents, it being given that one cubic foot of water weighs 1000 oz., and that brick is one and a half times as heavy as water.

51. Divide (*a*) 8·468 by 0·0292 and (*b*) 0·8 by 3·175; state the answer true to 5 significant figures. What is the value of (*c*) 2·778125 of 6*s.* 8*d.* + 0·75 of 2*s.* 6*d.* − 0·935 of 3*s.* 4*d.*?

52. A piece of carpet 72 yards long and 27 inches wide, costing £12. 12*s.*, is used in covering a room. Find the cost per square yard.

53. The table below shows the difference between clock time and apparent noon (as determined by observations on the sun) at different times of the year:

Find, true to the nearest minute, the average clock time at apparent noon during each half (*i.e.* Jan. to June; and July to Dec.) of the year.

Clock Time at Apparent Noon.		Clock Time at Apparent Noon.	
	hrs. min.		hrs. min.
January 1 -	12 4	July 1 -	12 3
February 1 -	12 14	August 1 -	12 6
March 1 -	12 12	September 1	11 50
April 1 -	12 4	October 1 -	11 44
May 1 -	11 57	November 1	11 50
June 1 -	11 58	December 1	12 2

32. VULGAR FRACTIONS.

The term **decimal fraction** has already been used to express the portion that one quantity is of another. The same result may be expressed in another way. Thus dealing with 3 oz. as compared with 1 lb., we might suppose 1 lb. divided into 16 equal parts, of which 3 are taken, and say

3 oz. is three-sixteenths or $\frac{3}{16}$ of 1 lb.

Similarly, if we speak of $\frac{5}{18}$ of a yard, the meaning is:—that the yard is supposed to be divided up into 18 equal lengths, and of these 5 are taken to form the required portion.

So by the **vulgar fraction** $\frac{a}{b}$, we mean, that a certain quantity regarded as a unit is divided up into b equal parts, and of these a are taken to form the fraction. a is styled the **Numerator**, b the **Denominator**.

EXAMPLE.— *What is the value of* $\frac{3}{16}$ *of* 1 *cwt.?*

1 cwt. could be divided up into 16 equal parts each of 7 lbs. $\frac{3}{16}$ of a cwt. would be 3 such amounts, or 21 lbs.

EXAMPLE.— *What is the value of* $\frac{13}{25}$ *of* £3. 2*s.* 6*d.?*

We can divide £3. 2*s.* 6*d.* into 25 equal amounts of 2*s.* 6*d.* 13 such parts will amount to £1. 12*s.* 6*d.*

∴ $\frac{13}{25}$ of £3. 2*s.* 6*d.* = £1. 12*s.* 6*d*

	£.	s.	d.
5	3	2	6
5		12	6
		2	6

EXAMPLE —*Evaluate* $\frac{14}{27}$ *of* 2 *tons* 3 *cwt.* 17 *lb.*

Here we have to divide the 2 tons 3 cwt. 17 lb. by 27, and then to multiply the answer by 14.

And it is clearly just the same if we multiply by 14 first and divide by 27 afterwards. The operation which is done first depends upon the particular example.

Here 2 tons 3 cwt. 17 lb. $\div 27$ = 1 cwt. 2 qr. 11 lb.
1 cwt. 2 qr. 11 lb. $\times 14$ = 1 ton 2 cwt. 1 qr. 14 lb.
$\therefore \frac{14}{27}$ of 2 tons 3 cwt. 17 lb. = 1 ton 2 cwt. 1 qr. 14 lb.

EXAMPLE.—*Evaluate* $\frac{12}{13}$ *of* £2. 3*s.* 4*d.*

£2. 3*s.* 4*d.* $\times 12$ = £26,
£26 $\div 13$ = £2;
$\therefore \frac{12}{13}$ of £2. 3*s.* 4*d.* = £2.

EXERCISES XLIII. a.

Mental or Oral.

State the value of the following:

1. $\frac{1}{3}$ of 1*s.*
2. $\frac{2}{3}$ of 1*s.*
3. $\frac{3}{4}$ of 1*s.*
4. $\frac{5}{8}$ of £1.
5. $\frac{1}{6}$ of 10*s.* 6*d.*
6. $\frac{4}{5}$ of £2.
7. $\frac{2}{3}$ of 1 ft.
8. $\frac{3}{4}$ of 1 ft. 8 in.
9. $\frac{7}{12}$ of 3*s.* 6*d.*
10. $\frac{3}{8}$ of 1 lb.
11. $\frac{13}{14}$ of 1 cwt.
12. $\frac{10}{14}$ of 2 cwt. 2 stones.
13. $\frac{11}{25}$ of £5.
14. $\frac{7}{15}$ of 1 qr. 2 lbs.
15. $\frac{13}{26}$ of 3 cwt.
16. $\frac{10}{12}$ of 3 yds.
17. $\frac{9}{24}$ of £2. 8*s.*
18. $\frac{10}{22}$ of a furlong.

EXERCISES XLIII. b.

Find the value of the following:

1. $\frac{15}{28}$ of 4 cwt.
2. $\frac{6}{11}$ of £12. 10*s.* 3*d.*
3. $\frac{13}{18}$ of £19. 17*s.* 6*d.*
4. $\frac{7}{15}$ of 5 tons 9 cwt. 17 lbs.
5. $\frac{11}{32}$ of 7 lbs. 12 oz.
6. $\frac{17}{66}$ of 93 miles 18 chains.
7. $\frac{12}{25}$ of 132 yds. 1 ft. 11 ins.
8. $\frac{23}{32}$ of 9 m. 8 cm. 8 mm.
9. $\frac{4}{15}$ of 8 kilogs. 4 Dekagr.
10. $\frac{8}{15}$ of 40 ac. 2 ro.
11. $\frac{7}{8}$ of a gross $+\frac{3}{4}$ dozen $-\frac{7}{10}$ score.
12. $\frac{35}{96}$ of £3. 4*s.* $-\frac{4}{7}$ guinea.
13. $\frac{8}{9}$ of 43 yds. 2 ft. 3 in. $+\frac{5}{12}$ of 1 chain.
14. $\frac{21}{31}$ of 136 tons 8 cwt. $+\frac{11}{21}$ of 344 cwt. 49 lbs. $-\frac{41}{56}$ of 1 ton.

Graphic Illustrations of Vulgar Fractions.

EXAMPLE.—*Draw a rectangle* 8 *cm. by* 9 *cm., regard this as a unit; divide and shade portions to represent*

(*a*) $\frac{1}{8}$, (*b*) $\frac{1}{9}$, (*c*) $\frac{1}{72}$, (*d*) $\frac{15}{72}$, (*e*) $\frac{5}{24}$.

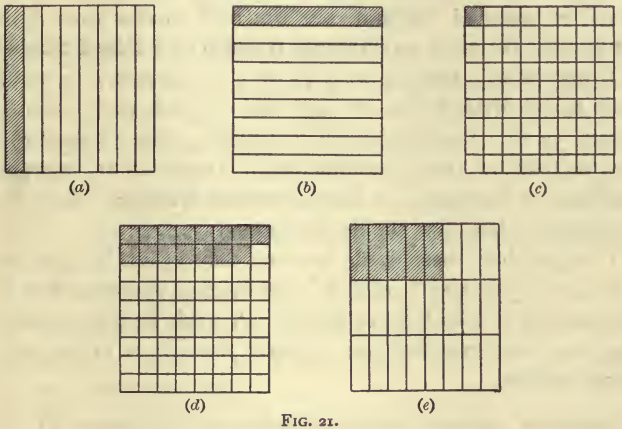


FIG. 21.

EXERCISES XLIII. c.

Graphic.

[*Apparatus: Squared Paper.*]

1. Draw 5 rectangles 12 cm. by 8 cm., each of which you may regard as a unit; divide and shade portions to represent the following fractions:

(*a*) $\frac{1}{8}$. (*b*) $\frac{1}{12}$. (*c*) $\frac{13}{96}$. (*d*) $\frac{25}{48}$.

2. Taking a line AB as unit, draw lines to represent the following fractions:

(i) $\frac{5}{6}$. (ii) $\frac{3}{4}$. (iii) $\frac{2}{3}$. (iv) $\frac{5}{8}$.

33. IMPROPER FRACTIONS AND MIXED NUMBERS.

If by means of a vulgar fraction it is wished to compare the magnitude of some quantity A with that of another quantity B which is taken as the unit, but which is smaller than A, the result must be either an **improper fraction** or a **mixed number.**

It may be said that 33 oz. is $\frac{33}{16}$ of a lb., because a lb. would have to be divided into 16 parts and 33 such parts taken to obtain 33 oz.; the fraction $\frac{33}{16}$, in which 33 (the numerator) is greater than 16 (the denominator), is known as an **improper fraction,** to distinguish it from a **proper fraction,** where the numerator is less than the denominator.

It might have been said, however, that 33 oz. is 2 lb. and 1 oz., *i.e.* is 2 lb. and $\frac{1}{16}$ of a lb.; the fraction $\frac{33}{16}$ would then be represented by 2 and $\frac{1}{16}$, or usually $2\frac{1}{16}$; this fraction, consisting of a *whole* number and a *proper fraction*, is known as a **mixed number.**

EXAMPLE.—*Express as mixed numbers the following:* $\frac{28}{12}$, $\frac{39}{8}$.

(1) $\frac{28}{12}$. Divide by 12, obtain 2 and 4 over,
i.e. $\frac{28}{12} = 2\frac{4}{12}$.

(2) $\frac{39}{8} = 4\frac{7}{8}$. Divide by 8, the quotient 4 gives the whole number, and the remainder 7 gives the numerator, while the denominator is unaltered.

EXAMPLE.—(*a*) *To convert a mixed number, say* $6\frac{5}{7}$, *to an improper fraction.*

6 units are the same as $7 \times 6 = 42$ sevenths. Therefore altogether, there are $42 + 5 = 47$ sevenths, and the corresponding improper fraction is $\frac{47}{7}$.

(*b*) *Convert* $11\frac{7}{12}$ *to an improper fraction.*

$$11\frac{7}{12} = \frac{12 \times 11 + 7}{12} = \frac{139}{12}.$$

EXAMPLE.—*Find* $5\frac{3}{7}$ *of* 3 *cwt.* 2 *qr.*

i.e. we have 5 times 3 cwt. 2 qr. = 17 cwt. 2 qr.
and $\frac{3}{7}$ of 3 cwt. 2 qr., *i.e.* 3 times 2 qr. = 1 cwt. 2 qr.
$\therefore$ $5\frac{3}{7}$ of 3 cwt. 2 qr. = 19 cwt.

EXERCISES XLIV.

1. Express as improper fractions:

(*a*) $3\frac{17}{18}$. (*b*) $5\frac{7}{9}$. (*c*) $18\frac{3}{16}$. (*d*) $29\frac{1}{25}$. (*e*) $62\frac{3}{40}$.
(*f*) $182\frac{5}{12}$. (*g*) $19\frac{61}{108}$. (*h*) $35\frac{73}{96}$. (*i*) $84\frac{19}{240}$. (*j*) $52\frac{11}{112}$.

2. Express as mixed numbers the following:

(*a*) $\frac{362}{25}$. (*b*) $\frac{185}{71}$. (*c*) $\frac{288}{17}$. (*d*) $\frac{93}{32}$. (*e*) $\frac{1874}{295}$.
(*f*) $\frac{7013}{112}$. (*g*) $\frac{9503}{1780}$. (*h*) $\frac{15713}{841}$. (*i*) $\frac{245}{144}$. (*j*) $\frac{735}{36}$.

3. A spends $2\frac{1}{8}$ times as much as B, who spends 5*s.* 5*d.* What does A spend?

4. How much greater is $3\frac{7}{12}$ of £4 than $4\frac{5}{6}$ of £2. 10*s.*?

5. What must be added to $3\frac{7}{15}$ of 4 tons 10 cwt. to make 20 tons?

6. When $7\frac{5}{8}$ lengths of 168 yards have been cut from $\frac{3}{4}$ of a mile, what length remains?

7. What must be taken from $18\frac{13}{56}$ cwt. to leave $7\frac{4}{7}$ of 3 qrs. 14 lbs.?

8. Find the value of $3\frac{9}{16}$ of £9. 9*s.* 4*d.* + $8\frac{7}{24}$ of 19*s.* 6*d.* − $13\frac{19}{21}$ of £1. 2*s.* 9*d.*

One Quantity as a Vulgar Fraction of Another.

EXAMPLE.—*Suppose it is desired to express* £2. 13*s.* 6*d. as a vulgar fraction of* £4. 15*s.*

£2. 13*s.* 6*d.* = 533*s.* 6*d.* = 6420*d.*
£4. 15*s.* 0*d.* = 95*s.* = 1140*d.*

£2. 13*s.* 6*d.* could therefore be obtained by dividing £4. 15*s.* into 1140 equal parts (each 1*d.*) and then taking 642 of these.

£2. 13*s.* 6*d.* is accordingly $\frac{642}{1140}$ of £4. 15*s.*

A slightly different procedure is as follows:

£2. 13s. 6d. = 53s. 6d. = 107 sixpences.
£4. 15s. 0d. = 95s. = 190 „

£2. 13s. 6d. could be therefore obtained by dividing £4. 15s. into 190 equal parts each equal to 6d. and taking 107 of these.

£2. 13s. 6d. = $\frac{107}{190}$ of £4. 15s.

$\frac{107}{190}$ and $\frac{642}{1140}$ here express the same portion—these fractions are therefore said to be equivalent.

34. EQUIVALENT FRACTIONS.

To express the part that 3 cwt. is of 1 ton, we can use the fraction $\frac{3}{20}$, for 1 ton could be divided up into 20 equal parts, termed cwt., of which 3 parts are taken; but the ton may also be divided into 80 qrs. and each cwt. into 4 qrs., the part taken being 12 qrs. out of 80 forming the whole ton, and therefore represented by the fraction $\frac{12}{80}$, *i.e.* $\frac{3 \times 4}{20 \times 4}$. Again, 3 cwt. is 112×3 lbs., 1 ton is 20×112 lbs., and the fraction could be also represented by $\frac{3 \times 112}{20 \times 112}$;

and therefore the fractions

$$\frac{3}{20}, \quad \frac{4 \times 3}{4 \times 20}, \quad \frac{112 \times 3}{112 \times 20} \text{ are equivalent;}$$

and so generally a fraction $\frac{a}{b}$ is equivalent to the fraction

$$\frac{n \times a}{n \times b}.$$

A fraction is unaltered if both numerator and denominator are multiplied by the same quantity, or if both numerator and denominator are divided by the same quantity.

When the numerator and denominator are the simplest numbers possible, a fraction is said to be in its **lowest terms.**

EXERCISES XLV. a.

1. (*a*) Write down the fraction expressing the part that 4 shillings is of a pound, in the same manner that the fraction $\frac{3}{20}$ was used on p. 162 to express the part that 3 cwt. is of 1 ton. Write down also the equivalent fractions possible on regarding a shilling as equivalent to (*b*) 2 sixpences, (*c*) 4 threepenny bits, (*d*) 12 pennies, (*e*) 24 halfpennies.

2. Express as equivalent fractions in their lowest terms :

(*a*) $\frac{8}{12}$, (*b*) $\frac{9}{12}$, (*c*) $\frac{6}{24}$, (*d*) $\frac{8}{24}$, (*e*) $\frac{5}{10}$, (*f*) $\frac{15}{100}$, (*g*) $\frac{27}{36}$.

3. Group together the equivalent fractions among the following :

$\frac{4}{12}$, $\frac{15}{16}$, $\frac{8}{16}$, $\frac{8}{24}$, $\frac{75}{80}$, $\frac{22}{24}$, $\frac{30}{32}$, $\frac{15}{45}$, $\frac{60}{120}$, $\frac{7}{14}$, $\frac{7}{21}$.

4. Write down 4 fractions equivalent to the following, (i) with smaller numerators, (ii) with larger numerators :

(*a*) $\frac{48}{96}$. (*b*) $\frac{120}{243}$. (*c*) $\frac{125}{375}$.

5. Fill in the missing parts (numerator or denominator) of the following fractions :

(i) $\frac{\cdots\cdots}{216}$, (ii) $\frac{30}{\cdots\cdots}$, (iii) $\frac{24}{\cdots\cdots}$, (iv) $\frac{\cdots\cdots}{72}$, (v) $\frac{\cdots\cdots}{144}$, (vi) $\frac{96}{\cdots\cdots}$,

so that they may be equivalent fractions to

(*a*) $\frac{2}{3}$. (*b*) $\frac{3}{4}$. (*c*) $\frac{1}{2}$.

6. What weight is the same fraction of 28 lbs. as (*a*) 84 lbs. is of 112 lbs., (*b*) 15*s.* is of £4. 4*s.*, (*c*) one yard is of 4 ft.?

7. What length is the same fraction of 36 inches as (*a*) an area of 2 sq. ft. is of 1 sq. yard, (*b*) a volume of 432 cubic inches is of a cubic foot?

8. One person has £360, another £108, to give away to charitable institutions. If the first gives a hospital £80, what does the hospital receive from the second person if he gives to it the same fraction of his total gifts as the first?

9. At one match a cricketer A made 54 runs out of a total of 297, and he made the same fraction of a total of 264 at a second match. What was the fraction, and what number of runs did he make at the second match?

We can see by means of equivalent fractions which of two or more fractions is the greatest. Thus:

EXAMPLE.—*To find out which of the two fractions* $\frac{13}{27}$, $\frac{17}{36}$ *is the greater.*

We try to find two fractions equivalent respectively to the given fractions, but with equal denominators.

Thus $\frac{13}{27}$, $\frac{26}{54}$, $\frac{39}{81}$, $\frac{52}{108}$, $\frac{65}{135}$ are all equivalent to $\frac{13}{27}$.

Also $\frac{17}{36}$, $\frac{34}{72}$, $\frac{51}{108}$, $\frac{68}{144}$ are all equivalent to $\frac{17}{36}$.

$\frac{52}{108}$ is clearly greater than $\frac{51}{108}$, and therefore $\frac{13}{27}$ is greater than $\frac{17}{36}$.

The numerators might have been made the same as in the following worked example:

EXAMPLE.—*To find which is the greater of the two fractions* $\frac{3}{113}$, $\frac{5}{188}$.

$\frac{3}{113}$, $\frac{6}{226}$, $\frac{9}{339}$, $\frac{12}{452}$, $\frac{15}{565}$, $\frac{18}{678}$ are all equivalent fractions.

$\frac{5}{188}$, $\frac{10}{376}$, $\frac{15}{564}$, $\frac{20}{752}$ ... form another group of equivalent fractions.

$\frac{15}{565}$ is clearly less than $\frac{15}{564}$, whence $\frac{3}{113}$ is less than $\frac{5}{188}$.

EXERCISES XLV. b.

1. Express the fractions $\frac{11}{15}$, $\frac{6}{7}$, $\frac{12}{17}$ with the same (least) common denominator; then arrange them in order of magnitude.

2. Write down the greater fraction of the following pairs:

(*a*) $\frac{13}{75}$, $\frac{12}{72}$. (*b*) $\frac{15}{27}$, $\frac{22}{41}$. (*c*) $\frac{17}{21}$, $\frac{21}{25}$. (*d*) $\frac{40}{73}$, $\frac{13}{24}$.

(*e*) $\frac{18}{29}$, $\frac{27}{38}$. (*f*) $\frac{120}{143}$, $\frac{150}{173}$. (*g*) $\frac{92}{185}$, $\frac{207}{415}$. (*h*) $\frac{19}{20}$, $\frac{49}{50}$.

(*i*) $\frac{11}{32}$, $\frac{12}{35}$. (*j*) $\frac{45}{58}$, $\frac{30}{38}$. (*k*) $\frac{4}{17}$, $\frac{12}{53}$.

Equivalent Fractions Illustrated Graphically.

Draw any line AB (Fig. 22), and regard it as a unit.

Bisect the line at C, then AC represents the fraction $\frac{1}{2}$. Divide AB into 4 equal portions. AC contains 2 of these, and hence AC also represents the fraction $\frac{2}{4}$. Divide AB into 6 equal portions, say at E, F, C, H, K. AC contains 3 out of the 6, and therefore represents the fraction $\frac{3}{6}$.

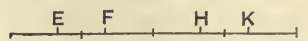


FIG. 22.

The fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ being all represented by the same length AC are hence all equivalent.

Again, draw a rectangle ABDC (Fig. 23) 6 cm. long, 4 cm. broad, and regard it as a unit.

FIG. 23.

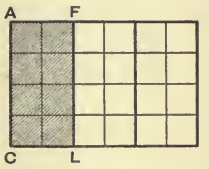


FIG. 24.

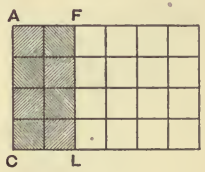


FIG. 25.

The rectangle may be divided into 6 equal portions by the 5 lines EK, FL, GM, HN, IO, and AFLC represents the fraction $\frac{2}{6}$.

In Fig. 24, AFLC represents the fraction $\frac{1}{3}$.

In Fig. 25, AFLC represents the fraction $\frac{8}{24}$.

Hence the fractions $\frac{2}{6}$, $\frac{1}{3}$, $\frac{8}{24}$ are all equivalent.

EXERCISES XLV. c.

Practical.

[*Apparatus: Squared paper. Scales. Set squares.*]

1. Draw a line 3″ in length and regard it as a unit; mark off a portion to represent the fraction $\frac{8}{12}$; show from your figure that the fractions $\frac{4}{6}$, $\frac{2}{3}$ are equivalent.

2. Take a line AB 4″ in length as a unit; cut off from it a portion AX to represent the fraction $\frac{3}{8}$ of the unit, and show that it is the same value as $\frac{6}{16}$.

3. Draw a square of 6 cm. side, and consider it as a unit area. Shade by diagonal lines $\frac{3}{4}$ of the area. Shade by cross diagonal lines $\frac{9}{12}$ of it. Note that these areas represent equivalent fractions.

4. Draw a rectangle 6 cm. long, 4 cm. broad; regarding it as a unit, mark by shaded lines an area representing the fraction $\frac{3}{4}$; show graphically that the fractions $\frac{6}{8}$, $\frac{9}{12}$, $\frac{18}{24}$ are equivalents.

5. Show graphically, by suitably divided squares or rectangles, that in the following sets the fractions in each set are equivalent:

(*a*) $\frac{4}{5}$, $\frac{8}{10}$, $\frac{12}{15}$. (*b*) $\frac{8}{12}$, $\frac{2}{3}$, $\frac{16}{24}$. (*c*) $\frac{16}{72}$, $\frac{2}{9}$, $\frac{4}{18}$.
(*d*) $\frac{12}{24}$, $\frac{4}{5}$. (*e*) $\frac{1}{4}$, $\frac{25}{100}$, $\frac{5}{20}$.

35. ADDITION AND SUBTRACTION OF FRACTIONS.

Two or more fractions with the same denominators can be easily added together.

EXAMPLE.—*To evaluate* $\frac{17}{24}+\frac{18}{24}+\frac{32}{24}$.

This addition means the sum of 17, 18, 32 equal parts each $(\frac{1}{24})^{\text{th}}$ of a unit, *i.e.* there are 67 such portions.

Hence the sum $= \frac{17+18+32}{24} = \frac{67}{24} = 2\frac{19}{24}$,

so $\frac{35}{42} - \frac{14}{42} = \frac{21}{42} = \frac{1}{2}$.

If the fractions have different denominators, they may be changed into equivalent fractions in which the denominators are the same.

EXAMPLE.—*To evaluate* $\frac{5}{8}+\frac{7}{12}+\frac{3}{16}$.

$\frac{5}{8}$, $\frac{10}{16}$, $\frac{15}{24}$, $\frac{20}{32}$, $\frac{25}{40}$, $\frac{30}{48}$ are all equivalent fractions; so are $\frac{7}{12}$, $\frac{14}{24}$, $\frac{21}{36}$, $\frac{28}{48}$, and also $\frac{3}{16}$, $\frac{6}{32}$, $\frac{9}{48}$ ....

The sum $\frac{5}{8}+\frac{7}{12}+\frac{3}{16}$ is therefore the same as

$$\frac{30}{48}+\frac{28}{48}+\frac{9}{48}=\frac{67}{48}=1\frac{19}{48}.$$

EXAMPLE.—*To find the difference between* $\frac{38}{35}$ *and* $\frac{4}{5}$.

$$\frac{38}{35}-\frac{4}{5}=\frac{38}{35}-\frac{28}{35}=\frac{10}{35}=\frac{2}{7}.$$

If the fractions be mixed numbers, the whole numbers should be considered separately.

EXAMPLE.—*Evaluate* $3\frac{1}{6}+2\frac{1}{2}+1\frac{3}{4}+\frac{2}{3}$.

$$\begin{aligned}3\tfrac{1}{6}+2\tfrac{1}{2}+1\tfrac{3}{4}+\tfrac{2}{3}&=3+2+1+\tfrac{1}{6}+\tfrac{1}{2}+\tfrac{3}{4}+\tfrac{2}{3}\\&=6+\tfrac{2}{12}+\tfrac{6}{12}+\tfrac{9}{12}+\tfrac{8}{12}\\&=6+\tfrac{25}{12}\\&=6+2\tfrac{1}{12}\\&=8\tfrac{1}{12}.\end{aligned}$$

EXAMPLE.—*Subtract* $3\frac{7}{9}$ *from* $6\frac{3}{7}$, *i.e. evaluate* $6\frac{3}{7}-3\frac{7}{9}$.

$$6\tfrac{3}{7}-3\tfrac{7}{9}=6+\tfrac{3}{7}-3-\tfrac{7}{9}=6-3+\tfrac{3}{7}-\tfrac{7}{9}=3+\tfrac{27}{63}-\tfrac{49}{63}$$

where $\frac{3}{7}$ and $\frac{7}{9}$ are changed into equivalent fractions with the same denominator.

Since $\frac{49}{63}$ is greater than $\frac{27}{63}$, proceed either thus:

$$3+\tfrac{27}{63}-\tfrac{49}{63}=2+1+\tfrac{27}{63}-\tfrac{49}{63}=2+\tfrac{90}{63}-\tfrac{49}{63}$$
$$=2+\tfrac{90-49}{63}=2\tfrac{41}{63},$$

or thus
$$3+\tfrac{27}{63}-\tfrac{49}{63}=3-\tfrac{22}{63}=2\tfrac{41}{63}.$$

EXERCISES XLVI. a.

1. One weight is $3\frac{1}{7}$ cwt. and another is $2\frac{13}{16}$ cwt. Express these in cwt. and lbs.; find their sum. State the preceding answer in cwt. and fractions of a cwt., and verify your result by adding together the fractions $3\frac{1}{7}$ and $2\frac{13}{16}$.

2. Find the sum of £$2\frac{13}{15}$ and £$5\frac{1}{6}$; verify in a manner similar to that suggested in Exercise 1.

3. Add together the following:

(*a*) $\frac{2}{3}$, $2\frac{1}{6}$, $3\frac{7}{8}$, $\frac{1}{4}$.
(*b*) $2\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $3\frac{1}{3}$.
(*c*) $\frac{11}{16}$, $3\frac{3}{4}$, $6\frac{1}{8}$, $\frac{19}{24}$.
(*d*) $7\frac{1}{3}$, $8\frac{4}{5}$, $\frac{13}{15}$, $2\frac{7}{9}$.
(*e*) $3\frac{5}{9}$, $4\frac{11}{16}$, $1\frac{12}{36}$, $\frac{25}{48}$.
(*f*) $\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$, $5\frac{7}{20}$, $\frac{1}{8}$.
(*g*) $20\frac{17}{32}$, $15\frac{2}{3}$, $1\frac{7}{12}$, $\frac{13}{18}$, $19\frac{9}{64}$.
(*h*) $3\frac{5}{8}$, $4\frac{1}{2}$, $93\frac{11}{18}$, $2\frac{2}{5}$, $\frac{11}{15}$, $1\frac{19}{30}$.
(*i*) $2\frac{11}{51}$, $18\frac{13}{15}$, $\frac{2}{3}$, $\frac{14}{17}$, $\frac{567}{765}$.
(*j*) $3\frac{8}{13}$, $72\frac{1}{3}$, $8\frac{3}{4}$, $20\frac{1}{2}$, $\frac{28}{39}$, $16\frac{3}{5}$.

Find the difference between the following pairs of fractions:

4. (*a*) $\frac{7}{9}$ and $\frac{11}{45}$, (*b*) $\frac{5}{8}$ and $\frac{4}{9}$, (*c*) $\frac{11}{12}$ and $\frac{13}{18}$, (*d*) $\frac{7}{20}$ and $\frac{3}{5}$.

5. (*a*) $\frac{19}{36}$ and $\frac{11}{24}$, (*b*) $\frac{47}{56}$ and $\frac{6}{7}$, (*c*) $\frac{31}{63}$ and $\frac{5}{14}$, (*d*) $\frac{509}{1008}$ and $\frac{13}{72}$.

6. (*a*) $\frac{57}{88}$ and $\frac{29}{33}$, (*b*) $\frac{906}{1225}$ and $\frac{17}{25}$, (*c*) $\frac{9}{16}$ and $\frac{7}{15}$, (*d*) $\frac{46}{81}$ and $\frac{1353}{1809}$.

7. (*a*) $19\frac{7}{64}$ and $13\frac{19}{96}$, (*b*) $20\frac{161}{192}$ and $17\frac{1923}{2240}$, (*c*) $18\frac{3}{13}$ and $40\frac{1}{4}$, (*d*) $760\frac{1}{8}$ and $63\frac{5}{13}$.

8. Evaluate $\frac{2}{3}+\frac{17}{36}-\frac{4}{6}+\frac{1}{2}-\frac{47}{72}$.

9. Evaluate $\frac{9}{16}+\frac{13}{18}-\frac{2}{3}+\frac{5}{6}-\frac{7}{8}$.

10. Evaluate $7\frac{4}{5}+18\frac{6}{7}-(6\frac{19}{35}+18\frac{13}{14})$.

11. Find the value of $£7\frac{5}{8}+4\frac{4}{9}$ guineas $-18\frac{5}{7}$ half guineas $+14\frac{7}{20}$ crowns.

12. To the sum of $6\frac{7}{8}$ crowns and $\frac{17}{32}$ of a sovereign, add the difference between $8\frac{2}{3}$ shillings and $£\frac{37}{96}$.

13. What is the value of $13\frac{3}{5}$ tons $+16\frac{5}{7}$ cwt. $-31\frac{11}{14}$ qr. -143 lb.?

14. Take $17\frac{5}{44}$ of a furlong from $5\frac{1}{8}$ miles.

15. When $23\frac{5}{9}$ yds. of a rope had been used, it was noted that $19\frac{5}{12}$ ft. were left. What was the length of the rope?

EXERCISES XLVI. b.

Practical.

[*Apparatus: Squared paper. Set squares. Scales.*]

1. Draw a line AB say 2″ long, divide it up into 3 equal parts; take *two* of these parts to represent the fraction $\frac{2}{3}$; next mark off on your paper a line twice as long as AB, and divide into 3 equal parts; compare the length of one of these parts with the line representing the fraction $\frac{2}{3}$.

2. Draw a line AB 5″ long; divide it into 4 equal parts; let the points of division be A_1, A_2, A_3. Next divide the portion AA_3 into 3 equal parts, and give the value of $\frac{2}{3}$ of this. Note that it coincides with $\frac{1}{2}$ of original, and hence $\frac{2}{3}$ of $\frac{3}{4}$ = ...?

3. Draw a line 5″ long. Divide it into 4 equal parts by repeated bisecting; mark off $\frac{3}{4}$ of the line, call it XY. Divide this portion XY into 5 equal parts, and mark off a part containing 2 of these; letter it XZ. Then XZ is $\frac{2}{5}$ of $\frac{3}{4}$. Divide the original line into 10 equal parts and show that XZ is 3 of these or $\frac{3}{10}$ of original line. Hence $\frac{2}{5}$ of $\frac{3}{4} = \frac{3}{10}$.

4. Draw a square of 9 cm. side and letter it ABCD. Divide AD into 3 equal parts at F, G. Through G draw GE parallel to AB. Then ABEG is $\frac{2}{3}$ of the square.

Divide AB into 9 equal parts at H, K, L, M, N, P, Q, R. Through N draw NO parallel to AD. Then ANOG is $\frac{5}{9}$ of ABEG, and ABEG is $\frac{2}{3}$ of the square. If the number of square centimetres in ANOG be reckoned, they will number 30, and ∴ ANOG is $\frac{30}{81}$ of the whole ABCD. So $\frac{5}{9}$ of $\frac{2}{3} = \frac{10}{27}$ or $\frac{30}{81}$.

5. Draw a rectangle 2″ by 1″; divide it up into two unit squares (each square of 1 sq. in. being regarded as a unit). Divide up the rectangle also into 3 equal portions by 2 lines parallel to the 2″ side; show from your figure that $\frac{1}{3}$ of 2 is $\frac{2}{3}$.

6. Draw figures to show that $\frac{3}{4}$ is equivalent to $\frac{1}{4}$ of 3; that $\frac{4}{5}$ is equivalent to $\frac{1}{5}$ of 4; and that $\frac{1}{6}$ of 3 is the same as $\frac{3}{6}$.

36. CONVERSION OF FRACTIONS.

Conversion of a Decimal into a Vulgar Fraction.

EXAMPLE.—*To convert* 0·216 *to a vulgar fraction.*

0·2 we know when multiplied by 10 = 2, and is therefore the tenth part of 2, or $\frac{2}{10}$ or $\frac{20}{100}$ or $\frac{200}{1000}$, these being equivalent fractions.

0·01 becomes 1 when multiplied by 100, *i.e.* it is the hundredth part of unity or $\frac{1}{100}$.

0·216 when multiplied by 1000 becomes 216, and is therefore the thousandth part of 216, which is the same as $\frac{216}{1000} = \frac{216}{10^3}$.

Similarly, (*a*) $32{\cdot}3452 = 32\frac{3452}{10000} = 32\frac{3452}{10^4}$. (*b*) $8{\cdot}3 = 8\frac{3}{10}$.

From this process it is seen that, when decimals are expressed as mixed fractions,

(i) whole numbers remain whole numbers;

(ii) the numerator of the fractional part consists of the decimal figures;

(iii) the denominator contains 10 raised to a power equal to the number of decimal figures.

When the decimal is to be expressed as an improper fraction, the complete decimal is written down without the decimal point to form the numerator, and the denominator consists of 10 raised to a power equal to the number of figures after the decimal point. Thus:

$$518{\cdot}237 = \frac{518237}{10^3} = \frac{518237}{1000}.$$

Conversion of a Vulgar Fraction into a Decimal.

EXAMPLE.—*To convert* $\frac{24}{60}$ *into a decimal fraction.*

The fraction means that a certain quantity regarded as a unit is divided up into 60 equal parts, and of these 24 are taken.

But we have seen that the same result is obtained by dividing a quantity consisting of 24 units into 60 equal parts.

Hence, to convert $\frac{24}{60}$ into a decimal, we merely divide 24 by 60 by the ordinary methods used in decimals; and similarly, in converting any vulgar fraction $\frac{a}{b}$ into a decimal, we merely divide a by b.

Recurring Decimals.

EXAMPLE.—*Convert* $\frac{24}{45}$ *into a decimal.*

$\frac{24}{45}$ when reduced to lowest terms $= \frac{8}{15}$.
Dividing 8 by 15, we obtain

$$15 \mid 800 \quad 0{\cdot}533$$
$$50$$
$$50$$
$$50$$

The remainder is *continually* 5 and the process will never terminate; the decimal quotient is represented by $0{\cdot}5\dot{3}$, where the dot above the 3 indicates that the 3 can be repeated indefinitely.

Similarly, $21{\cdot}8\dot{7}64\dot{3}$ would represent a non-terminating or recurring decimal, in which the figures 7643 recur indefinitely.

$$21{\cdot}8764376437643764376437643 \ldots .$$

EXERCISES XLVII. a.

N.B.—In no case need the decimal equivalents of any of the fractions stated be carried to more than five figures.

1. Express the following decimals as vulgar fractions, reducing the latter always to lowest terms:

(*a*) 0·6.	(*b*) 0·06.	(*c*) 0·006.
(*d*) 0·125.	(*e*) 0·75.	(*f*) 0·32.
(*g*) 0·375.	(*h*) 0·55.	(*i*) 0·0675.
(*j*) 0·664.	(*k*) 0·875.	(*l*) 0·0015625.
(*m*) 7·64.	(*n*) 590·875.	(*o*) 0·06325.
(*p*) 0·78575.	(*q*) 0·0525.	(*r*) 0·9816.
(*s*) 41·1875.	(*t*) 0·09375.	(*u*) 0·7925.
(*v*) 0·003875.	(*w*) 18·6875.	(*x*) 62·01025.

2. Arrange the following vulgar fractions in descending order of magnitude by first converting them into decimals; check your results by the method of equivalent fractions:

(*a*) $\frac{7}{33}$, $\frac{2}{9}$, $\frac{27}{131}$, $\frac{3}{14}$.
(*b*) $\frac{17}{24}$, $\frac{25}{36}$, $\frac{8}{12}$, $\frac{6}{8}$.
(*c*) $\frac{13}{25}$, $\frac{194}{375}$, $\frac{326}{625}$.
(*d*) $\frac{5}{8}$, $\frac{3}{4}$, $\frac{47}{64}$, $\frac{11}{32}$, $\frac{9}{16}$.
(*e*) $\frac{4}{15}$, $\frac{7}{27}$, $\frac{2}{9}$, $\frac{29}{135}$.

EXERCISES XLVII. b.

Graphic.

[*Apparatus: Squared paper. Diagonal and other Scales.*]

1. Draw straight lines to represent the following lengths: $\frac{11}{12}''$, $\frac{17}{8}''$, $\frac{14}{3}''$, $2\frac{3}{4}''$; measure the distances off also (to two decimal places) using your diagonal scale; check your results by converting the given fractions into decimals.

2. Draw four parallel horizontal lines having their left ends vertically over one another, and measure off from the left the following lengths: $3\frac{5}{8}''$, $3\frac{2}{3}''$, $3\frac{7}{12}''$, $3\frac{63}{100}''$. Arrange the lengths in descending order of magnitude and write opposite to each its length expressed as a decimal.

3. Perform the following operations graphically:

$$(a)\ 1\tfrac{3}{4}-2\tfrac{5}{8}+2\tfrac{1}{2}+3\tfrac{4}{5}.$$

Check your result by evaluating the sum as a vulgar fraction, and also by expressing each fraction as a decimal and calculating as a sum in decimals.

Also $(b)\ 2\frac{2}{3}+3\frac{1}{2}-2\frac{4}{5}-2\frac{27}{100}$.

Also $(c)\ 2\frac{1}{2}-2\frac{3}{4}+3\frac{2}{5}-1\frac{67}{100}$.

37. MULTIPLICATION AND DIVISION OF VULGAR FRACTIONS.

Multiplication by a Whole Number.

EXAMPLE.—*To multiply* $\frac{3}{4}$ *by* 5.

$\frac{3}{4}$ represents 3 portions, each portion being one-fourth of a unit.

$\frac{3}{4}\times 5$ represents therefore 15 portions, each portion being one-fourth of a unit, and the result is therefore represented by $\frac{15}{4}$, *i.e.* by $\frac{3\times 5}{4}$; similarly, any vulgar fraction $\frac{a}{b}\times$ any whole

number $n = \frac{n \times a}{b}$, *i.e.* to multiply a fraction by an integer we may multiply the numerator by that integer and keep the denominator unchanged.

EXAMPLE.—*To multiply* $\frac{11}{15}$ by 5.

As before, the result is $\frac{11 \times 5}{15}$; becoming, however, $\frac{11}{3}$ on reducing it to its lowest terms; *i.e.* we have divided the denominator by 5.

EXAMPLE.—*To multiply* $2\frac{3}{11}$ *by* 7.

This is the same as multiplying $\frac{3}{11}$ by 7 and adding the result to 7 times 2;

$$\therefore \quad = 14 + \frac{21}{11} = 14 + 1\frac{10}{11} = 15\frac{10}{11}.$$

The operations usually are performed mentally if the numbers are small.

Thus 7 times 3 = 21; 11 into 21 yields 1 and 10 remainder.
7 „ 2 = 14, and this with the 1 yields 15.
And the answer is $15\frac{10}{11}$.

Division by a Whole Number.

EXAMPLE.—*To divide* $\frac{5}{8}$ *by* 3.

Here it is required to divide into 3 equal parts an amount made up of 5 equal portions, each portion one-eighth of the complete unit.

If each one of these 5 portions be divided into 3 and one part taken from each portion, there will now be 5 parts each one twentyfourth of the unit, *i.e.* the result of the division is $\frac{5}{24}$.

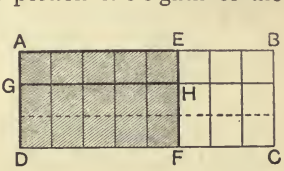


FIG. 26.—To illustrate the division of a fraction.

This operation can be illustrated by a diagram in which ABCD represents a certain unit, while AEFD represents $\frac{5}{8}$ of that unit. AGHE is $\frac{1}{3}$ of AEFD, and is clearly $\frac{5}{24}$ of the whole.

EXAMPLE.—*To divide* $\frac{6}{11}$ *by* 3.

In this case it is clear that when we divide by 3 the portion consisting of 6 parts (11 of which parts make the complete unit) the result will be 2 of these parts, *i.e.* $\frac{6}{11} \div 3 = \frac{2}{11}$.

Hence to divide any vulgar fraction $\frac{a}{b}$ by an integer c, either

(i) multiply the denominator by c,
(ii) or divide the numerator by c,

i.e. $\frac{a}{b} \div c = \frac{a}{b \times c}$ or $= \frac{a \div c}{b}$.

In the case of mixed numbers, it is frequently most convenient to reduce to improper fractions before division.

EXAMPLE.—*To divide* $1\frac{7}{12}$ by 3.

$$1\tfrac{7}{12} \div 3 = \tfrac{19}{12} \div 3 = \tfrac{19}{36}.$$

EXERCISES XLVIII. a.

Practical.

[*Apparatus: Squared paper. Scales. Set squares.*]

1. Draw a line $\frac{7}{12}$ of an inch in length; mark it off 5 times on another line about 3″ long; show by using a suitable scale that $\frac{7}{12} \times 5$ is $2\frac{11}{12}$.

2. Find graphically the results of the following operations:

(*a*) $3\frac{1}{8} \times 2$. (*b*) $1\frac{3}{4} \times 3$. (*c*) $2\frac{1}{2} \times 3$. (*d*) $1\frac{5}{6} \times 3$.
(*e*) $\frac{7}{8} \times 7$. (*f*) $\frac{3}{5} \times 10$. (*g*) $\frac{5}{6} \times 4$. (*h*) $\frac{3}{4} \times 7$.
(*i*) $\frac{7}{12} \times 7$. (*j*) $1\frac{11}{12} \times 3$.

3. Draw a line $1\frac{7}{8}$″ long; divide it into (*a*) 5, (*b*) 3, equal parts, and find graphically (*a*) $1\frac{7}{8} \div 5$, (*b*) $1\frac{7}{8} \div 3$.

4. Draw a line $3\frac{3}{8}$″ long; and find graphically (*a*) $3\frac{3}{8} \div 3$, (*b*) $3\frac{3}{8} \div 9$.

5. Draw a line $5\frac{3}{4}$″ long; divide it into 4 equal parts, finding thus, $5\frac{3}{4} \div 4$.

6. Draw a rectangle 8 cm. long and 4 cm. broad ; consider it as a unit ; mark off an area representing $\frac{39}{32}$ of the unit ; find graphically the result of dividing the fraction by 5.

7. Use the method suggested by Ex. **6**, to perform the following operations :

(*a*) $\frac{27}{35} \div 9$. (*b*) $\frac{36}{49} \div 6$. (*c*) $\frac{32}{80} \div 8$. (*d*) $\frac{25}{32} \div 5$.

EXERCISES XLVIII. b.

1. Find the value of $\frac{11}{24} \times 5$, and to verify your answer consider the unit as £1, find the value of £$\frac{11}{24}$, multiply the result by 5 and express the answer as a fraction of a £.

2. Evaluate

(*a*) $\frac{13}{20} \times 7$. (*b*) $\frac{8}{25} \times 5$. (*c*) $\frac{12}{8} \times 14$.

To prove your results replace the vulgar fractions by their equivalent decimals ; then multiply and change the answer back again to vulgar fractions.

3. Evaluate

(*a*) $\frac{21}{20} \div 7$. (*b*) $\frac{15}{28} \div 5$. (*c*) $\frac{19}{40} \div 6$. (*d*) $\frac{36}{13} \div 9$. (*e*) $\frac{13}{32} \div 4$.

Prove your result by a method similar to that suggested in Ex. **2.**

Multiplication of one Fraction by another.

EXAMPLE.—*To multiply one vulgar fraction* $\frac{a}{b}$ *by another* $\frac{c}{d}$.

We have to divide $\left(\frac{a}{b}\right)$ into d equal portions ; each of which must be $\frac{a}{b \times d}$, and then take c such portions. The answer therefore $= c \times \frac{a}{b \times d} = \frac{a \times c}{b \times d}$.

Or, the numerators are multiplied together to form the new numerator, and the denominators are multiplied together to

form the denominator; the resulting fraction is generally expressed in its lowest terms. Thus:

EXAMPLE.—$\frac{128}{243} \times \frac{81}{64}$

$$= \frac{128 \times 81}{243 \times 64} = \frac{2 \times 1}{3 \times 1} = \frac{2}{3}.$$

Dividing numerator and denominator by 64, and also by 81.

EXAMPLE.—$\frac{264}{175} \times \frac{35}{99}$

$$= \frac{264 \times 35}{175 \times 99}.$$

In simplifying it is well to note the common factors.

5 clearly divides 35 and 175; 11 and 3 both clearly divide 264 and 99.

Thus the fraction $= \frac{\cancel{11} \times \cancel{3} \times 8 \times \cancel{5} \times 7}{\cancel{5} \times 7 \times \cancel{5} \times \cancel{3} \times \cancel{11} \times \cancel{3}} = \frac{8}{15}$.

The operation of simplifying by cancelling is frequently conducted as follows:

$$\frac{\overset{8}{\cancel{8}}}{\cancel{2}\cancel{8}\cancel{4} \times \cancel{3}\overset{7}{\cancel{5}}} = \frac{8}{15}.$$
$$\cancel{11} \times \cancel{3}\cancel{5} \times \cancel{9}\cancel{9}$$
$$\underset{5}{\cancel{3}\cancel{5}} \quad \underset{3}{\cancel{9}\cancel{3}}$$

This is a clumsy process, and is productive of errors if carried at all far.

EXERCISES XLIX. a.

Practical.

[*Apparatus: Squared paper. Scales.*]

1. Rule your paper into squares of $\frac{1}{8}$ inch side, and find the areas of the following rectangles:

(*a*) Length $\frac{7}{8}''$, breadth $\frac{5}{8}''$.	(*b*) Length $\frac{1}{2}''$, breadth $\frac{3}{8}''$.
(*c*) „ $\frac{3}{8}''$, „ $\frac{6}{8}''$.	(*d*) „ $1\frac{3}{8}''$, „ $\frac{3}{8}''$.
(*e*) „ $\frac{7}{8}''$, „ $\frac{1}{4}''$.	(*f*) „ $\frac{1}{4}''$, „ $\frac{1}{4}''$.
(*g*) „ $\frac{3}{8}''$, „ $\frac{1}{4}''$.	(*h*) „ $\frac{9}{8}''$, „ $\frac{1}{4}''$.

2. Rule paper into squares of $\frac{1}{10}$ inch side, and find the areas of the following rectangles: (*a*) $\frac{9}{10}''$ long and $\frac{7}{10}''$ broad; (*b*) $\frac{3}{5}''$ long and $\frac{1}{10}''$ broad. Find also the area of a square, (*a*) $\frac{3}{10}''$ side; (*b*) $\frac{7}{10}''$ side.

3. From a square of 10 cm. side, a rectangle 9 cm. long by 8 cm. has been taken away. What area remains, and what fraction of the whole is it?

EXERCISES XLIX. b.

1. Multiply (*a*) $\frac{2}{5}$ by $\frac{10}{21}$, (*b*) $\frac{3}{8}$ by $\frac{24}{35}$, (*c*) $\frac{18}{19}$ by $\frac{17}{36}$, (*d*) $\frac{51}{49}$ by $\frac{63}{85}$.

2. „ (*a*) $\frac{12}{36}$ by $3\frac{3}{5}$, (*b*) $\frac{45}{66}$ by $16\frac{1}{2}$, (*c*) $\frac{74}{109}$ by $27\frac{1}{4}$, (*d*) $\frac{143}{156}$ by $\frac{65}{77}$.

3. „ (*a*) $\frac{7}{8}$ of $4\frac{4}{5}$ by $\frac{3}{11}$ of $\frac{7}{9}$, (*b*) $5\frac{5}{19}$ by $\frac{7}{50}$ of $\frac{19}{20}$, (*c*) $3\frac{1}{3}$ by $4\frac{1}{4}$.

4. „ (*a*) $\frac{27}{64}$ of $\frac{28}{15}$ of $\frac{3}{9}$ by $2\frac{3}{4}$ of $4\frac{2}{3}$, (*b*) $\frac{34}{325}$ of $\frac{125}{442}$ by $\frac{108}{153}$.

5. Find the continued product of (*a*) $7\frac{3}{7}$, $16\frac{4}{5}$, $1\frac{12}{13}$; (*b*) $\frac{3}{8}$, $\frac{8}{11}$, $12\frac{14}{19}$.

6. Find the continued product of $\frac{128}{189}$, $\frac{252}{357}$, $\frac{81}{948}$, $\frac{711}{640}$.

EXERCISES XLIX. c.

Practical.

[*Apparatus required: Mathematical instruments. Scales. Squared paper.*]

1. Using your scale, find how many times $\frac{1}{4}$ of an inch is contained in 3 inches. Hence, write down the result of the operation $3 \div \frac{1}{4}$.

2. Similarly, obtain

(*a*) $\frac{5}{2} \div \frac{1}{4}$. (*b*) $\frac{7}{8} \div \frac{1}{8}$. (*c*) $\frac{3}{4} \div \frac{1}{8}$.
(*d*) $3\frac{3}{4} \div \frac{5}{12}$. (*e*) $5\frac{1}{5} \div \frac{13}{10}$. (*f*) $3\frac{1}{2} \div \frac{7}{8}$.
(*g*) $3\frac{1}{2} \div \frac{7}{12}$. (*h*) $3\frac{3}{5} \div \frac{6}{10}$. (*i*) $4\frac{1}{6} \div \frac{5}{12}$.
(*j*) $3\frac{1}{3} \div \frac{5}{12}$.

3. Draw two rectangles, each 7 cm. long and 6 cm. broad; regarding each of these as a unit, rule them off so as to represent the fractions $\frac{2}{7}$ and $\frac{1}{8}$; and from your figures compare these fractions, *i.e.* find what multiple $\frac{2}{7}$ is of $\frac{1}{8}$, or find $\frac{2}{7} \div \frac{1}{8}$.

4. Draw two rectangles, each 15 cm. long and 8 cm. broad; regarding each of these as a unit, rule them off so as to represent the fractions $\frac{4}{15}$ and $\frac{3}{8}$; and from your figures compare these two fractions, *i.e* find what multiple $\frac{4}{15}$ is of $\frac{3}{8}$.

5. Similarly, find by drawing suitable figures the multiples that (*a*) $\frac{13}{16}$ is of $\frac{3}{8}$, and also of $\frac{5}{8}$; (*b*) $\frac{11}{14}$ is of $\frac{5}{6}$, and also of $\frac{1}{6}$.

6. Find graphically, (*a*) $\frac{12}{13} \div \frac{4}{5}$; (*b*) $\frac{8}{13} \div \frac{2}{5}$.

EXERCISES XLIX. d.

1. How many lb. are there in (*a*) $\frac{3}{7}$ of a cwt., (*b*) $\frac{5}{8}$ of a cwt.? What multiple is (*c*) $\frac{5}{8}$ of a cwt., of $\frac{3}{7}$ of a cwt.?

2. What multiple is ($\frac{5}{9}$ of a yard) of ($\frac{2}{3}$ of a yard)?

3. What multiple is ($\frac{5}{9}$ of a yard) of ($\frac{3}{4}$ of a yard)?

4. How many times is $\frac{5}{8}$ of a £ contained in $\frac{7}{3}$ of a £?

5. How often is £$\frac{3}{5}$ contained in $1\frac{5}{8}$ of a £?

6. How many times must £$\frac{7}{32}$ be repeated to make $\frac{5}{9}$ of a guinea?

7. How often may $2\frac{3}{4}$ qr. be taken from $4\frac{2}{5}$ ton?

8. How many lengths of $4\frac{5}{8}$ of a yard can be cut from $38\frac{1}{2}$ yd.?

9. What part of $3\frac{1}{2}$ miles is 220 yd.?

10. How often is $\frac{2}{3}$ ft. contained in 5 yd.?

11. How often is $\frac{5}{4}$ pint contained in $3\frac{1}{8}$ gal.?

12. How often is $\frac{5}{6}$ of a dozen contained in $3\frac{1}{2}$ scores.

Division of One Fraction by Another.

EXAMPLE.—*To divide* $\frac{5}{7}$ *by* $\frac{3}{8}$.

Put differently the question is:

What multiple is $\frac{5}{7}$ of $\frac{3}{8}$?

i.e. ,, ,, ,, $\frac{5 \times 8}{7 \times 8}$ of $\frac{3 \times 7}{8 \times 7}$ *i.e.* $\frac{40}{56}$ of $\frac{21}{56}$.

Now, it is clear that if $\frac{21}{56}$ be divided up into 21 equal parts, each $\frac{1}{56}$, and if 40 of such parts be taken, the result will be $\frac{40}{56}$.

Accordingly, $\frac{40}{56}$ is $\frac{40}{21}$ times $\frac{21}{56}$,

$$i.e.\ \frac{5}{7} \div \frac{3}{8} = \frac{5 \times 8}{7 \times 8} \div \frac{3 \times 7}{8 \times 7} = \frac{5 \times 8}{3 \times 7} = \frac{40}{21}.$$

EXAMPLE.—$\frac{13}{20} \div \frac{9}{7}$

$$= \frac{13 \times 7}{20 \times 7} \div \frac{9 \times 20}{7 \times 20}.$$

$$= \frac{13 \times 7}{140} \div \frac{9 \times 20}{140}.$$

And (13×7) one-hundred-and-fortieths is $\frac{13 \times 7}{9 \times 20}$ times (9×20) one-hundred-and-fortieths.

$$\therefore \frac{13}{20} \div \frac{9}{7} = \frac{13 \times 7}{20 \times 9} = \frac{91}{180}.$$

Similarly, any vulgar fraction $\frac{a}{b}$ may be divided by another vulgar fraction $\frac{c}{d}$. Thus

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times d} \div \frac{b \times c}{b \times d} = \frac{a \times d}{b \times c},$$

whence is obtained the following rule:

To divide one quantity by a fraction, invert the divisor and multiply.

EXAMPLE.—$\frac{156}{225} \div \frac{13}{15}$

$$= \frac{156}{225} \times \frac{15}{13} = \frac{12 \times 13 \times 15}{15 \times 15 \times 13} = \frac{12}{15} = \frac{4}{5}.$$

If one of the fractions be mixed, it should be first converted into an improper fraction.

EXERCISES L.

1. Divide $1\frac{45}{128}$ by (*a*) $\frac{9}{16}$, (*b*) $\frac{3}{32}$, (*c*) $\frac{15}{64}$, (*d*) $3\frac{1}{8}$, (*e*) $1\frac{57}{112}$.

2. Evaluate:

(*a*) $2\frac{2}{4} \div \frac{3}{2}$. (*b*) $\frac{144}{225} \div \frac{36}{25}$. (*c*) $\frac{64}{117} \div \frac{21}{78}$.

(*d*) $\frac{162}{205} \div 1\frac{11}{25}$. (*e*) $1\frac{17}{108} \div 15\frac{5}{8}$.

3. Evaluate, checking your results by decimals when convenient:

(*a*) $1\frac{7}{8} \div \frac{5}{4}$. (*b*) $3\frac{2}{5} \div \frac{8}{25}$. (*c*) $4\frac{9}{10} \div 1\frac{24}{25}$.

(*d*) $2\frac{13}{16} \div 1\frac{3}{5}$. (*e*) $\frac{153}{160} \div \frac{9}{10}$.

4. Evaluate:

(*a*) $\frac{57}{84} \div (1\frac{27}{49}$ of $\frac{7}{12})$. (*b*) $(\frac{57}{84} \div 1\frac{27}{49})$ of $\frac{7}{12}$.

(*c*) $1\frac{1}{27}$ of $[\frac{48}{187} \div (1\frac{13}{68}$ of $1\frac{5}{11})]$. (*d*) $(1\frac{1}{27}$ of $\frac{48}{187}) \div (1\frac{13}{68}$ of $1\frac{6}{11})$.

(*e*) $(1\frac{33}{72}$ of $\frac{18}{41}) \div (1\frac{151}{164}$ of $1\frac{54}{81})$.

EXERCISES LI.

Miscellaneous.

1. Express 10*s.* 6*d.* as a fraction of 15*s.* State the fraction in its lowest terms.

2. Express £2. 2*s.* 6*d.* as a fraction of £7. 1*s.* 8*d.*

3. Express three and a half guineas as a fraction of £5. 8*s.* 6*d.*

4. When 4*s.* 7*d.* has been paid from a bill of 17*s.* 5*d.*, what fraction of the bill remains?

5. Express 7 ton 17 cwt. 2 qr. as a fraction of 18 tons.

6. Express 20 yds. 28 in. as a fraction of 1 chain.

7. On an estate plan a rectangular block $12\frac{1}{2}'' \times 8\frac{1}{4}''$ represents 56 sq. yds. 24 sq. in.; what fraction is the plan, of the area it represents?

8. Express 2 qts. 1 pint as a fraction of 36 quarts.

9. A person travelled 56 miles by rail, 24 miles by coach and completed the journey of 112 miles by riding. Express as fractions of the entire length of the journey, the distances covered by each method of travelling.

10. When a cargo of 252 tons had been loaded in a ship, it was found that $\frac{5}{18}$ of the available tonnage was used; what was the total tonnage?

11. What fraction is 8 lb. 3 oz. of 2 lb. 14 oz.? Express the answer both as a mixed number and as an improper fraction.

12. What fraction of £2. 8*s.* is £1. 6*s.* 8*d.*?

13. How often is $\frac{1}{14}$ contained in the sum of $\frac{1}{7}$ and $\frac{5}{8}$?

14. Divide the sum of $\frac{4}{15}$ and $\frac{7}{9}$ by the product of $3\frac{1}{8}$ and $\frac{16}{25}$.

15. What is the difference between the product of $\frac{11}{12}$ and $7\frac{1}{5}$, and the sum of $2\frac{3}{5}$ and $3\frac{2}{7}$?

16. What must be added to the product of $\frac{1}{27}$ and $\frac{45}{56}$ to make $\frac{3}{2}$?

17. Arrange the following in ascending order of magnitude:

(*a*) $\frac{2}{4}+\frac{13}{14}$. (*b*) $2\frac{2}{3}-1\frac{1}{12}$. (*c*) $\frac{7}{4}\times\frac{19}{21}$. (*d*) $3\frac{1}{7}\div 1\frac{5}{7}$.

18. By what quantity must the sum of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{9}$ be multiplied to yield $1\frac{31}{54}$?

Evaluate the following expressions:

19. (*a*) $\dfrac{\frac{2}{3}(1\frac{1}{2}+2\frac{1}{4})}{\frac{3}{5}(2\frac{1}{4}-1\frac{1}{2})}$. (*b*) $\frac{7}{9}\times\frac{21}{25}\div(1\frac{4}{15}-\frac{4}{5})$. (*c*) $\dfrac{\frac{7}{15}+\frac{9}{35}}{\frac{5}{14}\times\frac{12}{15}}$.

20. (*a*) $\dfrac{(7\frac{4}{5}\text{ of }\frac{6}{13})-\frac{7}{15}}{\frac{6}{11}+\frac{17}{33}-\frac{5}{22}}$. (*b*) $\dfrac{6\frac{5}{6}+3\frac{7}{9}-8\frac{11}{18}}{4\frac{1}{6}\div 4\frac{1}{6}}$.

21. (*a*) $\dfrac{4\frac{2}{7}\text{ of }(\frac{13}{20}+\frac{11}{15})}{(4\frac{2}{7}\text{ of }\frac{13}{20})+\frac{11}{15}}$. (*b*) $\dfrac{7\frac{1}{8}+13\frac{1}{12}\text{ of }(3\frac{4}{15}\div 1\frac{2}{5})}{8\frac{2}{3}-4\frac{5}{8}}$.

22. (*a*) $\dfrac{4\frac{3}{7}-\frac{5}{9}\text{ of }4\frac{2}{13}+6\div 4\frac{3}{13}}{18\frac{99}{91}\div 8-\frac{2}{3}\text{ of }1\frac{6}{7}}$. (*b*) $\dfrac{6\frac{3}{17}\text{ of }3\frac{2}{5}-\frac{4}{11}\text{ of }12\frac{5}{8}}{11\frac{39}{41}\div(7\frac{1}{8}+3\frac{1}{12})}$.

23. Find the average Bank of England minimum rate of discount for each of the years from 1893 to 1903. State your answers as mixed numbers.

BANK OF ENGLAND MINIMUM RATE OF DISCOUNT, 1893 TO 1903.

MONTHS.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	1901.	1902.	1903.
January	$2\frac{9}{10}$	3	2	2	$3\frac{7}{8}$	3	$3\frac{3}{4}$	$4\frac{15}{16}$	$4\frac{15}{16}$	$3\frac{17}{20}$	4
February	$2\frac{1}{2}$	$2\frac{3}{8}$	2	2	$3\frac{1}{8}$	3	3	4	$4\frac{7}{16}$	$3\frac{1}{11}$	4
March	$2\frac{1}{2}$	2	2	2	3	3	3	4	4	3	4
April	$2\frac{1}{2}$	2	2	2	$2\frac{7}{8}$	$3\frac{13}{16}$	3	4	4	3	4
May	$3\frac{1}{2}$	2	2	2	$2\frac{1}{4}$	$3\frac{7}{8}$	3	$3\frac{13}{16}$	4	3	$3\frac{4}{5}$
June	3	2	2	2	2	3	3	$3\frac{3}{16}$	$3\frac{1}{4}$	3	$3\frac{1}{4}$
July	$2\frac{1}{2}$	2	2	2	2	$2\frac{1}{2}$	$3\frac{5}{16}$	$3\frac{3}{8}$	3	3	3
August	4	2	2	2	2	$2\frac{5}{8}$	$3\frac{1}{2}$	4	3	3	3
September	$4\frac{1}{4}$	2	2	$2\frac{7}{15}$	$2\frac{1}{8}$	$2\frac{5}{8}$	$3\frac{1}{2}$	4	3	3	$3\frac{9}{10}$
October	3	2	2	$3\frac{1}{3}$	$2\frac{5}{8}$	$3\frac{5}{8}$	$4\frac{1}{8}$	4	3	$3\frac{30}{31}$	4
November	3	2	2	4	3	4.	5	4	4	4	4
December	3	2	2	4	3	4	6	4	4	4	4
Average											

24. Evaluate (*a*) $\frac{2055}{3961} \div \frac{2466}{1631}$. (*b*) $\frac{6023}{12493} \div \frac{5117}{2015}$.

25. Evaluate (*a*) $\frac{5}{1143} - \frac{7}{1651}$. (*b*) $\frac{10}{1096} - \frac{7}{1644} + \frac{121}{3288}$.

26. Reduce to their lowest terms:

(*a*) $\frac{52275}{2743}$. (*b*) $\frac{58899}{4981}$. (*c*) $\frac{67334}{93995}$.

38. GREATEST COMMON MEASURE.

When a fraction has to be reduced to its lowest terms, the numerator and denominator are divided by *all* the factors which may be common to both. In most of the fractions dealt with so far, the process has been simple. We will now consider cases where the factors are not quite so obvious.

If both numerator and denominator can be split completely into their "prime" factors, it will be easy to recognize those which are common.

Prime Factors are those which are incapable of being split up any further.

We must begin, therefore, by stating rules which determine whether or not, some of the more common numbers are factors.

(*a*) **2.** All even numbers are divisible by 2.

(*b*) **4.** If the number expressed by the last two digits is divisible by 4, the number itself is divisible by 4.

(*c*) **8.** If the number expressed by the last three digits is divisible by 8, the number itself is divisible by 8.

(*d*) **3 or 9.** If the sum of the digits be a multiple of 3, the number itself is a multiple of 3, and this rule applies also to 9. Any number is a multiple of 9 if the sum of the digits is a multiple of 9.

(*e*) **11.** Find the difference between the sum of the digits in the units, hundreds, tens of thousands, and other odd number of places, and the sum of those in the tens, thousands, and other even places. If this difference be 0 or a multiple of 11, then the number is divisible by 11.

(*f*) **5.** Every number ending in 5 or 0 is a multiple of 5.

(*g*) **25.** Every number ending in 00, 25, 50 or 75 is a multiple of 25.

Proof of (*e*).

Consider any number such as 31742.

It may be regarded as

$$3 \times 10000 + 1 \times 1000 + 7 \times 100 + 4 \times 10 + 2.$$

Thus $= 3(9999 + 1) + 1(1001 - 1) + 7(99 + 1) + 4(11 - 1) + 2$

$= 3(11 \times 909) + 1(11 \times 91) + 7(11 \times 9) + 4(11)$

$+ 3 - 1 + 7 - 4 + 2.$

$= 11(3 \times 909 + 1 \times 91 + 7 \times 9 + 4) + 3 - 1 + 7 - 4 + 2.$

On dividing by 11, the remainder is the same as that obtained on dividing $3 - 1 + 7 - 4 + 2$, and the number can be divided exactly if $3 - 1 + 7 - 4 + 2$ can be divided exactly by 11, and similarly for any other number.

EXAMPLE.—*To simplify* $\frac{10296}{12420}$, *i.e. to reduce the fraction to its lowest terms.*

$10296 = 8 \times 1287 = 2^3 \times 1287$ — 8 divides the number expressed by the last 3 digits, *i.e.* 296.

$= 2^3 \times 3^2 \times 143$ — The sum of the digits in 1287 is clearly a multiple of 9.

$= 2^3 \times 3^2 \times 11 \times 13.$ — 143 is seen to be a multiple of 11 $\because 1 - 4 + 3 = 0$.

The expression is now split up into its prime factors.

$12420 = 4 \times 3105 = 2^2 \times 3105$ — 4 divides the number expressed by the last 2 digits, *i.e.* 20.

$= 2^2 \times 5 \times 621$ — 3105 ending in 5 is divisible by 5.

$= 2^2 \times 5 \times 3^2 \times 69$ — 621 has the sum of its digits equal to 9.

$= 2^2 \times 5 \times 3^3 \times 23.$ — 12420 is now split up into its prime factors.

$$\therefore \frac{10296}{12420} = \frac{2^3 \times 3^2 \times 11 \times 13}{2^2 \times 5 \times 3^3 \times 23} = \frac{2 \times 11 \times 13}{3 \times 5 \times 23} = \frac{286}{345}.$$

EXERCISES LII. a.

1. Split up into prime factors the following numbers:

(*a*) 225. (*b*) 132. (*c*) 792.
(*d*) 1375. (*e*) 504. (*f*) 1024.
(*g*) 256. (*h*) 384. (*i*) 320.

2. Simplify the following fractions:

(*a*) $\frac{2^3 \times 3 \times 5}{3^2 \times 2 \times 5^2}$. (*b*) $\frac{9 \times 11 \times 7}{3^3 \times 8 \times 11}$. (*c*) $\frac{3960}{5445}$.

(*d*) $\frac{1221}{2332}$. (*e*) $\frac{960}{528}$. (*f*) $\frac{524}{1179}$.

(*g*) $\frac{936}{715}$. (*h*) $\frac{999}{2035}$. (*i*) $\frac{99}{999}$.

Simplify the following fractions:

(*j*) $\frac{99}{9999}$. (*k*) $\frac{999}{9999}$. (*l*) $\frac{570}{912}$.

(*m*) $\frac{162}{567}$. (*n*) $\frac{1792}{3328}$. (*o*) $\frac{1125}{3125}$.

(*p*) $\frac{1287}{5049}$.

DEFINITION.—The greatest number which can divide exactly any given numbers is called their **Greatest Common Measure.**

EXAMPLE.—*To find the Greatest Common Measure of* 15000 *and* 4400.

Resolve the numbers into their prime factors, *i.e.* $2^3 \times 3 \times 5^4$ and $2^4 \times 5^2 \times 11$. Clearly the common factors are 2^3 and 5^2, and the Greatest Common Measure $= 2^3 \times 5^2 = 200$.

EXERCISES LII. b.

1. Find the Greatest Common Measure (G.C.M.) of the following:

(*a*) $2^3 \times 3^2 \times 5$ and $2^2 \times 3^3 \times 11$. (*b*) $2^4 \times 3^3 \times 5^2$ and $5^4 \times 3^3 \times 2^2$.

(*c*) 111 and 370. (*d*) 1728 and 1584.

(*e*) 1325 and 3125. (*f*) 252 and 297.

(*g*) 384, 480 and 576.

2. The frequencies of the vibrations of the notes comprising the major chord C, E, G are:

C, 256. E, 320. G, 384.

What are the three simplest numbers which can be employed in finding the ratio of these frequencies?

3. Draw up a table of all the prime numbers between 1 and 100.

4. A number lying between 1000 and 10000 is set up in type, the type becomes mixed, one figure is lost, the others being 3, 6 and 4; the number is, however, known to be divisible by 72. What was the missing figure, and what the actual number?

When in endeavouring to find the G.C.M. of two numbers, the factors are not easily found, a more general method has to be adopted. Its principle will be seen from the following graphical examples:

Graphical Introduction to General Method.

Suppose ABCD (Fig. 27) represents the plan of a room 84 feet long, 57 feet wide, and that we wish to cover the floor of the room with large square tiles none of which are to be cut. What is the biggest tile which can be employed?

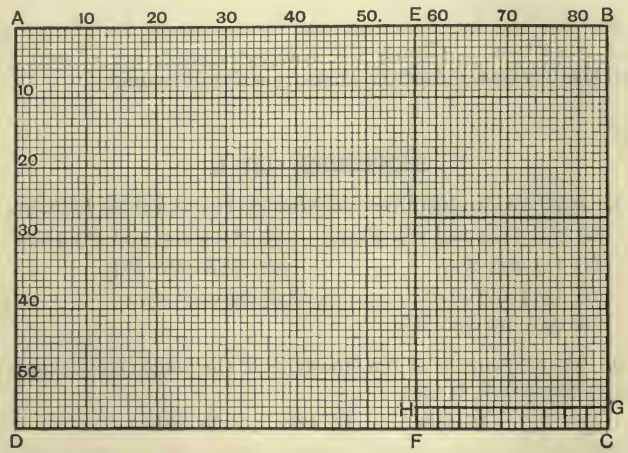


FIG. 27.

Mark off a distance equal to the breadth AD along the length AB, reaching as far as E say. Then, since the tiles fit exactly as far as D, they will extend exactly as far as E, and the edges of some of the tiles will therefore form a line down from E parallel to AD; draw this line; let it be EF. Then EBCF can be floored exactly with the same sort of tiles as the floor ABCD.

Set off distances, equal to EB, along BC—once, twice—reaching as far as G; through G draw GH parallel to CF, meeting EF at H. Then the tiles must cover GCFH exactly.

Mark next FH along FC; this can be done once—twice ... 9 times exactly, and GCFH can be floored exactly with 9 tiles, each 3 ft. in length; and 3 ft. is the side of the greatest tile which can be employed to cover the floor and 3 is the greatest common measure of 57 and 84.

EXERCISES LII. c.

Graphic.

[*Apparatus: Squared paper.*]

1. (*a*) A room is 11·7 feet long and 9·1 feet broad. If its floor is to be covered by square parquets, find the greatest size of a parquet in order that no parquet need be cut. Show on your drawing (to any convenient scale) the plan of the floor with its parquetry. Find also the largest parquet which could be used for a room, (*b*) 13·6 ft. long 11·9 ft. broad, (*c*) 15·4 ft. long, 12·6 ft broad.

2. A room is 16·5 feet long by 10·5 feet broad. Find the length of the largest pattern that may be employed on a frieze, in order that the pattern may repeat an exact number of times on the frieze.

3. The circumference of one wheel is 114 millimetres, the circumference of a second is 84 millimetres. If each wheel has to be cut into teeth of the same size, what is the greatest width of a tooth; state also how many teeth there will be on each wheel?

4. Find the G.C.M. of
(*a*) 87 and 63. (*b*) 64 and 76. (*c*) 135 and 215.

5. A rectangle is 15·5 cm. long and 11·5 cm. broad; find the least number of exact squares into which it might be cut.

General Method.

The general method for finding arithmetically the G.C.M. of two numbers is now clear.

EXAMPLE.—*To find the* G.C.M. *of* 1794 *and* 4069.

```
1794 ) 4069 ( 2
       481
       ----
481 ) 1794 ( 3
       351
       ----
351 )  481 ( 1
       130
       ----
130 )  351 ( 2
        91
        --
 91 )  130 ( 1
        39
        --
 39 )   91 ( 2
        13
        --
 13 )   39 ( 3
```

Divide 4069 by 1794; the quotient is 2 and remainder 481. The required G.C.M. is also the G.C.M. of 481 and 1794.

Divide 481 into 1794; the quotient is 3, remainder 351. The G.C.M. is therefore also the G.C.M. of 351 and 481.

Proceeding in this way, we see, finally, that the required G.C.M. is the Greatest Common Measure of 13 and 39.

But 13 divides 39 *exactly*, and hence 13 is the required Greatest Common Measure.

NOTE.—The quotients need not be written down, and the whole work may be shortened into the following form:

```
1794 | 4069
 351   481
  91   130
  13    39
```

The work may frequently be further shortened. When at any stage factors which are clearly not common can be recognized, these may be disregarded and discarded. Thus, in the preceding example, at the stage 91 130 the irrelevant factor **10** can be discarded, and 13 is at once seen to be the G.C.M.

EXERCISES LII. d.

1. A boy has two sticks, one 144 inches long, the other 78 inches long; he wants to cut them up into an exact number of pieces, each of the same length. What is the length of the longest piece that he can obtain in this way? How many such pieces will he then have?

2. Two persons A and B are distributing sums of money to poor people, each person receiving the same amount. If A distributed £21. 15*s.* and B distributed £8. 5*s.*, what is the highest possible amount that a poor person could have received?

3. Find the G.C.M. of

(*a*) 1965 and 2227. (*b*) 1703 and 3799. (*c*) 1631 and 2796.
(*d*) 1635 and 1853. (*e*) 7625 and 8175.

4. Reduce to their lowest terms the following fractions:

(*a*) $\frac{1903}{2595}$. (*b*) $\frac{2548}{2912}$. (*c*) $\frac{2537}{3354}$. (*d*) $\frac{3887}{3542}$.

(*e*) $\frac{3780}{5904}$. (*f*) $\frac{1356}{4181}$. (*g*) $\frac{1024}{2112}$. (*h*) $\frac{951}{6023}$.

(*i*) $\frac{2805}{4301}$. (*j*) $\frac{772}{965}$. (*k*) $\frac{204}{595}$. (*l*) $\frac{336}{1001}$.

(*m*) $\frac{1073}{2117}$. (*n*) $\frac{3612}{10668}$. (*o*) $\frac{2117}{3431}$. (*p*) $\frac{2923}{5587}$.

5. There are three glasses of volumes 336 c.c., 90 c.c., 528 c.c., respectively. Find the greatest volume of a measure (known but not graduated) which could be employed to determine their volumes if these volumes had not been known.

6. What is the smallest number of equal weights which can be obtained by dividing up completely, a weight of 1572 lbs., a second of 1392 lbs., a third of 1140 lbs. and a fourth of 1044 lbs.?

7. A piece of timber 375·0 inches long, 34·5 inches broad and 24·0 inches thick is to be cut completely up into a number of equal cubes. Find the size of the greatest possible cube.

8. What is the least number of equal cubes which can be arranged so as to form a block 473 inches long, 176 inches broad, 143 inches thick?

9. A portion of a breakwater is to be 121·6 ft. long, 53·2 ft. broad, 22·8 ft. thick, and is to be built of the largest cubical blocks of granite possible. Find the size of the blocks to be used.

10. There are three heaps of bricks numbering respectively 6194, 2282 and 7498, and they are to be carted away by taking away each time the same number of loads from each heap. What is the greatest number of bricks that can be taken per load, and what will be the total number of loads?

11. The perimeter of one garden is 1222 inches, the perimeter of another is 2162 inches. What is the length of the longest piece of string which may be used to compare their perimeters?

12. I wish to distribute 480 oranges and 1184 apples between some school children, giving an equal number of oranges or apples to each child. What is the greatest number of each fruit a child can have?

13. What is the greatest weight which can be taken an exact number of times from 15 cwt. 2 qrs. 16 lbs. and 10 cwt. 8 lbs. without leaving any remainder?

14. Two companies of soldiers numbering respectively 1632 and 3072 are to be arranged in parallel rows with the same number of men in each. What is the greatest number to a row so as to admit of this being done, and how many rows will there be?

15. What is the greatest amount which can be paid an exact number of times from each of the sums, £4. 15*s*., £26. 5*s*. 8*d*. and £13. 6*s*. without leaving any remainder?

16. There are four heaps of balls, A, B, C and D, numbering respectively 1071, 567, 2583, 1449. They are to be re-arranged in groups so as to have the same number in each group, and without mixing balls from different heaps. What is the greatest number which can be put to a group, and how many such groups will there be?

39. LEAST COMMON MULTIPLE.

Suppose we wish to add the three vulgar fractions $\frac{5}{12}$, $\frac{3}{4}$ and $\frac{4}{15}$. Each fraction must be changed into an equivalent fraction, and these three equivalent fractions must have the same denominator, and it is clearly important that this Common Denominator should be the smallest possible. Thus, in the case cited, we change

$$\frac{5}{12}, \frac{3}{4} \text{ and } \frac{4}{15}$$

into

$$\frac{25}{60}, \frac{45}{60} \text{ and } \frac{16}{60}.$$

The sum is therefore $\frac{86}{60}$; and 60 is the least number which is a multiple of 12, and 4, and 15; in other words, their **Least Common Multiple.**

In the same way we would proceed in adding—or subtracting—other fractions.

EXAMPLE.—*To find the Least Common Multiple of* 240, 128, 225.

Begin by splitting each number up into its prime factors thus:

$$240 = 8 \times 30 = 2^3 \times 2 \times 3 \times 5 = 2^4 \times 3 \times 5.$$
$$128 = 8 \times 16 = 2^3 \times 2^3 \times 2 \quad = 2^7.$$
$$225 = 25 \times 9 = 5^2 \times 3^2.$$

The multiple must therefore contain a 2, repeated 4 times so far as is necessary for 240 to be a factor; but 128 contains 2 repeated as a multiple 7 times, and therefore the Lowest Common Multiple (frequently represented by L.C.M.) must contain 2^7 as a factor.

Similarly, it must contain 5^2 and 3^2, and its value therefore

$$= 5^2 \times 3^2 \times 2^7 = 28800.$$

EXAMPLE.—*To find the Least Common Multiple of* $3^2 \times 7^3 \times 2^4$ *and* $5^3 \times 7^2 \times 2$ *and* $3^3 \times 5^5 \times 7^1$.

The L.C.M. of $3^2 \times 7^3 \times 2^4$, and $5^3 \times 7^2 \times 2^1$, and $3^3 \times 5^5 \times 7^1$ will be $3^3 \times 7^3 \times 2^4 \times 5^5$, in which it will be noted that

(1) *Each* prime factor occurring in the given numbers, occurs in the L.C.M.

(2) The power associated with that factor in the L.C.M., is the *highest* power associated with that factor in any of the given numbers.

We may express these rules, using symbols instead of numbers. Suppose one number when split up into its prime factors $= a^x \times b^y \times c^z \times e^u$,

and that a second number $= a^X \times b^Y \times c^Z \times d^W$,

where X is greater than x,

Y ,, ,, y,

Z ,, ,, z.

Then the L.C.M. $= a^X \times b^Y \times c^Z \times e^u \times d^W$.

EXERCISES LIII. a.

Mental or Oral.

State or write down the L.C.M. of the following in factor form, *without evaluating*,

1. $2^2 \times 3$, $3^2 \times 2$.
2. $3^5 \times 5^3$, $3^4 \times 5^4$.
3. $2^4 \times 3 \times 5^2$, 2×3^2, $2^3 \times 5^3$.
4. $11^2 \times 2^4 \times 5^3$, $11 \times 2^1 \times 3^3$, $5 \times 3 \times 2$.
5. $7^4 \times 3^1 \times 2^3$, $7^1 \times 5^4 \times 2^3$, $7^1 \times 5^1 \times 3^7$, $5 \times 3^2 \times 2^2$.
6. $13 \times 4^2 \times 2^1$, $4^1 \times 2^2 \times 13^2$.
7. $8^1 \times 9$, $9^2 \times 8^3 \times 2^1$, $3^2 \times 9 \times 4$.
8. $5 \times 10^2 \times 4^2$, $4^2 \times 10 \times 5^2$.
9. 7×8, 4×7, 8×4, 3×6.
10. $9^2 \times 5$, 5×4^2, 10×3.

General Method.

Occasionally it may be difficult to split up each term into its prime factors. In these cases, and in these cases only, the Least Common Multiple is obtained by successive applications of the principles involved in Greatest Common Measure.

EXAMPLE.—*To find the* L.C.M. *of* 1391 *and* 1819.

The G.C.M. of 1391 and 1819 can be found in the ordinary way to be 107.

By division we see that

$$1391 = 107 \times 13$$
$$1819 = 107 \times 17$$
$$\text{And the L.C.M. } \therefore = 107 \times 13 \times 17$$
$$= 23647.$$

And generally, suppose A and B are any two numbers of which c is the G.C.M.

Then A contains c as one factor; let a contain the others, so that $A = a \times c$.

Then B contains c as one factor; let b contain the others, so that $B = b \times c$; and as there are no factors common to a and b, the L.C.M. $= a \times b \times c$, of which a common form is

$$\frac{a \times c \times b \times c}{c}, \text{ i.e. } \frac{AB}{c}.$$

If we wish to find the L.C.M. of three or more numbers A, B, C, D,

we find the L.C.M. of A, B, say E,
then ,, L.C.M. of E and C, say F,
and ,, L.C.M. of F and D, say G.

G will be the required L.C.M. of A, B, C, D.

EXERCISES LIII. b.

Least Common Multiple and Greatest Common Measure.

1. Find the Least Common Multiple of
$$3^4 \times 5^2 \times 2^4 \text{ and } 3^2 \times 5^3 \times 2 \times 7.$$

2. Find the Least Common Multiple of
$$7^2 \times 2^6 \times 3, \quad 7 \times 3^2 \times 2, \quad 5 \times 11 \times 3^3.$$

3. Find the Least Common Multiple of
   (*a*) 65, 78, 26. (*b*) 192, 256 and 600.
   (*c*) 111, 12, 37. (*d*) 4, 14, 18, 24.

4. Express as fractions with their denominators equal and as small as possible :

(*a*) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{5}$. (*b*) $\frac{11}{24}, \frac{12}{25}, \frac{3}{20}, \frac{7}{15}$. (*c*) $\frac{11}{39}, \frac{17}{91}, \frac{2}{7}, \frac{3}{5}$.

(*d*) $\frac{5}{17}, \frac{31}{68}, \frac{11}{12}, \frac{4}{119}$. (*e*) $\frac{6}{13}, \frac{4}{9}, \frac{3}{8}, \frac{11}{144}$.

5. Arrange in descending order of magnitude the following fractions :

(*a*) $\frac{52}{195}, \frac{209}{780}, \frac{125}{468}$. (*b*) $\frac{23}{56}, \frac{32}{77}, \frac{10}{24}, \frac{17}{42}$. (*c*) $\frac{85}{728}, \frac{109}{936}, \frac{59}{504}$.

6. What is the least sum of money which can be paid exactly with an exact number of florins and crowns ?

7. What is the least distance that can be measured exactly by rods 4 ft. and 6 ft. long ?

8. A vessel completely full of water can be emptied by filling from it a beaker of 54 c.c. content, pouring the water away from the beaker and repeating the process. This can also be done with a beaker of 36 c.c. content. Find the least possible volume of the vessel.

9. A fire hearth can be exactly covered by a number of 6 in. square tiles and of 8 inch square tiles. What is the least sized hearth which will admit of this, and how many tiles of each size will be needed ?

10. The floor of a recess may be covered by an exact number of bricks 9″ by 3″, or square tiles of 8″ side, or slabs 1 ft. square. What is the least area which will admit of this, and how many of each kind will be required ?

Evaluate the following :

11. $\frac{5}{57} - \frac{3}{76}$.

12. $\frac{8}{27} + \frac{7}{72} - \frac{3}{16}$.

13. $\frac{9}{32} - \frac{5}{9} + \frac{17}{24}$.

14. $\frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6} - \frac{6}{7}$.

15. $\frac{13}{15} - \frac{17}{18} + \frac{11}{27}$.

16. $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7}$.

17. How much is the largest of the following fractions greater than the least, $\frac{11}{24}, \frac{61}{128}, \frac{13}{27}, \frac{7}{16}$?

18. By how much must the larger of the following fractions be diminished to be equal to the smaller, $\frac{143}{289}, \frac{58}{119}$?

19. Find the least number which is an exact multiple of 51 and of 68.

20. A series of numbers is such that each number on being divided by 6 leaves a remainder 1, on being divided by 8 leaves a remainder 3; show that the numbers in the series increase by 24; find the smallest number of the series.

21. A scale graduated in twelfths of an inch is placed with its edge against that of another scale graduated in sixteenths of an inch. The first graduation on each scale is at its end, and the scales are both 2 feet long; where will the graduations coincide?

22. Find the smallest number which has a remainder 2 when divided by 126 and also by 432.

23. Find the smallest number which has a remainder 3 when divided by each of the numbers, 75, 24, 40, 15.

24. What is the smallest number which when increased by 5 is divisible by 28, 36, 63 and 108?

25. The children of a school may be arranged in classes of 24, 25, or 15, with none over. What is the least number of scholars admitting of this?

26. When a hogshead of sugar was packed in 2 qr. packets there were 12 lb. over, and when packed into 48 lb. packets there were also 12 lb. over. Find the least weight of sugar to allow this being done.

27. What is the least sum of money which can be paid using one kind only of any of the present silver coins of the kingdom?

28. A sum of money is to be arranged in piles containing £1. 12*s.* and so that there is a single 1*s.* over, and also into piles of £3 and again a single 1*s.* over. What is the smallest amount with which this can be done?

29. A number lies between 1500 and 2000; the number is known to be exactly divisible both by 102 and 36; find the number.

30. Find the smallest number lying between 1000 and 2000 which is divisible by 35, 112 and also by 280.

31. Three men in walking take paces of 32″, 30″, and 28″ respectively. They start together. What is the smallest distance walked when each man shall have reached it by an exact number of paces?

32. What is the smallest sum in English money which can be paid either by an exact number of francs (each $9\frac{1}{2}d.$) or by an exact number of shillings, and how many francs would there be in this sum?

33. Each of two rolling wheels has a pin on its circumference which strikes a bell as the wheel turns. Given that the circumferences are 54 in. and 198 in., and that each pin strikes its bell at the same instant in starting, how many times will the bells be struck simultaneously in going a distance of half a mile?

34. On dividing 740 by a certain number the remainder is 2, but on dividing 987 by the same number the remainder is 3; find the greatest value of the number.

35. A number divides both 1728 and 1296; find the number, knowing that it lies between 50 and 80.

36. A cubical block is made up of bricks each 28 cm. long, 16 cm. wide and 14 cm. thick, all laid the same way; find the smallest possible number of bricks in the block.

37. Two scales are placed with their edges alongside one another. In one of the scales the divisions are 0·05 inch apart, in the other scale the divisions are 0·045 inch apart. What is the smallest distance between divisions exactly opposite one another?

DEFINITION.—A number is said to be a **perfect square** when it can be split up into two equal factors: thus 49 is a perfect square, since $49 = 7 \times 7 = 7^2$. If the number consists of 3 equal factors, the number is a **perfect cube**, and so on.

38. Find the smallest numbers, the product of which with (*a*) 1008, (*b*) 11025, (*c*) 7936 will be a perfect square.

39. Find the smallest number which is a perfect square, and contains 6336 as a factor.

40. Find the smallest number containing in the same way as in Ex. 39, (*a*) 1728, (*b*) 31212.

41. Find the least number which is a perfect cube, and which contains (*a*) 1875 as a factor, also which contains (*b*) 432, (*c*) 8575, (*d*) 7986 as factors.

42. On dividing 21892 by 12, by 15, and by 35, the remainders are 6, 7 and 17 respectively. What is the number nearest to 21892, and less than it, which would leave the same remainders? How many numbers less than 21892 are there which would have these same remainders?

43. What is the greatest factor of 2475 which is also a perfect square?

44. Write down a series of numbers, less than 50, each of which when divided by 3 leaves a remainder 1; write down a series of numbers, also less than 50, each of which when divided by 7 leaves a remainder 6. Note the numbers common to the two series, and then write down a series formed by the 10 smallest numbers, such that when divided by 3 the remainder is 1, and when divided by 7 the remainder is 6.

40. PROPORTION.

The principle involved is best understood by considering a number of examples.

EXAMPLE.—*A grocer sells 4 lb. of tea for 9s.; what will be the price of 3 lb. of tea?*

Here it is clear that

(1) If no tea is sold no money would be received, and if no money be received no tea would be sold.

(2) For *each* lb. of tea sold, presumably the same amount of money would be received.

We reason, therefore, as follows:

The selling price of 4 lb. of tea is 9*s.*

„ „ 1 lb. of tea is $\frac{1}{4}$ the price of 4 lb., *i.e.* $\frac{1}{4}$ of 9*s.*

The selling price of 3 lb. of tea is 3 times the cost of 1 lb., *i.e.* $\frac{3}{4}$ of 9*s.*, *i.e.* 6*s.* 9*d.* ∴ 3 lb. of tea would cost 6*s.* 9*d.*

We should generally shorten the work in practice as follows:

Selling price of 4 lb. is 9*s.*

„ „ 3 lb. is $\frac{3}{4}$ of 9*s.*, *i.e.* 6*s.* 9*d.*

EXAMPLE.—*A man buys* 12 *cwt. of copper for* £48; *how much could he buy for* £56?

Again we satisfy ourselves that

(1) If no money be paid no copper could be obtained, and if no copper be received no money would be expended.

(2) For each £ expended, the same amount of copper would be received (if the amount of money expended be very large, this might not be the case).

We reason then as before:

£48 buys 12 cwt. of copper.

£56 „ $\frac{56}{48} \times 12$ cwt. of copper, *i.e.* $\frac{56 \times 1}{4}$ cwt., *i.e.* 14 cwt.

∴ £56 is the buying price of 14 cwt. of copper.

Care must be taken to see if the principle involved can really be applied. Consider the following case:

The distances through which a body falls vertically from rest are nearly as follows:

In the first second, 16 feet.

„ „ 2 seconds, 64 feet.

„ „ 3 seconds, 144 feet.

„ „ 4 seconds, 256 feet.

Incorrect results would therefore be obtained if, knowing simply the distance fallen through in 3 seconds, we tried to calculate the distance the body would fall through in other times by reasoning as follows:

A body falls through 144 feet in 3 seconds.

∴ „ „ 1 foot in $\frac{3}{144}$ sec. or $\frac{1}{48}$ sec.

„ „ 16 feet in $\frac{1}{48} \times 16$, *i.e.* in $\frac{1}{3}$ sec.

According to this calculation, a body falls 16 feet in $\frac{1}{3}$ sec., whereas it takes 1 sec. to fall that distance in reality.

The question has to be asked, whether, to each second added to the time, there corresponds always the same amount to be added to the distance, and also if the distance is nothing when the time is nothing.

EXAMPLE.—*The weight of a sheet of iron* 6 *ft. square is* 4·86 *lb.; what would be the weight of a similar sheet* 3·5 *ft. square?*

Here

(1) The weight becomes indefinitely small as the length of the side of the square becomes indefinitely small;

but it cannot be said

(2) That each increase or decrease of a foot in the length of the side gives the same increase or decrease in the weight;

and ∴ the weight of the sheet, 3·5 ft. sq., is *not* $\frac{3{\cdot}5}{6} \times 4{\cdot}87$ lb.

We could say, however, that

(2A) Each *square foot* increase or decrease in the area of the sheet means the same increase or decrease in the weight;

and put the question in this way:

36 sq. ft., *i.e.* (6×6), of the iron weigh 4·86 lb.; what will be the weight of 12·25 (*i.e.* $3{\cdot}5 \times 3{\cdot}5$) sq. ft. of iron?

36 sq. ft. weigh 4·86 lb.

12·25 „ „ $\frac{12{\cdot}25}{36} \times 4{\cdot}86$ lb.

$= (12{\cdot}25)\,(0{\cdot}135)$ lb.

$= 1{\cdot}65375$ lb.

EXAMPLE.—*A salesman's salary consists partly of a fixed amount and partly of a certain sum for every £ worth of goods sold. If his salary be £*150 *when he sells £*1000 *worth of goods, what will his salary be when £*2000 *worth of goods are sold?*

Here, it cannot be said that

1. When *no* goods are sold, no salary is received; although we can say,

2. For each £ increase in the value of the goods sold, the salesman's salary increases by the same amount.

The question cannot, in fact, be solved on the data above.

Suppose, however, we knew that his salary was £180 when the value of the goods sold was £1500.

The question could now be put differently. If the *increase* in his salary be £30 for £500 increase in the value of the goods sold, what will be the increase in his salary for £1000 increase in the value of the goods sold?

Increase in goods sold to the value £500 gives £30 increase of salary.

Increase in goods sold to the value £1000 gives $\frac{1000}{500} \times 30$ increase of salary, *i.e.* £60.

His salary is therefore £60 greater when he sells £2000 worth of goods than when he sells only £1000 worth, and therefore is £210.

EXAMPLE.—*Water consists of* 16 *parts by weight of oxygen to every* 2 *parts by weight of hydrogen. How much hydrogen can be obtained by decomposing* 12·474 *grams of water?*

Here, out of every (16 + 2) grams of water, 2 grams are hydrogen.

∴ out of every 12·474 grams of water $\frac{12 \cdot 474}{18} \times 2$ are hydrogen.

∴ amount of hydrogen $= \frac{1 \times 12 \cdot 474}{9}$ gr. = 1·386 gr.

EXAMPLE.—*How much oxygen will be used up in burning* 2·34 *grams of carbon, assuming that in the carbonic acid gas produced there are* 12 *parts by weight of carbon to every* 32 *parts by weight of oxygen?*

12 gr. of carbon unite with 32 gr. of oxygen.

2·34 gr. „ „ „ $\frac{2 \cdot 34}{12} \times 32$ gr. of oxygen.

∴ amount of oxygen

$$= \frac{2 \cdot 34}{12} \times 32 \text{ grams} = \frac{2 \cdot 34 \times 8}{3} \text{ grams} = 8 \times 0 \cdot 78 \text{ grams}$$

$$= 6 \cdot 24 \text{ grams.}$$

EXAMPLE.—*In a Railway the working expenses per mile per annum were as follows:*

	£.
Maintenance, - - - -	139
Locomotives and carriages, - -	390
Traffic charges, - - - -	223
Rates and duty, - - - -	49
Miscellaneous, - - - -	150

If on the line the total working expenses for the year were £219681, find the expenditure under each of the above heads.

The total working expenses per mile
$= £(139 + 390 + 223 + 49 + 150) = £951$;
∴ out of a total of £951 the 'Maintenance' amounts to £139.
Out of a total of £219681 the 'Maintenance' amounts to $\frac{219681}{951} \times 139£$.
Out of a total of £219681 the 'Maintenance'
$= 231 \times £139 =$ £32109
So out of a total of £219681 the cost of

Locomotives and carriages	$= 231 \times £390 =$	£90090
Traffic charges - -	$= 231 \times £223 =$	£51513
Rates and duty - -	$= 231 \times £49 =$	£11319
Miscellaneous - -	$= 231 \times £150 =$	£34650
	Total	£219681

EXAMPLE.—*Four people* A, B, C *and* D *put* £3000, £4500, £2000 *and* £2500 *into a business concern; the total profits available for division in the ratio of the share capital is* £3600. *Find the profit made by* A, B, C *and* D.

Here on £(3000 + 4500 + 2000 + 2500) the profit is £3600;
∴ on £3000 the profit is $\frac{3000}{12000} \times £3600 = £900$ = A's share.
on £4500 „ „ $\frac{4500}{12000} \times £3600 = £1350$ = B's „
on £2000 „ „ $\frac{2000}{12000} \times £3600 = £600$ = C's „
on £2500 „ „ $\frac{2500}{12000} \times £3600 = £750$ = D's „

EXERCISES LIV.

1. If 100 kilos. be the French equivalent of 220 lb., what will be the French equivalent of 55 lb.?

2. What also is the English equivalent of 350 kilos.?

3. If 220 gallons be the same as 1000 litres, how many litres will there be in 33 gallons?

4. How many gallons will there be in 45 litres?

5. If a train can describe 88 ft. in 1 sec., how long will it take to describe 1 mile (5280 ft.)?

6. A train goes at the rate of 45 miles per hour; how many feet does it describe per second?

7. If a train describes a journey of 125 miles in 2 hours and 10 minutes, how long would it (moving at the same speed) take to describe a journey of 92 miles?

8. A steamer moves at the rate of 300 knots per day; how many hours will it take to describe 175 knots?

9. The cost of 35 oranges being 2*s.* 4*d.*, how many can be bought for 3*s.* 4*d.*?

10. If a stream pours 18,000 cubic feet of water into the sea in 12 minutes, how much will it pour into the sea in 35 minutes?

11. If 4800 cubic feet of coal gas cost 12*s.*, what will be the cost of 2500 cubic feet?

12. The weight of 9 square feet of sheet lead, $\frac{9}{16}$ of an inch thick, being 300 lb., what will be the weight of a roof made of such sheet lead, the area being 1800 square feet?

13. If 4 lb. of tea cost 6*s.* 6*d.*, how much will 15 lb. cost?

14. How many lb. of apples can be bought for £1. 5*s.* 6*d.* if 12 lb. cost 1*s.* 5*d.*?

15. If 14 yards of cloth cost £2. 5*s.*, what will be the length of a piece of similar cloth, value £4?

16. Assuming that, as a rough calculation, the price of a sailing ship is proportional to its tonnage, find the value of a ship of

3420 tons displacement, if a ship of 1728 tons displacement cost £4800.

17. If the weight of a sheet of brass, 3 ft. long and 2 ft. broad, be 60 lb., what will be the area of a similar sheet of brass weighing 1001 lbs.?

18. If a cube of iron of 42 inch edge weighs 11 tons, what will be the weight of a block whose volume is 24·5 cubic feet?

19. In transmitting power by belting, one wheel makes 420 revolutions for every 300 made by a second; how many revolutions does the second make while the first makes 280?

20. If 84 lb. of sugar could be bought for 15*s.* 2*d.*, what would be the cost of 60 lb. of sugar?

21. How many lb. of sugar could be bought for 7*s.* 7*d.* if 84 lb. cost 15*s.* 2*d.*?

22. If one million cubic inches of air weigh 46·55 lb., what is the weight of 1 cubic foot (1728 cubic inches)?

23. If one million cubic inches of air weigh 46·55 lb., how many cubic inches will weigh 14·4 lb.? (State your answer to the nearest hundred.)

24. If the weight of a piece of copper wire 4000 ft. long be 130·88 lb., what will be the weight of a piece of similar wire 125 ft. long?

25. What length of a piece of wire similar to that in Ex. 24 will have a weight of 11·452 lb.?

26. If the cost of carrying a ship's load be the same for each ton carried, calculate the cost of carrying a load of 1440 tons when the cost of carrying 1800 tons is £2250.

27. If the wages paid to navvies for driving a tunnel be calculated at a certain amount for each yard that the tunnel is driven, what will be the wages paid when a length of 18 yards is driven, knowing that the wages paid for a length of 24 yards was £25. 4*s.*?

28. If the cost of covering with linoleum the floor of a room 14 feet square be £3. 13*s.* 6*d.*, how much will it cost to cover the floor of a room 16 ft. square with the same kind of linoleum?

29. If the cost of painting a cubical block of 8 ft. edge be 12*s.*, what will be the cost of painting a cubical block of 6 ft. edge?

30. The freight of a cargo being calculated according to its tonnage, what is the weight of a cargo whose freight is £860, if a cargo of 4200 tons could be sent for £3010?

31. If a rod of iron 4·2 square inches in section can sustain a pull equal to the weight of 121·8 tons, what pull could a rod of 6 square inches sustain?

32. If a steel wire, section $\frac{1}{16}$ of a square inch, can just hold a load of 3·25 tons without breaking, what is the section of a wire which can just hold a load of 2 tons?

33. If the weight of 25 feet of common lead piping of $\frac{3}{4}$ inch bore be 400 lb., what is the length of a coil of similar piping weighing 1 ton?

34. The weight of an anchor and 120 ft. of cable is 2088 lb., the weight of the anchor and 210 ft. of cable is 2898 lb.; what is the weight of the anchor?

35. A jug containing 258 cubic inches of a liquid weighs (with contents) 16 lb.; if 126 cubic inches of the liquid be poured away, the weight is only 12 lb. 1 oz.; what is the weight of the jug?

36. If a man can bore 78 inches per day through granite and 286 inches per day through limestone, what depth of a hole could a man bore in granite during the time that he bores 231 inches in limestone?

37. If in a town of 2268 people the average daily consumption of water is 6000 cubic feet, what, at the same rate, would be the average consumption in a town of 2079 inhabitants?

38. If 432 lb. of water can be changed into steam by burning 56 lb. of coke, what weight of coke must be burnt in order to evaporate 729 lb. of water?

39. In producing coke from coal, 1540 lb. of coke are obtained from 2240 lb. of coal; how much coal must be employed in the production of 5665 lb. of coke?

EXERCISES LV.

Practical.

[*Apparatus: Paper. Compass. Scales. Wooden Cylinders. Fine Thread. Pins.*]

Measurement of Lengths of Curved Lines.

1. Draw a circle 3 cm. radius.

(*a*) Set the dividers to a distance of say 4 cm.; mark it round as far as it goes; thus, from A to B, B to C, C to D, measure off the remaining distance DA, and find out the distance A, B, C, D, A. This is clearly less than the distance along the circumference.

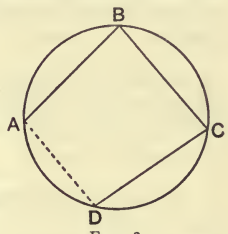


FIG. 28.

(*b*) Repeat the measurements, with the dividers set to 3 cm., 2 cm., 1 cm., 6 mm., 5 mm., 4 mm., 3 mm.

You observe that the distance round increases as the distance to which you set the dividers decreases, but that when the distance is small all the results become practically the same, and that a curved line may be regarded as made up of a very great number of indefinitely short straight lines.

2. Measure with your dividers the circumferences of circles with the following radii:

(*a*) 3 inches, 2 inches, 1 inch.

(*b*) 6 cm., 5 cm., 4 cm., 3 cm., 2 cm., 1 cm.

3. Repeat the measurements in Ex. 2, using a fine piece of thread instead of the dividers.

Start at any mark; put one end of the thread upon it, previously having knotted the thread, and hold it there with the nail of the first finger of your left hand. Make the thread coincide as nearly as you can with a small part of the curve, and place the nail of the first finger of your right hand upon it. Now release your left-hand finger and carefully place it at the point where your right-hand finger is held; then, using your right hand, go on to make

some more of the string exactly coincide with another small length of curve. Repeat this until you have completed the whole curve.

FIG. 29.—Measurement of a curved line with thread.

Measure on your scale the length of thread from the knot to the end.

4. Repeat the measurements of the circle in Ex. 2 by sticking a number of pins at close intervals along the circle; wrap cotton round these pins, and when the cotton overlaps cut it; measure the cut piece of cotton.

5. Wrap a strip of paper closely round a wooden cylinder, and make a small hole with a pin at a place where the paper overlaps. Unroll the paper and measure the distance round the cylinder.

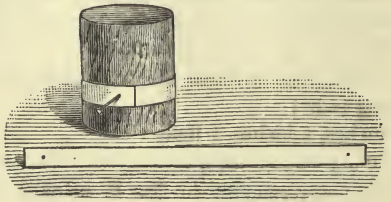


FIG. 30.—Measurement of the circumference of a cylinder.

6. From the results of your measurements, see if the length of the circumference divided by the length of the diameter is always the same.

The multiple that the circumference is of the diameter is usually denoted by the symbol π.

41. PERCENTAGES.

EXAMPLE.—*A particular kind of gun-metal alloy is composed of* 10 *per cent. tin and* 90 *per cent. copper. How much tin is there in* 3 *tons of this gun metal?*

Here, by the term 10 *per cent.* is meant 10 parts out of every 100, so that the amount of tin present is 10 hundredths or 0·10 of 3 tons = 6 cwt., and similarly the amount of copper present is 90 hundredths or 0·90 of 3 tons = 54 cwt., and it is clear that *per cent.* is used after a number to express a fraction.

EXAMPLE.—*The profit made by a tradesman is* 15 *per cent. on the selling price. What is the profit made on goods sold for* £5. 12*s.* 6*d.*?

The first part of the question means that

$$\begin{aligned}\text{The profit} &= \tfrac{15}{100} \text{ of the selling price}\\ &= 0{\cdot}15 \text{ of } £5.\ 12s.\ 6d.\\ &= 0{\cdot}15 \text{ of } £5{\cdot}625\\ &= £0{\cdot}84375 = 16s.\ 10\tfrac{1}{2}d.\end{aligned}$$

EXAMPLE.—*A commission agent obtains a commission of* £12. 5*s. on selling goods to the value of* £150; *at what rate per cent. does he earn commission?*

Here the commission on £150 is £12. 5*s.*, *i.e.* £12$\frac{1}{4}$.

$$\begin{aligned}£100 \text{ is } £12\tfrac{1}{4} \times \tfrac{100}{150}\\ = £\tfrac{2}{3} \times 12\tfrac{1}{4}\\ = £8\tfrac{1}{6}.\end{aligned}$$

$\therefore$ commission is earned at the rate of $8\frac{1}{6}$ per cent.

EXAMPLE.—*Type-metal is made of varying composition, ranging from* (*a*) 1 *part antimony to* 3 *parts lead, up to* (*b*) 1 *part antimony to* 7 *parts lead. Express each composition as a percentage composition.*

Here we wish to find the amounts of antimony and lead respectively in each 100 parts of alloy.

(*a*) In (1 + 3) parts of type-metal 3 parts are lead;
∴ in 100 „ „ $3 \times \frac{100}{4}$ parts are lead,
$= 75$ „ „
In (1 + 3) parts of type-metal 1 part is antimony;
∴ in 100 „ „ $1 + \frac{100}{4}$ parts are antimony,
i.e. 25 „ „
The composition of type-metal (*a*) is therefore
75 per cent. lead,
25 per cent. antimony.

(*b*) In 100 parts of type-metal $\frac{100}{7+1} \times 7$ parts are lead,
$= 87{\cdot}5$ „ „
and in „ „ „ $\frac{100}{7+1} \times 1$ parts are antimony,
$= 12{\cdot}5$ „ „
The composition of type-metal (*b*) is therefore
87·5 per cent. lead,
12·5 per cent. antimony.

N.B.—The symbol % is used to express *per cent.*

EXAMPLE.—*A tradesman allows his customers* 4% *off their bills when settled within one month of the purchase of the goods; a certain customer has* 4*s.* 7½*d. returned to him in this way; what was the amount of his bill?*

On a bill of £100, the amount allowed off is £4; the bill is $\frac{100}{4}$ times the amount allowed off, and in the example the bill is $\frac{100}{4} \times 4s.\ 7\frac{1}{2}d. = £5.\ 15s.\ 7\frac{1}{2}d.$

EXAMPLE.—*By selling goods at* £4. 15*s.* 4*d., a tradesman experienced a loss of* 12% *on the cost price; what should be the selling price, so as to effect a gain of* 16% *on the cost price?*

Suppose the goods had originally cost £100; then the corresponding selling price would have been £(100 − 12), *i.e.* £88; whereas to effect a gain of 16% he should have sold at £(100 + 16), *i.e.* £116.
∴ the required price is $\frac{116}{88}$ times the actual selling price
$= \frac{116}{88} \times £4.\ 15s.\ 4d.$
$= \frac{29}{22} \times £4.\ 15s.\ 4d.$
$= 29 \times 4s.\ 4d.$
$= £6.\ 5s.\ 8d.$

EXERCISES LVI.

Where no other data are given, the ratio of the circumference of a circle to its diameter may be assumed as 3·14 : 1.

1. The diameter of a halfpenny is 1 inch, its circumference is 3·14 inches approximately; what is the circumference of a penny—its diameter being known to be 1·1875 inches?

2. The driving wheel of a locomotive is 5 ft. 6 in. in diameter; find its circumference, knowing that the circumference of its trailing wheel is 9·42 ft. and the diameter 3 feet?

3. If a driving wheel makes 336 revolutions in a journey of 1·25 miles, how many revolutions would the wheel make in a journey of 3 miles?

4. A carriage wheel is 1·26 metres in diameter and it makes 160 revolutions per minute; what is the speed of the carriage in metres per minute?

5. One horse on a round-about is at a distance of 12 feet from the central axis, another horse is at a distance of 15 ft. If the inner horse describes a distance of 840 feet in a certain time, how many feet will the outer horse have described in the same time?

6. One toothed wheel of 2 feet diameter drives another of 3 inches *radius*; the second wheel has a third of 2′ 6″ diameter mounted on the same axis, and the third turns a fourth wheel of 2 inches radius.

If a point on the circumference of the first wheel is moving at the rate of 4·4 ft. per sec., at what speed is a tooth on the last wheel moving?

7. An endless strap passes round two pulleys, one being of 2 ft. 3 in. diameter, the other of 7 inches diameter. The large pulley makes 5 revs. per second. What is the speed of the strap and how many revolutions does the smaller pulley make in 28 seconds?

8. A cage is raised from the bottom of a mine 157 fathoms deep by means of a cable and a winding drum 10 feet in diameter;

how many revolutions will the drum make in raising the cage from the bottom of the mine to the top?

9. The diameter of the earth being assumed to be 8000 miles, find the number of statute miles in 60 geographical miles or knots.

(60 knots form the $\frac{1}{360}$ part of the circumference of the equator. Assume the ratio of circumference to diameter to be 3·141 in this and the next example.)

10. Find the distance between two points on the parallel of latitude 60°, but differing in longitude by 24·3 secs., the radius of the circle on the earth at latitude 60° being 2000 miles.

(The complete parallel would correspond to $360 \times 60 \times 60$ secs.)

11. A mass of wood 16 lb. in weight displaces 0·256 cubic ft. of water; how much water will be displaced by a portion of the wood 7·3 lb. in weight?

12. If the displacement of a ship of 3200 tons be 112000 cubic feet, how much less water would be displaced if the ship was lightened to the extent of 480 tons?

13. Chalk consists of 56 parts lime to 44 parts carbon dioxide. On strongly heating the chalk the carbon dioxide is evolved and the lime remains. How much lime can thus be obtained from 3 tons of chalk?

14. The percentage composition of anthracite coal is as follows:

92·56 parts carbon.
3·33 „ hydrogen.
2·53 „ oxygen.
1·58 „ ash.

How much pure carbon is there in 13 tons 4 cwt. of anthracite coal?

15. Coal gas is obtained, together with other products, by distilling coal. The following is a yield obtained from a ton of Newcastle coal:

Coal gas,		9500 cubic feet.
By-products	coke,	1500 lb.
	tar,	70 lb.
	ammoniacal liquor,	80 lb.

How much of such coal must be treated when the by-products weigh 2310 lb.? Find also what gas, coke, and tar will be produced.

16. Britannia metal consists of 140 parts by weight of tin, 3 parts copper and 9 parts of antimony. Find the amount of tin necessary to make a can of Britannia metal, the weight of the can being 2·337 kilo.

17. Pure Epsom salts, without the water associated with it, contains

24 parts magnesium
32 „ sulphur
64 „ oxygen
} by weight.

Find the amount of magnesium contained in 12·3 grams of Epsom salts.

18. Marine glue is made by heating gently 1 part by weight of india-rubber and 12 of naphtha, and adding 20 parts by weight of powdered shellac. How much naphtha will be required to make 1·87 lb. of marine glue?

19. The receipts per mile per annum on a Railway are made up as follows:

	£
Passengers,	1511.
Goods,	1575.

If the total receipts be £351804, what are the receipts from passengers, what from goods?

20. The cost of running a locomotive, per train mile, is as follows:

	d.
General charges,	0·4.
Running expenses,	5·5.
Repairs, etc.,	3·3.

Find the cost under each of the above heads during a period when the expenditure altogether was £2300.

21. The subscription to the 'Games Club' in a school of 200 boys is 6*s.* 6*d.* per term, the money being expended on Cricket, Football, and Athletic Sports. If Cricket receives 2*s.* for every 9*d.*

received by Football, and for every 6*d.* received by Sports, find the expenditure under each head per year?

22. Three persons have 3 shares, 4 shares, and 7 shares respectively in a business concern, the total number of shares held by other people being 42. The profits amount to £1400; what do the three persons severally receive?

23. The perimeter of a given triangle (*i.e.* the total length of the three sides) is 49 inches, the triangle being of the same shape as another triangle whose sides are 17·22 inches, 17·91 inches and 8·62 inches respectively; what are the lengths of the sides of the given triangle?

24. The gold coinage of this country is 22 carats fine, *i.e.* contains 22 parts of pure gold and 2 parts alloy. If the weight of a sovereign be 123·274 grains, what weight of pure gold was required for the coinage of 7500700 sovereigns?

25. The rateable values of three adjoining parishes are severally £15,345. 10*s.*, £18,924. 10*s.* and £12,330. A rate of £2425 is made upon them. What proportion has each parish to contribute?

26. Three towns have respectively 14,001, 11,628 and 4845 males capable of serving as soldiers. A company of 482 men has to be raised from the three towns. Apportion the number of men to be found by each town.

27. A, B, C and D join to buy a ship valued at £7400. A takes 9 shares, B 11 shares, C 17 shares and D 27 shares. Apportion the purchase money between them.

28. Two farmers rent a meadow for £2. 15*s.*; one puts in 37 cattle and the other 43. What amount of rent should each pay?

29. A specimen of brass consists of 13 parts of tin to every 112 parts of copper. Find its percentage composition.

30. The percentage composition of a mortar is as follows:

32·5 lime.
67·5 river sand.
100·0 mortar.

How much sand is there in 4 ton 3 cwt. of the mortar?

31. Find the amount per cent. of nitro-glycerine in dynamite consisting of 73 parts nitro-glycerine, 2 parts coal dust and 290 parts of clay.

32. Standard gold consists of 22 parts of pure gold to 2 parts of copper. Find the percentage composition of standard gold, *i.e.* find, out of every 100 parts of standard gold, how much pure gold there will be and how much copper. (State the answer true to the second decimal place.)

33. Tough brass, suitable for engine work, is made by alloying together 15 parts of tin, 100 parts of copper and 15 parts of zinc. Find the percentage composition of the brass.

34. Pewter consists of 100 parts of tin and 17 parts of antimony. Find its percentage composition.

35. Blue vitriol, or sulphate of copper, is a chemical compound, made up of 63 parts copper, 32 parts sulphur, 64 parts oxygen. Find its percentage composition.

36. 'Stone lime' is made up of 1 bushel of lime to 3 of sand and 1·5 bushels of water. Find its percentage composition by volume.

37. Average limestone consists of 112 grs. of carbonate of lime to 9 of clay and 29 of sand. Find its percentage composition. Find out also how much pure carbonate of lime could be obtained from 20 ton 5 cwt. of limestone.

38. Dynamite used for blasting is made up of 287 parts of clay, 2 parts of coal dust and 75 parts of nitro-glycerine. Find its percentage composition. What weight of nitro-glycerine will there be in 32·2 kilo. of dynamite?

39. Find the percentage composition of ordinary caustic soda which contains

23 parts by weight of sodium.
16 parts by weight of oxygen.
1 part by weight of hydrogen.

40. The copper ore generally employed in smelting consists of about 30% iron and 13% of copper. How much copper could be obtained by smelting 14 tons of the ore?

41. A man buys a horse at a fair, selling it again later for £24, thereby making a profit of 10% on the buying price. What did he pay for the horse?

42. If by selling goods at a profit of 8% on the buying price, a tradesman makes a profit of £3. 6*s*. 8*d*., what was the buying price of the goods?

43. Express as convenient vulgar fractions the following percentages: $3\frac{1}{3}\%$, $33\frac{1}{3}\%$, 25%, $12\frac{1}{2}\%$.

44. What percentages are the equivalents of the following vulgar fractions: $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{40}$, $\frac{1}{50}$, $\frac{3}{4}$, $\frac{1}{6}$?

45. Most rocks are porous and can absorb water. The following table gives data as to this property for a number of rocks. Supply the figures which are missing in the table (true to 3 significant figures).

	Geological Formation.	Locality.	Vol. of water absorbed by 100 vol. of rock.	Gallons of water absorbed per cub. ft. of rock.
(*a*)	Old Red Sandstone	Bristol -	10·30	0·642
(*b*)	Limestone - -	Clifton -	.....	1·018
(*c*)	Freestone - -	Cheltenham	35·29	.....

46. Complete the following table:

TABLE OF CRIMES AND OFFENCES IN COUNTIES IN 1880.

	County.	Population. Census 1881.	Number committed for Trial.	Number per 1000.
(*a*)	Bedford -	149,000	94	.....
(*b*)	Berks -	218,000	.....	0·47
(*c*)	Bucks -	176,000	86	.....
(*d*)	Cambridge -	185,000	56	.....

47. A very rich kind of limestone contains 95·89% of carbonate of lime; assuming that in burning this limestone into lime 56%

of the carbonate yields lime, what quantity of this limestone is required to yield 1 ton of lime?

48. The wages of mechanics in a district are reduced from an average of £2. 8*s.* 9*d.* per week to £2. 5*s.* 6*d.* After a time they have an advance of $7\frac{1}{2}$ % on their later wages. Find the reduction per cent. in the first instance, and the average weekly wages in the second case.

49. By selling a book for 3*s.* 9*d.*, ten per cent. of the cost is loss; at what price should it be sold to make a gain of 10%?

50. A certain publishing firm gives a discount of $33\frac{1}{3}$ % off the published price, counts 25 books as 24, and gives a further discount of $2\frac{1}{2}$ % on the reduced price. What does the publisher get for a book the published price of which is 16*s.*?

51. A market woman buys 200 eggs at five for threepence and an equal number at eight for sixpence. She sells the whole at a uniform price and gains $33\frac{1}{3}$ % on her outlay. What is the selling price per score?

52. A house was sold for £560, and thereby a loss of 9 per cent. was made. If it had been sold for £650, what would then have been the loss or gain per cent.?

53. A trading concern in 1 yr. made a profit of £1875 on a turnover of £23,500; in the 2nd year a loss of £710 was sustained on a turnover of £22,360; while in the 3rd year a profit of £935 was made on a turnover of £24,140. What was the average profit per cent. on the 3 years' turnover?

54. A mass of ore of 3 ton 2 cwt. 2 qr. loses weight during smelting to the extent of 17 cwt. Express the loss as a percentage.

55. If, when goods are sold at £26. 7*s.*, a profit of 24 % is made on the buying price, what did the goods cost?

56. What is the cost price of goods on which, when sold at £3. 8*s.* gain, a profit is made of 15 % on the selling price?

57. The population of a town increases in two years from 23,280 to 23,862; find the gain % during that time.

58. In calculating the area of a rectangle, the real length of which is 47 ft. 3 in. and breadth 28 ft. 7 in., the computer writes 49 ft. 6 in. instead of 47 ft. 3 in.; what percentage error does he make?

59. A merchant allows a customer 5 % discount on a bill, and receives a net amount £182. 17*s.* 6*d.*; what was the bill?

60. Eight trams contain 7 tons 5 cwt. of coal from a seam A. When the coal is screened it yields 3 tons 5 cwt. 1 qr. of large coal; and an equal weight from a seam B when similarly screened yields 4 tons 14 cwt. 1 qr. of small. Compare the percentages of large coal in each seam.

61. A man, whose gross income is £300, pays income tax at 1*s.* in the £ on the excess of his income over £160; what percentage is the amount paid of his gross income?

62. A rod of brass is 425 cm. long at 15° C., and at 100° C. its length is 425·065025 cm.; find the percentage gain in length.

63. 35 % of an estate of 2 sq. miles 24 ac. is set apart for building purposes; how much is left?

64. The value of land near a town increases by $33\frac{1}{3}$ % during a certain period. Find the increase in the value of an estate of 660 acres, originally worth £45 an acre.

65. When a metal is cast in a mould the portion of the casting near the surface has generally to be chiselled off. If an amount of 4800 gr. is removed in this way, and the remaining portion be 200 kilo., what percentage of the original casting has been taken away?

EXERCISES LVII. a.

Graphic.

[*Apparatus: Scales. Mathematical Instruments. Squared Paper.*]

1. On the squared centimetre paper mark off two straight lines at right angles to one another, letter them Ox and Oy (Fig. 31), O being at the meeting place of the lines.

Along Ox set off a distance ON = 10 cm.; measure upwards a line NP = 9 cm. Join OP.

At the end of each cm. in ON, starting from O, say at the points N_1, N_2, N_3, etc., draw the verticals N_1P_1, N_2P_2, N_3P_3, ..., etc., and test the lengths with regard to the two fundamental relations.

1. Do they alter by equal amounts as the horizontal distances ON_1, etc., alter by constant amounts?

2. Do they become indefinitely small as ON_1, etc., become indefinitely small?

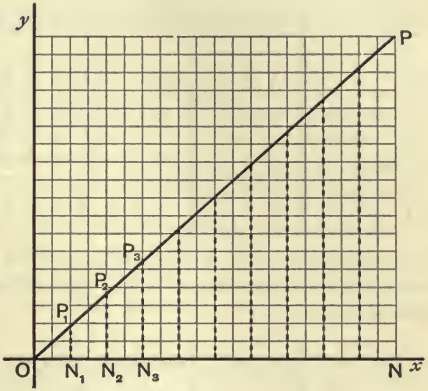


FIG. 31.

Test also OP with regard to ON, and OP with regard to NP.

2. Draw a number of other lines and test in the same way.

3. Obtain points P_1, P_2, ... as before by setting off ON_1, ON_2, ON_3, etc., along Ox, and N_1P_1, N_2P_2, etc., parallel to Oy, where ON_1, etc., and N_1P_1, etc., are given by the following tables (values given in cm.). Join the points P_1, P_2, etc.

	ON_1	N_1P_1	ON_2	N_2P_2	ON_3	N_3P_3	ON_4	N_4P_4	ON_5	N_5P_5	ON_6	N_6P_6
(*a*)	1	1·2	2	2·4	3	3·6	4	4·8	5	6	6	7·2
(*b*)	0·8	0·7	1·6	1·4	2·4	2·1	3·2	2·8	4	3·5	4·8	4·2
(*c*)	0·4	1	0·8	2	1·2	3	1·6	4	2·0	5	2·4	6
(*d*)	1·2	0·6	2	1	2·8	1·4	3·6	1·8	4·4	2·2	5·2	2·6

In each case notice that N_1P_1, etc., increase by equal amounts for equal increases in ON_1, etc., and that they vanish together.

4. Obtain points representing pairs of quantities, where, although equal increases occur in one for equal increases in the other, they do not vanish together.

	ON_1	N_1P_1	ON_2	N_2P_2	ON_3	N_3P_3	ON_4	N_4P_4	ON_5	N_5P_5	ON_6	N_6P_6
(*a*)	0	2·2	1	4	2	5·8	3	7·6	4	9·4	5	11·2
(*b*)	0	0·6	1	1·2	2	1·8	3	2·4	4	3·0	5	3·6
(*c*)	2	1	3	1·4	4	1·8	5	2·2	6	2·6	7	3
(*d*)	1	2	2	3	3	4	4	5	5	6	6	7

5. Obtain points where the quantities concerned are not so related that equal changes in the one correspond to equal changes in the other.

	ON_1	N_1P_1	ON_2	N_2P_2	ON_3	N_3P_3	ON_4	N_4P_4	ON_5	N_5P_5	ON_6	N_6P_6
(*a*)	0	0	1	0·2	2	0·8	3	1·8	4	3·2	5	5
(*b*)	0	0	1	0·3	2	1·2	3	2·7	4	4·8	5	7·5
(*c*)	0	0	0·5	0·2	1	1·6	1·2	5·4	2	12·8		

EXERCISES LVII. b.

1. If the rail of a railroad remains straight, and if in moving along it for a distance of 460 yards horizontally, a vertical rise of 2·3 ft. is experienced, after what horizontal distance will the rise be 3 ft.?

2. In driving a slant of 440 yds. through a seam of coal, there is a uniform dip of 4·5 inches in each yard. How much lower is the bottom of the slant than the top?

3. If the rise in a straight railroad be 1 vertical in 114 horizontal, what vertical rise will occur in a horizontal displacement of 3249 ft.?

4. A yacht makes a straight tack in a certain direction, and afterwards finds it has moved 4·08 miles to the east and 3·89 miles to the north. How much further north will it have gone when it is 11·73 to the east of its original position—the same tack being pursued all the time?

5. A yacht sails for 5·22 miles along a tack, making 3·1 miles east for every 4·65 miles measured in the direction of the tack; it then changes its course, now making 3 miles west for every 8 miles along the new course; after pursuing this new tack for 5·8 miles, how far will it have moved east of its original position?

6. A road rises at the rate of 12·32 yards for each mile of road for a distance of 872 yards from a place A; the inclination then increases to 23·6 yards per mile until B is arrived at, distant 1 mile 91 yards from A, measured along the road. How much higher is B than A?

7. After going down the 'incline' in a mine for 183 yards, a point is reached 43·26 yards below the surface. The total length of the incline is 280·6 yards. What is the depth of the bottom of the incline below the surface?

8. A ladder consists of 40 steps, each 32 cm. apart. One end of it rests on the ground, the other against a wall at the height of 9·32 metres. A man ascends the ladder; how high will he be when he has gone up 28 steps?

9. When a pile of dry sand is made, the sand begins to slip once the steepness is greater than that corresponding to a rise of 71·4 cm. for each horizontal distance of 91 cm. Find the greatest height of a sand castle (without patting), the base of which is 213·2 cm. in diameter.

10. A wall is 8·262 feet high. Dry sand is thrown against it until it forms a slope just reaching to the top of the wall. Find the least distance of the bottom of the slope from the wall. (Data as in Exercise **9.**)

Scale Drawing.

Very few objects can be drawn of exactly the actual size; they have to be shown either reduced or enlarged to a certain **scale.**

Thus, a large scale Ordnance Map may be drawn to a scale of 6 inches to the mile, where a real distance of 1 mile is represented by a distance of 6 inches on the map. A map of England may be shown to a scale of 100 miles to the inch. The fraction that the distance on the map is of the distance it represents is known as the **Representative Fraction,** and is frequently represented by the letters R.F. A scale 6 inches to the mile could therefore be called a scale where R.F. $= 6/63360$, *i.e.* R.F. $= 1/10560$.

EXAMPLE.—*The real distance between two towns—we may call them* A *and* B*—is known to be* 232 *miles; on a map they appear to be* 3·16 *inches apart.* C *and* D*, two other towns, appear to be* 4·74 *inches apart on the map. What is the real distance between* C *and* D?

A distance of 3·16″ on the map represents an actual distance of 232 miles.

∴ a distance of 4·74″ on the map represents an actual distance of

$$232 \times \frac{4{\cdot}74}{3{\cdot}16} \text{ miles}$$

$$= \frac{232 \times 3}{2} \text{ miles}$$

$$= 348 \text{ miles.}$$

The distance from C to D is therefore 348 miles.

EXAMPLE.—*Find the Representative Fraction on a map where a distance of* 7 *fur.* 8 *ch. is represented by a line* 5·148 *inches long.*

Here 7 fur. 8 ch. $= 78$ ch. $= 78 \times 22$ yds. $= 78 \times 22 \times 36$ inches.

The fraction which represents the ratio of the distance on the map to the actual length represented is:

$$\frac{5{\cdot}148}{78 \times 22 \times 36} = \frac{5{\cdot}148}{12 \times 5148} = \frac{1}{12000}.$$

In other words, the R.F. $= 1/12000$.

EXAMPLE.—*A scale is constructed of* 6 *inches to the mile; a comparative scale of kilometres has also to be constructed. What would be the length of line on the new scale which represents a distance of* 2·31 *kilometres?*

By a Comparative Scale, with regard to a given scale, is meant one with the same Representative Fraction but in which different units are employed; and therefore, by using the two scales, a distance may be read off in different units, say in miles or in kilometres, etc.

A scale of 6 in. to the mile has $\frac{6}{63360}$ for its R.F. (p. 220).

A new scale is required by means of which distances can be read off, not in miles but in kilometres.

Since the R.F. is still $\frac{6}{63360}$, a distance of 2·31 kilometres is represented by

$$2{\cdot}31 \times \frac{6}{63360} \text{ kilometre} = \frac{2{\cdot}31 \times 1000 \times 100}{10560} \text{ cm.}$$

$$= \frac{23{\cdot}1}{1{\cdot}056} \text{ cm.}$$

$$= 21{\cdot}875 \text{ cm.}$$

EXAMPLE.—*In testing the magnifying power of a lens, it is found that* 6 *divisions on a millimetre scale appear enlarged and to coincide with divisions on a centimetre scale* 5·6 *cm. apart. What is the real radius of a circle which appears to be of* 1·26 *inches radius?*

Here the lens magnifies $\frac{5{\cdot}6}{0{\cdot}6}$ times,

or the magnification is $\frac{5{\cdot}6}{0{\cdot}6}$ diameters.

The apparent radius = The real radius multiplied by the magnification.

The real radius of the circle, therefore, is $1{\cdot}26 \times \frac{0{\cdot}6}{5{\cdot}6}$ inch

$$= \frac{0{\cdot}756}{5{\cdot}6} \text{ inches}$$

$$= 0{\cdot}135 \text{ inch.}$$

EXERCISES LVIII.

Practical.

[*Apparatus: Mathematical Instruments. Squared Paper.*]

1. In the accompanying plan of a room the real distance between the points H, M is 23 feet 9 inches. Measure off the distance on

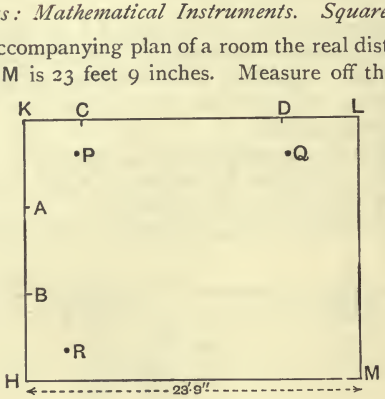


FIG. 32.

the plan between the points H, M, and then, measuring also the distances on the plan between the points mentioned below, calculate by proportion the real distance between them.

(*a*) A to B, (*b*) C to D, (*c*) L to M, (*d*) K to M,
(*e*) P to R, (*f*) P to Q, (*g*) R to Q.

2. Draw a new plan in which HM is represented by a line 3 inches long, the other lines being altered in proportion.

3. The accompanying map (Fig. 33) is without a scale, but it is known that the distance from Stockton to Doncaster is 61 miles. Construct a diagonal scale for use with the map; use it to find the distances between (*a*) York and Lincoln, (*b*) Doncaster and Skegness, (*c*) Sheffield and Scarboro.

4. The distance from London to Exeter is 150 miles; find, by means of measurements on the accompanying map (Fig. 34), the distance from London to (*a*) Bristol, (*b*) Reading, (*c*) Gloucester, (*d*) Birmingham, (*e*) Norwich, (*f*) Derby, (*g*) Northampton.

Durham
Stockton
Scarboro
York
Leeds
Selby
Hull
Wakefield
Doncaster
Sheffield
Lincoln
Skegness
Nottingham

FIG. 33.

Derby
Norwich
Birmingham
Northampton
Gloucester
London
Bristol
Reading
Exeter

FIG. 34.

5. The sea passage from Newhaven to Dieppe is 75 miles; find the length of the passage from (*a*) Dover to Calais, (*b*) Folkestone to Boulogne.

FIG. 35.

EXERCISES LIX.

Miscellaneous.

1. Define a prime number. When are numbers said to be prime to one another? Write down all the prime numbers below 60. Find the prime factors of 14348907.

2. If a man pays £14. 13*s.* 5*d.* as income tax at 7*d.* in the £, what is his full income before paying the tax?

3. Find the cost of 4217 articles at 8*s.* 5*d.* each.

4. Subtract $\frac{15}{16}$ of £16. 2*s.* 4*d.* from 0·0125 of £1626. 15*s.*, and find by what decimal the result must be multiplied to produce £1. 6*s.* 1½*d.*

5. £11. 18*s.* 9*d.* is paid for a piece of work done by A, B and C. A has worked 42 hours, B 77 hours and C 48 hours. If C does in 2 hours as much as A or B does in 3 hours, what should each man receive?

6. Convert $\frac{189}{625}$ into a decimal (5 places), and 0·00003125 into a vulgar fraction.

7. What fraction, the denominator of which is 580, equals $\frac{2}{5}$?

8. A rectangular block of stone is 5 yds. 7 in. long, 12 ft. 6 in. broad and 3 ft. 9 in. thick. Find how many cubic inches it contains.

9. What weight must be added to $\frac{3}{8}$ of $\frac{5}{4}$ of half a cwt. to make it equal to $\frac{3}{10}$ of $3\frac{3}{4}$ quarters avoirdupois?

10. Gunpowder is composed of $\frac{1}{10}$ sulphur, $\frac{3}{20}$ charcoal, $\frac{3}{4}$ nitre by weight. How many lbs. of each will be required to make $1\frac{1}{8}$ tons of gunpowder?

---

11. The sum of three fractions is $\frac{3}{4}$. The first is $\frac{3}{64}$ and the second is $\frac{2}{3}$. What is the third?

12. How much is (*a*) $15\frac{1}{4}+\frac{2}{3}$ of $7\frac{3}{4}$; (*b*) 0·8125 of 2 tons 4 cwt.; (*c*) 78·125 × 40·96?

13. If 3 fires burn 65 cwt. of coal in 26 days, in how many days will the fires burn 4 tons?

14. The population of a town fell from 9600 in 1881 to 8544 in 1891; how much per cent. was the decrease?

15. Of 21 tons 5 cwt. of coal, 19 per cent. is wasted. What weight is wasted?

16. A sum of money was divided among 3 men. If the first had $\frac{1}{3}$, the second $\frac{5}{8}$ and the third 38*s.*, what did the money amount to?

17. What is a *prime* number. Give one example to show that two numbers which are *prime* to each other are not necessarily *prime* numbers.

Resolve 17424 into prime factors, and hence (or otherwise) find the side of a square field which contains 3 acres 2904 square yards, and reduce it to the decimal of a mile.

18. What is the value of 3 lb. 7 oz. 5 dwt. 6 grs. of silver at 3*s.* 4*d.* per oz.?

19. Find the lowest whole number which will convert $0{\cdot}02\dot{3}\dot{0}\dot{9}$ into a finite decimal, by subtracting the decimal from one a hundred times as great and studying the result.

20. If a person who invests £9000 obtains 4 per cent. per annum on two-thirds of it and 5 per cent. per annum on the rest, what income do the investments yield him, and what percentage does he obtain for his money?

---

21. Find the value of 0·384375 of £5.

22. Simplify (*a*) $\dfrac{3\frac{2}{7}+4\frac{2}{5} \text{ of } \frac{20}{33}}{6\frac{2}{3}-1\frac{5}{6} \text{ of } \frac{20}{33}}$. (*b*) $\dfrac{12{\cdot}32-7{\cdot}56}{20{\cdot}35+3{\cdot}45}$.

23. (*a*) How much must I add to $1\frac{1}{36}$ to make $2\frac{7}{130}$? and (*b*) find a simple fraction which is equal to $\dfrac{3\frac{7}{8}-4\frac{1}{3}+2\frac{1}{12}}{9\frac{1}{36}-7\frac{19}{27}}$.

24. A can dig a garden in $4\frac{1}{2}$ days, while B can dig the same in 6 days. How long will A and B together take to dig half the garden?

25. A man buys 87 horses for £1740. He sells 25 at 7 per cent. profit on the cost price, and 40 at $12\frac{1}{2}$ per cent. profit; he loses 3 by disease, and sells the rest at cost price. How much does he gain?

26. If 40 men earn £36. 8*s.* in 8 days, how many men will, at the same rate of wage, earn £40. 19*s.* in 8 days?

27. At an election a candidate polled $6\frac{1}{4}$ per cent. less than his opponent, who obtained 2160 votes; how many votes were given for the unsuccessful candidate?

28. What is the debt, 71 per cent. of which is £301. 15*s.*?

29. If a cubic foot contains $6\frac{1}{4}$ gallons, how many gallons of water are there on a field of $2\frac{1}{2}$ acres, when it is flooded to a uniform depth of an inch?

30. The carriage of 18 cwt. 3 qrs. for a distance of 14 mls. 2 fur. amounts to £9. What should the carriage of 5 tons for 19 mls. amount to at the same rate?

---

31. Take the sum of $2\frac{2}{3}$ and $\frac{3}{20}$ from the sum of 6, $4\frac{2}{5}$ and $1\frac{5}{12}$.

32. A man's income is £459. 17*s.* 6*d.* per year, and of this he saves £15. 6*s.* 7*d.*; what percentage of his income is this?

33. Find all the prime numbers that divide both 1287 and 1144 without any remainders.

34. A tradesman marks his goods 20 per cent. above cost price, but makes a reduction of 10 per cent. on the marked price for ready money; what is his gain per cent. on ready money transactions?

35. A and B ride a race of 31 miles on bicycles. The driving wheel of A's machine makes 3410 revolutions per hour, and has a circumference of 168 inches; that of B makes 3520 revolutions per hour, and has a circumference of 162 inches: which will win, and by how much?

36. If a person who invests £5000 obtains $4\frac{1}{2}$ per cent. per annum on three-fourths of it, and $3\frac{1}{2}$ per cent. per annum on the rest, what income do the investments yield him?

37. If $16\frac{3}{4}$ tons of coal, bought for £7. 5*s.* 2*d.*, are carried 324 miles at a further charge of $\frac{1}{8}d.$ per ton per mile, what is the total cost of the coal per ton to the purchaser?

38. Find the value of 0·0625 of £1. 4*s.* + 0·025 of 5 guineas + 1·125 of 10*s.* 8*d.*, then reduce the sum to the fraction of £3. 4*s.* 6*d.*

39. A, B and C rent a field between them for £20. 10*s.* A puts in 25 sheep for 8 months, B 24 sheep for 9 months and C 40 sheep for 6 months. How much should each have to pay?

40. One gallon of spirit which contains 11 per cent. of water is added to 3 gallons containing 7 per cent. of water, and to this mixture half a gallon of water is added. Find the percentage of water in the mixture.

---

41. Find (by fractions) a number such that, if one twenty-ninth part of it be taken away, the remainder is 377.

42. Add together $£5\frac{14}{81}$ and $13\frac{2}{27}s.$

43. Find the square root of the least integer that is a common multiple of $1\frac{1}{121}$, $22\frac{1}{49}$, and $1\frac{9}{16}$.

44. A watch is offered for sale at £5. 15*s.* 0*d.*; and, if that price is reduced by 5 per cent., the dealer who is selling it will still make $9\frac{1}{4}$ per cent. profit on the cost price; how much did the watch cost him?

45. If to every quart of milk a milkman adds half-a-pint of water, and if he sells the mixture at the same price per quart as he gave for the milk alone, what is his profit per cent.?

46. Explain the meaning of a prime factor.

Write down all the prime factors common to 3003 and 8778.

47. Divide (by decimals) 72 thousandths by 25, and also by nine ten-thousandths.

48. Taking the values of zinc and copper to be £17. 14*s.* 0*d.* and £73. 15*s.* 0*d.* per ton respectively, find how much of each metal there will be in a mass compounded of the two, which weighs 14 cwt., and is worth £23. 12*s.* 0*d.*?

49. A man loses 30 per cent. of his property and has £1400 left; how much had he at first?

50. Explain the rule for finding the Least Common Multiple. Find the smallest sum of which 14*s.* 7*d.*, £1. 11*s.* 6*d.* and £3. 15*s.* are exact parts.

---

51. Eight bells, which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with a clock striking; how many hours must elapse before they all toll simultaneously again with the clock striking? The clock is supposed to strike at the hour only.

52. Three railway tickets, a 1st, a 2nd and half a third class were purchased for 16*s.* $10\frac{1}{2}d.$ The 1st class ticket costs $1\frac{2}{3}$ times as much as the 2nd, and the 2nd class $1\frac{1}{2}$ times as much as a whole

3rd class ticket. The distance travelled was 45 miles. Find the cost of each ticket and the rate per mile for each class.

53. What number is that from which if $\frac{7}{1\frac{5}{8}} - \frac{3}{1\frac{1}{2}}$ be deducted, and to the remainder $\frac{4}{1\frac{1}{4}}$ be added, the sum will be $3\frac{2}{5}\frac{3}{6}$?

54. If with sales amounting to £1000 per month a tradesman gains £100 in 7 months, what sales would he have to make in order to gain £60. 10*s*. in 11 months?

55. Distribute £630 among A, B, C and D; so that B and C may together have half what A and D have together, that B's money may be $\frac{7}{8}$ of C's, and D's money $\frac{2}{5}$ of A's.

56. Find the square root of the Least Common Multiple of $7\frac{1}{9}$, $19\frac{9}{25}$ and $20\frac{1}{4}$.

57. Express 6·72 oz. as the decimal of $2\frac{1}{2}$ cwt., and explain the process by which you obtain your answer.

58. A population of 84,700 people loses during a pestilence 5929 persons; what is the loss per cent.?

59. If air contains 23·01 % of its weight of oxygen, and if a cubic ft. of air weighs 1·23 oz., find to the nearest ounce the weight of oxygen in a room 18 ft. wide, 27 ft. 4 in. long and 12 ft. high.

60. Show that the difference between any improper fraction and unity is always greater than the difference between unity and the reciprocal of the fraction.

[*Take any improper fraction which you like to select, and show that the proposition is true for that fraction; then extend your reasoning to improper fractions generally.*]

---

61. The residue of an estate was left to be divided between three persons, A, B, C, in such proportion that A's share was to be to B's share as 4 : 5, and B's share to C's as 9 : 16; the residue realised £2415. How much was each person entitled to?

62. If you invest a sum of money in such a way that on one-third of it you gain 3 per cent., on one-fifth you lose 4 per cent., and on the remainder gain 6 per cent.; what average rate per cent. do you make on the whole sum invested?

63. The value of a certain house in 1880 has increased 35 per cent. since 1877. The house was rated in 1877 at two-thirds of its value, and in 1880 it is rated at three-fifths of its value, the rate in the £ remaining the same. Compare the rate paid in 1877 with that paid in 1880.

64. What is the greatest number that will divide 1035 and 1665 without remainders?

65. After deducting income tax at 8*d.* in the £ from a sum of money, £508. 9*s.* 4*d.* is left; what is the sum?

66. Find the difference between $10\frac{2}{3}$ and $1\frac{5}{7}$, and then divide it by the sum of $7\frac{1}{8}$ and $3\frac{3}{10}$.

67. Divide $19\frac{111}{133}$ by $4\frac{27}{71}$.

68. I have to collect £21 for a society. In the 1st week I collected £4. 7*s.* 6*d.* and in the 2nd week £8. 16*s.* 3*d.* What fraction of the whole sum was collected, and what fraction remains uncollected?

69. A druggist buys a certain commodity by avoirdupois weight and sells it by troy weight. The buying price is 3*d.* per oz., and he sells at 6*d.* per oz. What is his gain per cent. on the buying price?

70. What is the rateable value of a parish if a rate of 3*s.* $5\frac{1}{2}$*d.* in the £ produces a sum of £14,352. 1*s.* 8*d.*?

---

71. A person sold a horse at a loss of 20 per cent. on the cost price; if he had received £10 more for it, he would have gained 10 per cent. on the cost price. Find the cost of the horse.

72. A man leaves £32,818 to be divided among his four sons in the proportion of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$. Find the share of each.

73. What is the value of $4\frac{1}{2}$ per cent. of £1350?

74. Goods sold for £207. 14*s.* $9\frac{1}{2}$*d.* gave a profit of three-eighths of their cost. Find their cost.

75. Find the dividend on £4146. 12*s.* 6*d.* at 11*s.* 8*d.* in the £.

76. How often is each of the quantities $19\frac{1}{8}$, $19\frac{5}{6}$, and $21\frac{1}{4}$ contained in their Least Common Multiple?

77. Reduce £1. 17*s.* $1\frac{1}{2}$*d.* to the fraction of £22. 5*s.* 6*d.*

78. Reduce $1\frac{1}{2\frac{1}{5}} \times 3\frac{5}{6} \div 6\frac{3}{11}$ to a decimal.

79. If the cost of a piece of cloth, 28 inches wide, be 8 guineas, what should be paid for a piece twice as long and 25 inches wide?

80. If a man buys eggs at 10 for a shilling, and sells them at 8 for a shilling, what rate per cent. profit does he make on his outlay?

---

81. After spending $\frac{5}{8}$ of my money at A, then $\frac{3}{4}$ of the remainder at B, and next $\frac{4}{1\frac{2}{3}}$ of what still remained at C, I had 3*s.* $9\frac{1}{2}$*d.* over. How much had I at first?

82. When 2 cwt. 1 qr. 2 lb. of sugar are taken away from a ton seven times, what decimal of a ton still remains, and what is the value of the remainder at £20·8 per ton?

83. Simplify each of the following: (*a*) $\frac{2}{7}\frac{8}{8}\frac{8}{2}$, (*b*) $\frac{3\frac{1}{3}}{5\frac{5}{7}}$, (*c*) $(2\frac{5}{8} + 1\frac{3}{7}) \div \frac{9}{56}$.

84. A heap of cannon balls can be arranged in groups of 34, 51, or 170, leaving 10 over from each group. What is the least number of balls the heap can contain?

85. At noon I had completed $\frac{2}{11}$ of my journey, at 2.15 I had completed $\frac{5}{8}$ of it, and had travelled 63 miles since noon. Find (*a*) the total length of my journey, (*b*) the uniform rate of travelling, (*c*) the times of starting and arriving.

86. Find the cost of 389 pairs of boots at £3. 7*s.* 6*d.* for 4 pairs.

87. State the rule for the multiplication of fractions, and give the reason for it.

Which is the greatest of the fractions $\frac{7}{15}$, $\frac{9}{17}$, $\frac{13}{25}$?

88. A person, after spending half of his money, gives away 17*s.* 6*d.* He has then 17*s.* 6*d.* left. How much had he at first?

89. Divide £21. 7*s.* 6*d.* between a man, a woman, and a boy, so that for every 5*s.* the man gets, the boy may get 3*s.*, and for every 5*s.* the woman gets, the boy may get 3*s.* 9*d.*

90. A salary of 600 rupees a month (12 months to the year) was once worth £720 a year; but, owing to the depreciation of silver, it is now reduced by $31\frac{1}{4}$ per cent. What is the present value in English money of a rupee?

---

91. Simplify $\frac{4\frac{3}{7}-\frac{5}{9} \text{ of } 4\frac{2}{13}+6\div 4\frac{2}{5}}{18\frac{99}{91}\div 8-\frac{2}{5} \text{ of } 1\frac{6}{7}}$.

92. A man pays a tenth of his income in rates and taxes and a twelfth in insurances, then he has left £492. 13*s.* 1*d.* What was his original income?

93. A person directed in his will that $\frac{1}{3}$ of his property (£38,657. 6*s.* 6*d.*) should go to A, $\frac{1}{4}$ to B, $\frac{1}{5}$ to C and $\frac{1}{6}$ to D. Show that these directions could not be carried out. Show how it should be divided so that the shares may have to one another the ratio he intended.

94. A builder (A) sells to an agent (B) a house for £4860, by which he loses 19 per cent. on the cost price; B disposes of it to C at a price which would have given A 17 per cent. on the cost price profit. Find B's total gain.

95. The children of a school can be arranged either in classes of 32, 48 or 40 with none over. What is the least number of scholars which will permit this, and how many classes will there be of each number?

96. If gold is worth £3. 17*s.* 6*d.* an ounce, and a nugget of gold weighing 1 lb. 6 oz. 15 dwt. contains 15% of gold, what is the value of the gold?

97. If $3\frac{1}{2}$ tons be carried a certain distance for £7. 15*s.* 2*d.*, how much should be charged for 95 cwt. for the same distance?

98. A, B and C have a pasture. A puts 500 sheep on it for 4 months, B 600 for 3 months and C 1200 for 2 months. If the whole rent be £279, how much should each pay?

99. The sum of three fractions is $\frac{3}{4}$. The first was $\frac{9}{64}$, the second $\frac{19}{336}$. What was the third?

100. Find the prime factors of 495, 429 and 195, and employ them to find the square root of the product of these numbers.

---

101. A farmer bought a flock of sheep numbering 864 for £1720, and kept them for 6 months at a cost of 14*s.* 9*d.* per head: he then sold them at £3. 1*s.* 8*d.* each. What profit or loss was made?

102. A farmer has 14 milking cows, each giving 2 gallons 1 pint of milk per milking. They are milked morning and evening, and the milk sold at $1\frac{1}{4}d.$ per pint. What is the value of the milk sold in the month of May?

103. A man buys 45 bushels of apples (40 lb. to the bushel) for £6. One third of them is spoilt; he sells a third of the remainder at 2*d.* a lb. and two thirds at 4*s.* a bushel. What profit is made, and what fraction of the whole cost is profit?

104. A mother receives £2. 10*s.* $9\frac{1}{2}d.$ weekly, and spends in 5 weeks £7. 12*s.* $4\frac{1}{2}d.$ How long will she be in saving £45. 14*s.* 3*d.*?

105. Three tons of goods were bought at the rate of $6\frac{1}{2}d.$ per lb., and the carriage was 5*d.* a lb., what is gained by selling them at £5. 12*s.* per cwt.?

106. How many turkeys, each worth 15*s.* $7\frac{3}{4}d.$, are equivalent in value to 1502 geese at 9*s.* 5*d.* each?

107. If the product of $\frac{7}{8}$ and $2\frac{1}{4}$ be added to the sum of $21\frac{5}{7}$ and $3\frac{6}{11}$, by how much will the result differ from 100?

108. A man spends 65 % of his income and saves £434. What is the amount of his income, and what sum does he spend?

109. The population of a town on Jan. 1, 1901, was 64,000. It increased 5 per cent. in each year; what was the population on Jan. 1, 1904?

110. An English sovereign contains 113 grains pure gold: 15·432 grains make 1 gramme; a 20 franc piece contains 5·8 grammes of pure gold. What number of francs should £1 be equal to?

---

111. A merchant sells an article at an advance of 12 per cent. on the cost price; how much per cent. of the selling price is profit?

112. The gross rental of a farm is £520. The owner pays income-tax at 7*d.* in the £, and then 5 % of the remainder to the collector of the rent. How much will the owner himself receive?

113. An army of 145,600 men loses 12 % of its number through disease and 15 % of the remainder in battle. How many men are left?

114. A solicitor charges a certain percentage for negotiating a mortgage of £1450 on a house. His bill is £23, of which 15 guineas are law costs. What does he charge per cent. for negotiating?

115. If 12 % of an army be disabled in the first month of a campaign, and 15 % of the remainder in the second month, and 10 % of the latter remainder in the third month, it then numbers 33,660. What was the original number of men, and the number disabled each month.

116. In coining shillings at the Mint, a composition is used consisting of 37 parts of silver and 3 parts of copper. The price of the silver is 5*s.* 2*d.* an ounce, and the weight of a shilling is $\frac{2}{11}$ of an ounce. What is the value of the pure silver in the shilling?

117. 40 lbs. troy of standard gold can be coined into 1869 sovereigns, the proportion of pure gold to alloy in standard gold being 22 to 2. What weight of pure gold is there in a sovereign?

118. A, B, C agree to pay their Hotel Bill in the proportion of 4 : 5 : 6. A pays the first day's bill, which is £2. 10*s.* 10*d.*; B the 2nd day's, which is £3. 12*s.* 6*d.*; and C the 3rd, which is £3. 17*s.* How must they settle their accounts?

119. Three men A, B, C have a grazing farm and pay £42. 12*s.* rent: A puts in 90 oxen, B puts in 96 and C puts in 98. What proportion of the rent should each pay?

120. A farmer bought 4 horses and 7 cows for £238, and the price of 2 horses was equal to the price of 5 cows. How much did he give for each?

---

121. A works for 6 days of 8 hours a day; B works for 5 hrs. on the first day, and on each of the 5 subsequent days 1 hr. longer than on the preceding day. A does as much in 4 hrs. as B does in

5 hrs. The total sum paid to A and B for wages for the work is £2. 2*s*. How much should each receive?

122. A legacy of £2420 is left to three persons in such proportions that after the payment of a legacy duty of 15 per cent. the first receives twice as much as the second, the second three times as much as the third. What are their respective shares?

123. A bankrupt, whose estate is worth £698, has 4 creditors, A, B, C and D. He owes A £270, B £300, C £150 and D £100. The legal expenses of his bankruptcy are £83. How much will each creditor receive?

124. A person bought an estate and subsequently sold it for £625 less than he gave for it, thereby losing $1\frac{1}{4}$ per cent. on the cost price. What should he have received in order to gain $12\frac{1}{2}$ per cent. on the cost price?

125. If a grocer gains 10 per cent. by selling tea at 2*s*. 3*d*. per lb., what will he gain per cent. on the buying price by selling it at 2*s*. 9*d*. per lb.?

126. If 4 per cent. be lost by selling silk at 10*s*. per yard, at what price per yard should it be sold in order to gain 5 per cent.?

127. Find the value of $\frac{3}{8}$ of 7*s*. 6*d*. + 0·625 of 10*s*. $-\frac{5}{9}\frac{4}{9}$ of 9*s*. 2*d*., and express the result as the decimal of £4.

128. Multiply £15 by 7·8498, giving the answer true to the nearest penny.

129. If 45 sheep cost £20, how much will 27 cost?

130. What dividend will be paid on a debt of £906. 10*s*. at 15*s*. 1*d*. in the £?

131. What percentage of £1625 is £52. 16*s*. 3*d*.?

132. What is the lowest number divisible exactly by 33, 22, 55 and 30?

42. CALCULATIONS AND APPROXIMATIONS.

In actual business transactions answers and results rarely work out exactly, and the figures are only approximations.

EXAMPLE.—*A man asks his butcher for a piece of beef to weigh about* 10 *lb. A piece is cut off, and on weighing is found to be* 9 *lb.* 13 *oz.—or nearer* 9 *lb.* 13 *oz. than* 9 *lb.* 12 *oz. The butcher calculates the price at say* 11*d. per lb. on the assumption that the weight is* 9 *lb.* 13 *oz. What is the cost of the beef?*

The price is therefore $11d. \times 9\frac{13}{16} = 8s.\ 11\frac{15}{16}d.$, but $8s.\ 11\frac{15}{16}d.$ is nearer $9s.$ than $8s.\ 11\frac{1}{2}d.$, because $\frac{15}{16}d.$ is nearer to $1d.$, *i.e.* $\frac{16}{16}d.$, than to a halfpenny, *i.e.* $\frac{8}{16}d.$

Accordingly, the price charged is $9s.$

NOTE.—It would probably be found, however, that the butcher would charge $9s.$ for anything over $8s.\ 11\frac{1}{2}d.$ and less than $9s.$, whether nearer to $9s.$ or not.

EXAMPLE.—*Find the cost of* 115 *tons* 8 *cwt.* 25 *lb. of coal at* 12*s.* 6*d. per ton.*

Coal would be weighed to the nearest quarter probably, and the price calculated accordingly. Thus, when sold at 12*s.* 6*d.* per ton, an amount of coal 115 tons 8 cwt. 25 lb. in reality would have only been weighed to the nearest quarter, and the price calculated on 115 tons 8 cwt. 1 qr.

			s.	*d.*	
The cost of the 115 tons would be			1437	6	
,,	,,	8 cwt.	,,	5 . 0	
,,	,,	1 qr.	,,	0 . 2,	though really the fourth part of $7\frac{1}{2}d.$

The total cost therefore would be 1442*s.* 8*d.*, *i.e.* £72. 2*s.* 8*d.*

EXERCISES LX. a.

Mental or Oral.

Find true to the nearest $\frac{1}{2}d.$ the cost of the following :

1. Shoulder of mutton (*a*) 4 lb. 10 oz., (*b*) 5 lb. 2 oz., (*c*) 4 lb. 12 oz., (*d*) 6 lb. 3 oz., at 9*d.* per lb.

2. Sirloin of beef (*a*) 7 lb. 12 oz., (*b*) 4 lb. 6 oz., (*c*) 7 lb. 2 oz., (*d*) 8 lb. 9 oz., at $11\frac{1}{2}d.$ per lb.

3. Piece of cheese (*a*) 5 lb. 3 oz., (*b*) 4 lb. 5 oz., (*c*) 6 lb. 8 oz., (*d*) 12 lb. 6 oz., at 11*d.* per lb.

4. A length of silk (*a*) 2 yds. 34 in., (*b*) 3 yds. 16 in., (*c*) $4\frac{1}{8}$ yds. long, at 4*s.* 3*d.* per yd.

5. A piece of cloth 4 yds. 8 in. long, at 5*s.* per yard.

EXERCISES LX. b.

Find true to the nearest $\frac{1}{2}d.$ the cost of the following :

1. A truck load of coal, 7 tons 14 cwt. 2 qrs., at 11*s.* 4*d.* per ton.

2. A mass of 34 cwt. 27 lb. of copper, at £63 per ton. (Weighed and calculated true to the nearest lb.)

3. A length of cloth 3 yds. and an eighth, at 3*s.* 3*d.* per yd.

4. A piece of timber 7 ft. 8 in. long, at 8*d.* per foot run.

5. Two pounds three ounces of tea, at 1*s.* $7\frac{1}{2}d.$ per lb.

Find the weights of the following masses :

6. $25\frac{1}{4}$ cubic feet of water—assuming that 1 cubic foot of water weighs 62·5 lb. (State answer true to the nearest lb.)

7. A brick wall $67\frac{1}{2}$ feet long, 9 inches thick and $7\frac{1}{2}$ feet high. (Assume 1 cubic foot of brickwork weighs 1 cwt., and state your answer true to the nearest cwt.)

8. A prism of iron 10 ft. 3 in. long and 87 sq. inches in cross section. State your answer true to the nearest quarter. (1 cubic foot of iron to be assumed as weighing 450 lb.)

9. A cable length of 120 fathoms, each foot of which weighs $2\frac{2}{3}$ lb. (State your answer true to the nearest qr.)

10. The yield of a vein of coal, 2 ft. 9 in. thick, 6 acres in extent. (State your answer in tons true to the nearest hundreds digit, assuming that approximately 1 cubic yard in the mine yields one ton of coal.)

11. What quantity of coal per acre can be obtained from three seams, severally 2 ft. 3 in., 2 ft. 6 in. and 2 ft. 9 in. thick, assuming the same data as in Ex. 10? (True to the nearest hundred.)

12. The yearly output of a limestone quarry being 23,536 tons, find the yield of lime to the nearest hundred when the stone yields 52 per cent. of lime.

13. What is the cost to the nearest penny of cutting 63 tons 11 cwt. 2 qrs. of coal, at 1*s.* 7¼*d.* per ton?

14. Find also the cost to the nearest penny of cutting 95 tons 15 cwt. 1 qr. if the price per ton be 30 % above that in Ex. **13.**

15. Find the cost to the nearest penny of 43⅜ gall. of oil at 1*s.* 2*d.* per gall.

16. Find the cost of 4 tons 17 cwt. 3 qrs. of pit wood, at 19*s.* 6*d.* per ton.

17. What is the value of 134 lb. 9 oz. of old brass at 6½*d.* per lb.?

18. What is the value of a copper ingot weighing 2 cwt. 21 lb., at 8¼*d.* per lb.?

19. 3000 fire-bricks are assumed to weigh 9 tons. What is the weight of a brick to the nearest lb.?

20. Find true to a thousand gallons what additional quantity of water is added to a reservoir whose area is 55 acres 1200 sq. yds. by a rainfall of 0·9 inch, assuming 277 c. inches to a gallon.

21. Find true to a ton the yield of hay from a meadow of 5 ac. 3 ro. 25 sq. po., at 4 tons to the acre.

22. Find true to the nearest quarter the yield of wheat from an acreage of 146 acres 2 ro. 16 po., at 3½ quarters to the acre.

23. Find to the nearest cubic yard the quantity of earth removed in forming a ditch 220 yds. long, 8 ft. wide and 3 ft. 3 in. deep.

24. Estimate to the nearest thousand the number of gallons of water pumped from a mine per day by an engine, when it makes

3 strokes per minute and discharges 19·3 cubic ft. per stroke, assuming 1 cubic ft. of water to equal 6·25 gall.

25. Find to the nearest acre the area of a rectangular park 2 m. 600 yds. long and 1 m. 800 yds. wide.

N.B.—It should be carefully noted that the degree of approximation varies in each particular case.

For example, a tax was made on salaries, investments, etc. The amount taxed was £608,606,903; the tax could be determined at a certain amount in the £, fixed exactly to a $\frac{1}{2}d.$ A variation of $\frac{1}{2}d.$ in the rate per £ makes, however, a difference of over £1,400,000 in the produce from the tax.

26. A town estimates its expenditure for the year as £26,000, true to the nearest thousand pounds. If the rateable value of the town be £170,000, find to what degree of accuracy (nearest penny or halfpenny, etc.) the rate per £ can be fixed.

27. A train is drawn along at a nominal speed of 80 ft. per sec. If the speed varies to the extent of 1 ft. per sec., what difference might it make in the distance described in 12 hours' continuous run?

28. If, owing to unequal arms, a balance is out by 3·5 lb. in a cwt., what is the amount of the error when 96 tons 18 cwt. 3 qrs. are weighed by it?

Decimalization of Money.

The fact that the money payment in exchange for goods and service is only approximate, is important in its bearing on the use of the decimal system.

$1s. = 0{\cdot}05$ of a £.

$1d. = 0{\cdot}0041666\ldots$£, *i.e.* $= 0{\cdot}00416\dot{}$£.

$\frac{1}{4}d. = 0{\cdot}00104166\ldots$£, $= 0{\cdot}00104166\dot{}$£.

So that, frequently, to express a sum of money as a decimal of a £ would involve the use of recurring decimals, and the decimal system might then be inconvenient for calculation.

If the calculations are given, however, to the *nearest* farthing, and it is noted that $\frac{1}{4}d. =$ £$0{\cdot}00104166\ldots$ and that

£0·001 = 0·96 farthing, it will often be sufficient to state the decimalized sum of money to the nearest figure in the third place of decimals.

EXAMPLE.—*To decimalize a sum of money, say* £14. 12*s.* 7¼*d.*

(*a*) When the methods of ordinary reduction are employed, the working will be as follows:

£14. 12*s.* 7¼*d.* = £14. 12*s.* 7·25*d.* = £14. 12·604166...*s.*
= £14·63020833....

Frequently, especially in mental and approximate calculations, it is more convenient to express the pence and farthings as decimals of a £, without first expressing them as decimals of a shilling. Thus:

1 farthing $= £\frac{1}{960} = £\left(\frac{1000}{960}\right)\frac{1}{1000} = £\left(\frac{960+40}{960}\right)\left(\frac{1}{1000}\right)$
$= £\left(1+\frac{40}{960}\right)\frac{1}{1000} = £\left(1+\frac{1}{24}\right)0{\cdot}001$;

and, therefore, if the *pence* and *farthings* be expressed as a number of farthings, and this number be *increased* by its 24*th part* (usually this will only be required to the nearest unit), the result will give the pence and farthings as *thousandths* of £1.

Applied to the same example, the working (mental) would be as under:

(*b*) 7¼*d.* = 29 farthings
= 0·030£ approx.
12*s.* = 12(0·05)£
= 0·60£.

Increase 29 by its 24th part, true to the nearest unit, and regard each unit as a thousandth of £.

£14. 12*s.* 7¼*d.* = £14·630 approx., but true to 3rd decimal place.

More accurately, £14. 12*s.* 7¼*d.* = £14·629 + £$\frac{1}{24}$(0·029).

EXAMPLE.—*To decimalize* £103. 13*s.* 2½*d.*

2½*d.* = 10 farthings = £0·010 approx.

$\left(\text{or accurately, } £0{\cdot}010 + \frac{£0{\cdot}010}{24}\right)$.

13*s.* = 13 × £0·05 = £0·650.

∴ £103. 13*s.* 2½*d.* = £103·660, true to the 3rd decimal place.

More accurately, £103. 13*s.* 2½*d.* = £103·660 + $\frac{£0{\cdot}01}{24}$.

EXAMPLE.—*To decimalize* £25. 8*s*. $4\frac{1}{4}d$.

$4\frac{1}{4}d. = 17$ farthings
$= £0{\cdot}018$
$8s. = £0{\cdot}4$,
$\therefore$ £25. 8*s*. $4\frac{1}{4}d. = £25{\cdot}418$ approx.

17 increased by its 24th part is 18, true to the nearest unit.

$$\left(= £25{\cdot}417 + \frac{£0{\cdot}017}{24} \text{ accurately}\right).$$

Conversely, the method of converting sums of money expressed in decimals of £, to pounds, shillings and pence, is illustrated by the following example:

EXAMPLE.—*To express* £36·243 *as* £. *s*. *d*.

Each 0·05 corresponds to a 1*s*.,
each 0·001 is a $\frac{1}{4}d$. nearly.

And £36·243 = £36. 4*s*. + 43 farthings nearly
= £36. 4*s*. + $10\frac{3}{4}d$. nearly.

To be more accurate, we should remember that in decimalizing the farthings we take each $\frac{1}{4}d$ as £0·001, and then increase the result by $\frac{1}{24}$.

Here, therefore, we should *diminish* the 43 farthings by their 24th part, and thereby obtain a close approximation.

$\therefore$ since $\frac{1}{24}$th of 43 = 2 approx.
£0·043 = 41 farthings approx.
and £0·2 = 4*s*.
$\therefore$ £36·243 = £36. 4*s*. $10\frac{1}{4}d$.

Evaluating £36·243 in the ordinary way,

£36·243 = £36 + 20(·243)*s*. = £36. 4·86*s*.
= £36. 4*s*. 10·32*d*.
= £36. 4*s*. $10\frac{1}{4}d$.
to the nearest farthing.

EXAMPLE.—*To express* £6·183 *as* £. *s*. *d*.

0·183 = 3(0·05) + 0·033
33 reduced by its 24th part = 32 approx.
$\therefore$ £6·183 = £6. 3*s*. + 32 farthings approx.
= £6. 3*s*. 8*d*. true to the nearest farthing.

EXERCISES LXI. a.

Decimalize the following, or express in £. *s.* *d.* (true to the nearest farthing or to £0·001), by the preceding methods, and verify in the manner employed on pp. 240-241 :

1. £26. 1*s.* 2½*d.*
2. £26. 1*s.* 5½*d.*
3. £35·121.
4. £35·108.
5. £98. 18*s.* 5¼*d.*
6. £15. 11*s.* 8*d.*
7. £83·696.
8. £34·112.
9. £17·857.

EXERCISES LXI. b.

Mental or Oral.

Express as decimals of £:

1. £2. 2*s.* 0*d.*
2. £2. 3*s.* 0*d.*
3. £5. 5*s.* 0*d.*
4. £5. 8*s.* 0*d.*
5. £3. 17*s.* 0*d.*
6. £4. 15*s.* 0*d.*
7. £102. 13*s.* 0*d.*
8. £116. 19*s.* 0*d.*
9. ¼*d.*
10. ¾*d.*
11. 1¼*d.*
12. 2¼*d.*
13. 5¾*d.*
14. £2. 0*s.* 8¼*d.*
15. £12. 0*s.* 9¾*d.*
16. £11. 10*s.* 8¼*d.*
17. £12. 8*s.* 7½*d.*
18. £12. 9*s.* 7½*d.*
19. £12. 1*s.* 9¾*d.*
20. £101. 3*s.* 4½*d.*
21. £13. 2*s.* 1½*d.*
22. £13. 7*s.* 3½*d.*
23. £14. 6*s.* 1¼*d.*
24. £114. 5*s.* 1¼*d.*

EXERCISES LXI. c.

Mental or Oral.

Express £. *s.* *d.* as decimals of a £, or *vice versa*, in the following :

1. £2. 13*s.* 6*d.*
2. £3. 8*s.* 4½*d.*
3. £101·123.
4. £6. 17*s.* 0¼*d.*
5. £3. 15*s.* 0¼*d.*
6. £7. 16*s.* 1*d.*
7. £3. 14*s.* 1¼*d.*
8. £3·287.
9. £6·102.
10. £12·301.
11. £8·293.
12. £16. 18*s.* 7*d.*
13. £3. 16*s.* 4*d.*
14. £1·726.
15. £2·318.
16. £100. 2*s.* 0*d.*
17. £100. 1*s.* 10*d.*
18. £23. 17*s.* 11*d.*
19. £15. 6*s.* 8¾*d.*
20. £3·167.
21. £2·135.
22. £2. 3*s.* 6½*d.*
23. £7. 1*s.* 2¼*d.*
24. £13·006.
25. £8·103.
26. £9. 2*s.* 7½*d.*

Possible Errors in Decimalization.

In the method of decimalizing sums of money that has already been explained, it must be noticed that frequently it may not be accurate enough to decimalize to the third place, when the sum of money is expressed in £.

EXAMPLE.—*To evaluate the cost of* 30,000 *articles at* £12. 2*s.* $4\frac{3}{4}$*d. each.*

£12. 2*s.* $4\frac{3}{4}$*d.* = £12·1 and 19 farthings
= £(12·119 + $\frac{1}{24}$ of 0·019)
= £12·120 true to the third decimal place.

Taking the cost of 30,000, at this assumed value for the single article, the cost works out
= 30,000 × £12·120 = £363,600;
whereas the correct cost
= 30,000 × £12·119 + 30,000 × $\frac{1}{24}$ × £0·019
= £363,570 + £$\frac{190}{8}$
= £363,570 + £23. 15*s.*
= £363,593. 15*s.* with an error of £6. 5*s.* 0*d.*

EXAMPLE.—*Evaluate the cost of* 3600 *articles at* £1. 18*s.* $7\frac{1}{2}$*d. each.*

£1. 18*s.* $7\frac{1}{2}$*d.* = £1·9 + 30 farthings = £1·9 + £0·030 + $\frac{£0·030}{24}$

3600 × £1. 18*s.* $7\frac{1}{2}$*d.* = 3600 × £1·930 + $\frac{1}{24}$ × 3600 × £0·030
= £694·8 + £4·5
= £699. 6*s.*

It does not always follow that the above is the quickest or most advantageous method of evaluating, and it is mentioned here in order to point out the possible error which might creep in when the approximate decimalized value is used. In dividing by numbers greater than 1, there is no danger in approximating, since the error gets reduced.

EXERCISES LXII.

1. Evaluate the following :
   (*a*) £1. 17*s*. 4*d*. × (i) 100, (ii) 1000, (iii) 800, (iv) 480.
   (*b*) £156. 4*s*. $2\frac{3}{4}d.$ ÷ (i) 100, (ii) 700, (iii) 300.
   (*c*) £2892. 0*s*. $7\frac{1}{2}d.$ × (i) 260, (ii) 125.

2. In finding the cost of 600,000 articles at 12*s*. $7\frac{1}{2}d.$ each, the result is calculated by reducing 12*s*. $7\frac{1}{2}d.$ to the decimal of a £, true to the third decimal figure. What is the error so produced?

3. Find the cost of
   (*a*) 125 articles at £3. 10*s*. 2*d*. each.
   (*b*) 200 articles at £4. 11*s*. $2\frac{1}{4}d.$ each.
   (*c*) 100 articles at £1. 11*s*. $0\frac{1}{2}d.$ each.
   (*d*) 450 articles at £3. 10*s*. $4\frac{1}{4}d.$ each.

4. What error would ensue if, in evaluating the following, the approximate values in decimalizing are used before multiplying?

	£.	*s*.	*d*.
(*a*) 1000 articles at	0 .	10 .	$1\frac{1}{4}$.
(*b*) 5600 „	0 .	7 .	$2\frac{1}{2}$.
(*c*) 84,000 „	0 .	1 .	$1\frac{1}{2}$.
(*d*) 96,000 „	1 .	3 .	$0\frac{1}{2}$.

43. PRACTICE.

Practice is a simple and convenient method of computation by means of aliquot parts. An **aliquot part** of a number or quantity is a part which will divide into that number or quantity without remainder. Thus, 2*s*. 6*d*. is an aliquot part of £1, for it is exactly $\frac{1}{8}$ of £1.

The method of finding cost by practice sums is often used by tradesmen, and will be evident from the following worked examples. It will be noted that the process admits of many variations.

EXAMPLE. — *What is the value of* 873 *articles at* £3. 5*s.* 10½*d. each?*

(*a*)

	£.	s.	d.	
	873	0	0	cost if articles were £1 each.
			3	
	2619	0	0	„ „ £3 „
5*s.* = ¼th of £1.	218	5	0	„ „ 5*s.* „ (*i.e.* ¼th the cost at £1 each).
7½*d.* = ⅛th of 5*s.*	27	5	7½	cost if articles were 7½*d.* each (*i.e.* ⅛th the cost at 5*s.* each).
3*d.* = 1/20th of 5*s.*	10	18	3	cost if articles were 3*d.* each (*i.e.* 1/20th the cost at 5*s.* each).
	2875	8	10½	cost at £3. 5*s.* 10½*d.* each.

(*b*)

	£.	s.	d.	
	873	0	0	
			3	
	2619	0	0	cost at £3 each.
6*s.* = 1/10th of £3.	261	18	0	„ 6*s.* „
	2880	18	0	„ £3. 6*s.* each.
1½*d.* = ⅛th of 6*s.*	5	9	1½	„ 1½*d.* each.
	2875	8	10½	By subtraction obtain cost at £3. 5*s.* 10½*d.*

(*c*) More frequently it is advisable to work in decimals.

	£.	
	873	
	3	
	2619	cost at £3 each.
5*s.* = ¼th of £1.	218·25	„ 5*s.* „
6*d.* = 1/10th of 5*s.*	21·825	„ 6*d.* „
3*d.* = ½ of 6*d.*	10·9125	„ 3*d.* „
1½*d.* = ½ of 3*d.*	5·45625	„ 1½*d.* „
	2875·44425	cost at £3. 5*s.* 10½*d.* each.

Cost, therefore, £2875. 8*s.* 10½*d.*

EXAMPLE.—*Find the cost of* $547\frac{3}{8}$ *cwt. at* £1. 2*s.* $8\frac{1}{2}$*d. per cwt., using the decimal method.*

	£.	
	547·375	cost at £1 per cwt.
2*s.* = $\frac{1}{10}$th of £1.	54·7375	,, 2*s.* ,,
8*d.* = $\frac{1}{3}$rd of 2*s.*	18·24583	,, 3*d.* ,,
$\frac{1}{2}$*d.* = $\frac{1}{16}$th of 8*d.*	1·14036	
	621·399	(To nearest figure in 3rd place of decimals, since 0·39869 is nearer to 0·399 than to 0·398.)

∴ cost = £621. 7*s.* $11\frac{3}{4}$*d.*

EXAMPLE.—*Find the value of* $237\frac{2}{3}$ *tons at* £0. 9*s.* 7*d. per ton.*

	£.				
	237	= cost of 237 tons at £1.			
5*s.* = $\frac{1}{4}$th of £1.	59·25	=	,,	,,	,, 5*s.*
4*s.* = $\frac{1}{5}$th of £1.	47·4	=	,,	,,	,, 4*s.*
6*d.* = $\frac{1}{8}$th of 4*s.*	5·925	=	,,	,,	,, 6*d.*
1*d.* = $\frac{1}{6}$th of 6*d.*	0·9875	=	,,	,,	,, 1*d.*
	103·5625				

= £103. 11*s.* 3*d.* = cost of 237 tons at 9*s.* 7*d.*
6*s.* 5*d.* = ,, $\frac{2}{3}$ ton at 9*s.* 7*d.*
£103. 17*s.* 8*d.* = ,, $237\frac{2}{3}$ tons at 9*s.* 7*d.*

EXERCISES LXIII. a.

Mental or Oral.

1. What aliquot parts of a £ are the following sums :
10*s.*, 6*s.* 8*d.*, 5*s.*, 4*s.*, 1*s.* 8*d.*, 2*s.*, 2*s.* 6*d.*, 3*s.* 4*d.*, 1*s.* 4*d.*, 1*s.*, 10*d.*, 1*s.* 3*d.*?

2. State as aliquot parts of a shilling :
6*d.*, 4*d.*, 3*d.*, 2*d.*, $1\frac{1}{2}$*d.*, 1*d.*

3. What aliquot parts of a ton are :
10 cwt., 5 cwt., 4 cwt., 2 cwt., 1 cwt.?

4. State as aliquot parts of a cwt. :
2 qrs., 1 qr., 56 lbs., 28 lbs., 16 lbs., 14 lbs., 7 lbs.

5. What aliquot parts of a quarter are :
14 lbs., 7 lbs., 4 lbs., $3\frac{1}{2}$ lbs., 2 lbs., 1 lb.?

EXERCISES LXIII. b.

Mental or Oral.

1. What aliquot part of a £ is each of the following?
(*a*) 3*s.* 4*d.*, (*b*) 5*s.*, (*c*) 2*s.*, (*d*) 1*s.* 4*d.*, (*e*) 1*s.* 3*d.*, (*f*) 6*s.* 8*d.*, (*g*) $7\frac{1}{2}d.$, (*h*) 5*d.*

2. What aliquot part of 10*s.* is each of the following?
(*a*) 2*s.* 6*d.*, (*b*) 2*s.*, (*c*) 5*s.*, (*d*) 1*s.* 3*d.*, (*e*) $7\frac{1}{2}d.$, (*f*) 5*d.*, (*g*) 3*s.* 4*d.*, (*h*) $1\frac{1}{2}d.$

3. What aliquot part of 4*s.* is each of the following?
(*a*) 1*s.* 4*d.*, (*b*) 2*s.*, (*c*) 8*d.*, (*d*) 3*d.*, (*e*) 2*d.*, (*f*) $1\frac{1}{2}d.$

4. What aliquot part of 2*s.* 6*d.* is each of the following?
(*a*) 10*d.*, (*b*) 6*d.*, (*c*) 5*d.*, (*d*) 3*d.*, (*e*) $2\frac{1}{2}d.$

What amounts should be placed in the following empty brackets?

5. 2*s.* 3*d.* is (*a*) $\frac{1}{4}$ of ( ), is (*b*) $\frac{1}{6}$ of ( ), is (*c*) $\frac{1}{8}$ of ( ).

6. 1*s.* 4*d.* is (*a*) $\frac{1}{7}$ of ( ), is (*b*) $\frac{1}{10}$ of ( ), is (*c*) $\frac{1}{9}$ of ( ).

7. 13*s.* 4*d.* is (*a*) $\frac{1}{3}$ of ( ), is (*b*) $\frac{1}{7}$ of ( ), is (*c*) $\frac{1}{8}$ of ( ).

8. $2\frac{1}{2}d.$ is (*a*) $\frac{1}{12}$ of ( ), is (*b*) $\frac{1}{10}$ of ( ), is (*c*) $\frac{1}{8}$ of ( ).

Find the values of the following articles :

9. (*a*) 15 at 1*s.* 4*d.*, (*b*) 30 at 1*s.* 4*d.*, (*c*) 31 at 1*s.* 4*d.*, (*d*) 29 at 1*s.* 4*d.*

10. (*a*) 6 at 3*s.* 4*d.*, (*b*) 18 at 3*s.* 4*d.*, (*c*) 24 at 3*s.* 4*d.*, (*d*) 25 at 3*s.* 4*d.*, (*e*) 23 at 3*s.* 4*d.*

11. (*a*) 36 at 4*d.*, (*b*) 27 at 4*d.*, (*c*) 333 at 4*d.*, (*d*) 48 at 4*d.*, (*e*) 49 at 4*d.*

12. (*a*) 24 at 10*d.*, (*b*) 48 at 10*d.*, (*c*) 50 at 10*d.*, (*d*) 46 at 10*d.*, (*e*) 97 at 10*d.*, (*f*) 121 at 10*d.*

13. (*a*) 32 at $7\frac{1}{2}d.$, (*b*) 64 at $7\frac{1}{2}d.$, (*c*) 65 at $7\frac{1}{2}d.$, (*d*) 96 at $7\frac{1}{2}d.$, (*e*) 100 at $7\frac{1}{2}d.$, (*f*) 190 at $7\frac{1}{2}d.$

14. (*a*) 12 at 1*s.* 8*d.*, (*b*) 24 at 1*s.* 8*d.*, (*c*) 26 at 1*s.* 8*d.*, (*d*) 48 at 1*s.* 8*d.*, (*e*) 50 at £1. 1*s.* 8*d.*, (*f*) 96 at £2. 1*s.* 8*d.*

15. (*a*) 8 at 2*s.* 6*d.*, (*b*) 8 at £1. 2*s.* 6*d.*, (*c*) 16 at 2*s.* 6*d.*, (*d*) 15 at £1. 2*s.* 6*d.*, (*e*) 48 at £1. 2*s.* 6*d.*, (*f*) 97 at £1. 2*s.* 6*d.*

EXERCISES LXIII. c.

Calculate the value of the following:

1. 1973 articles at (*a*) 2*d.*, (*b*) 3*d.*, (*c*) 4*d.*, (*d*) 6*d.*, (*e*) 8*d.*, (*f*) 10*d.* each, using in each instance but 1 aliquot part.

2. 2375 articles at (*a*) $1\frac{1}{2}d.$, (*b*) $2\frac{3}{4}d.$, (*c*) $3\frac{1}{4}d.$, (*d*) $4\frac{3}{4}d.$, (*e*) $5\frac{1}{2}d.$, (*f*) $7\frac{1}{4}d.$, (*g*) $8\frac{3}{4}d.$, (*h*) $6\frac{3}{4}d.$, (*i*) 9*d.*, (*j*) $10\frac{1}{2}d.$

3. 584 articles at (*a*) 2*s.* 3*d.*, (*b*) 3*s.* 5*d.*, (*c*) 4*s.* 8*d.*, (*d*) 5*s.* 6*d.*, (*e*) 6*s.* 10*s.*, (*f*) 7*s.* 4*d.*, (*g*) 8*s.* 9*d.*, (*h*) 9*s.* 8*d.*, (*i*) 10*s.* 8*d.*

4. 3719 articles at (*a*) 2*s.* $5\frac{1}{2}d.$, (*b*) 13*s.* $8\frac{1}{4}d.$, (*c*) 15*s.* $9\frac{1}{2}d.$, (*d*) 18*s.* $5\frac{1}{4}d.$, (*e*) 18*s.* $9d.$*, (*f*) 18*s.* $4d.$*, (*g*) 19*s.* 2*d.*, (*h*) 19*s.* $4\frac{1}{2}d.$ (* Using only 1 aliquot part.)

5. 6934 articles at (*a*) £1. 12*s.* 8*d.*, (*b*) £2. 14*s.* $7\frac{1}{2}d.$, (*c*) £3. 7*s.* $3\frac{3}{4}d.$, (*d*) £14. 13*s.* $6\frac{1}{4}d.$

6. 735 articles at (*a*) £23. 6*s.* $10\frac{1}{2}d.$, (*b*) £34. 12*s.* $9\frac{3}{4}d.$, (*c*) £42. 15*s.* $7\frac{1}{2}d.$, (*d*) £53. 17*s.* $8\frac{1}{4}d.$

7. 5931 articles at (*a*) £18. 0*s.* $10\frac{1}{2}d.$, (*b*) £27. 0*s.* $11\frac{1}{4}d.$, (*c*) £19. 3*s.* $1\frac{1}{2}d.$, (*d*) £45. 5*s.* $3\frac{3}{4}d.$

8. 7582 articles at (*a*) £9. 3*s.* 9*d.*, (*b*) £10. 12*s.* 6*d.*, (*c*) £8. 2*s.* 8*d.*, (*d*) £5. 8*s.* $4d.$, (*e*) £19. 19*s.* $4\frac{1}{2}d.$

9. 5327 articles at (*a*) £73. 7*s.* $3\frac{1}{2}d.$, (*b*) £28. 9*s.* $6\frac{3}{4}d.$, (*c*) £54. 16*s.* $4\frac{1}{2}d.$, (*d*) £76. 17*s.* 10*d.*

10. 6751 articles at (*a*) £3. 3*s.* 9*d.*, (*b*) £5. 6*s.* 3*d.*, (*c*) £8. 2*s.* 8*d.*, (*d*) £6. 13*s.* 4*d.*, (*e*) £2. 17*s.* 6*d.*, using only one aliquot part in each case, and then checking your result by using two.

EXERCISES LXIII. d.

Find the cost of the following, giving results true to the nearest penny:

1. $357\frac{3}{4}$ yds. at (*a*) 2*s.* 10*d.*, (*b*) 3*s.* 10*d.*, (*c*) 5*s.* 8*d.*, (*d*) 8*s.* 6*d.*, (*e*) 15*s.* 9*d.*, (*f*) 18*s.* 2*d.* each.

2. $61\frac{7}{12}$ articles at (*a*) £2. 4*s.* 3*d.*, (*b*) £5. 11*s.* $8\frac{1}{2}d.$, (*c*) £9. 14*s.* 7*d.*, (*d*) £36. 17*s.* 9*d.* each.

3. $395\frac{1}{16}$ cwt. at (*a*) £3. 5*s.* 4*d.*, (*b*) £1. 9*s.* 6*d.*, (*c*) £5. 13*s.* 2*d.*, (*d*) £9. 18*s.* 6*d.* each.

4. $59\frac{3}{8}$ miles at (*a*) £0. 4*s.* 10*d.*, (*b*) £2. 8*s.* 10*d.*, (*c*) £9. 12*s.* $5\frac{1}{2}d.$, (*d*) £14. 14*s.* $3\frac{3}{4}d.$ per mile.

5. $765\frac{2}{5}$ oz. at (*a*) 3*s.* $8\frac{1}{2}d.$, (*b*) £3. 17*s.* $10\frac{1}{2}d.$, (*c*) £5. 4*s.* 9*d.*, (*d*) £8. 0*s.* 9*d.* each.

6. 593 tons at (*a*) 15*s.* $4\frac{1}{3}d.$, (*b*) £2. 3*s.* $10\frac{2}{5}d.$, (*c*) £5. 13*s.* $3\frac{3}{8}d.$, (*d*) £9. 14*s.* $7\frac{7}{10}d.$ each.

7. 429 bushels at (*a*) 4*s.* $6\frac{2}{3}d.$, (*b*) £0. 5*s.* $8\frac{5}{12}d.$, (*c*) £1. 7*s.* $9\frac{3}{7}d.$, (*d*) £2. 5*s.* $8\frac{3}{16}d.$ each.

8. 1573 articles at (*a*) 14*s.* $7\frac{1}{7}d.$, (*b*) £3. 15*s.* $6\frac{3}{8}d.$, (*c*) £4. 7*s.* $5\frac{3}{8}d.$, (*d*) £7. 19*s.* $2\frac{3}{16}d.$ each.

9. 2015 articles at (*a*) 15*s.* $0\frac{1}{6}d.$, (*b*) £4. 2*s.* $3\frac{5}{8}d.$, (*c*) £7. 16*s.* $4\frac{1}{16}d.$, (*d*) £3. 12*s.* $8\frac{3}{5}d.$ each.

10. $673\frac{5}{8}$ quarters at (*a*) 4*s.* $2\frac{3}{4}d.$, (*b*) £0. 6*s.* $4\frac{3}{8}d.$, (*c*) £2. 10*s.* $9\frac{5}{16}d.$, (*d*) £5. 17*s.* $8\frac{7}{12}d.$ per qr.

In the case of Compound Practice the same remarks apply as in Simple Practice (see pp. 245-246).

EXERCISES LXIII. e.

Mental or Oral.

Find the value of:

1. (*a*) 5 cwt. at 10*s.* a ton, (*b*) 10 cwt. at 3*s.* 4*d.* a ton, (*c*) 4 cwt. at £2. 7*s.* 6*d.* a ton.

2. (*a*) 1 ton 10 cwt. at 9*s.* 6*d.* a ton, (*b*) 2 tons 5 cwt. at 18*s.* a ton, (*c*) 70 tons 2 cwt. at 5*s.* a ton.

3. (*a*) 16 lbs. at 1*s.* 9*d.* a cwt., (*b*) 14 lbs. at 2*s.* 8*d.* a cwt., (*c*) 8 lbs. at 28*s.* a cwt.

4. (*a*) 8 oz. avoirdupois at 1*s.* 10*d.* a lb., (*b*) 4 oz. at 1*s.* 5*d.* a lb., (*c*) 2 oz. at 10*d.* a lb.

5. (*a*) The rent of a stable for 3 months at £7. 10*s.* the year, (*b*) the rent of a house for 6 months at £11. 15*s.* 0*d.* the year, (*c*) the rent of an office for 73 days at £180 a year.

6. (*a*) 11 yd. at 3*s.* 8*d.* a fur., (*b*) 55 yd. at 19*s.* 6*d.* a fur., (*c*) 2 chains at 6*s.* 8*d.* a mile.

7. (*a*) 4 fur. at £23. 5*s.* 6*d.* a mile, (*b*) 9 inches at 2*s.* 3*d.* a yd., (*c*) 2 yd. 18 in. at 3*s.* a yd.

8. (*a*) 4 pints at 10*d.* a gal., (*b*) 1 peck at 3*s.* 6*d.* a bush., (*c*) 4 bush. at 45*s.* a quarter.

9. (*a*) 25 packets at 9*d.* per 100, (*b*) 48 pen nibs at 1*s.* 3*d.* per gross, (*c*) 250 bricks at 32*s.* 6*d.* per 1000.

10. (*a*) 12 grains at 2½*d.* per dwt., (*b*) 144 grains of gold at £45 per lb.

EXERCISES LXIII. f.

Find the value of each of the following quantities, giving the answers true to the nearest penny where necessary:

1. 17 cwt. 2 qr. 21 lb. at (*a*) £1. 10*s.* 4d., at (*b*) £3. 14*s.* 8*d.* and (*c*) £7 per cwt.

2. 85 cwt. 3 qr. 14 lb. at (*a*) £2. 16*s.*, at (*b*) £3. 17*s.* and (*c*) £5. 14*s.* 4*d.* per cwt.

3. 160 tons 14 cwt. 1 qr. at (*a*) £0. 11*s.* 8*d.*, at (*b*) £2. 4*s.* 2*d.* and (*c*) £9. 6*s.* 8*d.* per ton.

4. 1135 tons 17 cwt. at (*a*) 8*s.* 9*d.*, (*b*) 15*s.* and (*c*) £3. 5*s.* 10*d.* per ton.

5. 19 lb. 10 oz. at (*a*) 1*s.* 8*d.*, (*b*) 3*s.* and (*c*) 4*s.* 4*d.* per lb.

6. 34 miles 6 fur. at (*a*) £6. 13*s.* 6*d.*, (*b*) £18. 15*s.* and (*c*) £47. 11*s.* 3*d.* per mile.

7. 84 m. 968 yd. at (*a*) £5. 12*s.* 6*d.* and (*b*) £24. 6*s.* 3*d.* per mile.

8. 13 m. 65 ch. at (*a*) £9. 10*s.* 4*d.* and (*b*) £8. 14*s.* 6*d.* per mile.

9. 47 m. 101 poles at (*a*) £8. 10*s.* and (*b*) £17. 18*s.* 9*d.* per mile.

10. 59 yd. 25 in. at (*a*) 1*s.* 4*d.* per yd., at (*b*) 2*s.* 3*d.* per yd.

11. 74 ac. 3 ro. 27 sq. po. at (*a*) £2. 18*s.* 4*d.*, at (*b*) £4. 7*s.* 6*d.* per ac.

12. 1 ro. 35 sq. po. at (*a*) £1. 4*s.* 8*d.* and (*b*) £2. 10*s.* per ac.

13. 134 ac. 1331 sq. yd. at (*a*) £2. 5*s.* and (*b*) £3 per ac.

14. (*a*) 73 gal. 5 pt. and (*b*) 28 gal. 3 pt. at 1*s.* 8*d.* per gal.

15. (*a*) 3689 qr. 3 bush. and (*b*) 974 qr. 5 bush. 2 pk. at 4*s.* 8*d.* per qr.

16. (*a*) 935 pints at 10*d.* per gal. and (*b*) 495 bushels at 6*s.* 3*d.* per quarter.

17. (*a*) 53 hr. 36 min. labour at 10*d.* per hr. and (*b*) 4435 min. at 2*s.* 2*d.* per hr.

18. (*a*) 17650 bricks at £1. 7*s.* 6*d.* per 1000 and (*b*) 860 envelopes at 1*s.* 2*d.* per 100.

19. (*a*) 43 lb. 10 oz. of silver at 5*s.* 6*d.* per oz. and (*b*) 1390 grains of gold at £3. 18*s.* per oz.

EXERCISES LXIV.

Miscellaneous.

1. What dividend will be received on £485. 8*s.* 4*d.* at 4*s.* 6*d.* in the £?

2. Find the value of 763 mining shares at 15*s.* $4\frac{3}{8}$*d.* per share.

3. What is the weight of 546 yards of steel rails at 2 qr. 10 lb. per yd.?

4. What will be cost of covering 384 sq. ft. with lead, running 11 lb. per foot super., at $7\frac{1}{2}$*d.* per lb.?

5. Find the railway charge on the transit of 750 tons 14 cwt., for 90 miles, at $\frac{3}{4}$*d.* per ton per mile.

6. Find the amount of the rates in the year on an assessment of £39,456. 10*s.* at 2*s.* 10*d.* in the £ for the 1st half year, and 2*s.* 8*d.* in the 2nd half.

7. What is the income tax on an assessed income of £1135. 18*s.* 9*d.* at the rate of 7*d.* in the £?

8. A field of 24 ac. 3 ro. 30. sq. po. yields 4 quarters of wheat per acre. Calculate the value of the wheat at 32*s.* 6*d.* per qr.

9. A hay rick measures 1656 cubic ft.; find its value at 4 guineas a ton when 12 cubic yd. weigh 1 ton.

10. Find the estimated weight of coal in a taking of 54 ac. 3 ro. 18 sq. po. at 14,520 tons per acre.

11. Copper being sold at £72. 3*s.* 6*d.* per ton and brass at £48. 10*s.* 9*d.*, how much dearer is 6 cwt. 60 lb. of copper than brass?

12. Find the distance run by an express train starting at 10.56 a.m. and reaching its destination at 3.45 p.m. the same day, at the rate of 58 miles 800 yd. per hour.

13. What weight of water does a cistern hold when it contains 9 cubic ft. 648 cub. in.? (1 cubic foot of water weighs 62·5 lbs.)

14. What is the half-year's rent for 3 gardens, each 2 ro. 25 sq. poles, at £3. 3*s.* 4*d.* per acre per year?

15. A merchant's debts are £2375. 18*s.* 9*d.*, but he is only able to pay a dividend of 12*s.* 10*d.* in the pound. What is the value of his assets?

16. The shares of a company are quoted one day at £32$\frac{1}{4}$, and at the end of a week they have fallen to £29$\frac{7}{16}$. What difference in value does this represent to a man holding 96 shares?

17. What is the value of a gold chain weighing 1 oz. 12 dwt. 18 grs. at £4. 16*s.* 0*d.* per oz.?

18. What is the yield of a field of 12 ac. 36 sq. po. at the rate of 4 tons 13 cwt. 70 lb. per acre?

19. What is the cost of enclosing a square field of 436 yards side with wire rope, running 3 lb. to the yard, at 7*s.* per cwt.?

20. Find the duty on the following imported articles. (Employ whatever method you think most suitable.)

(*a*) 3752 lb. of coffee; duty 2*d.* per lb.

(*b*) 1796 gal. of brandy; duty 11*s.* 4*d.* per gal.

(*c*) 96,726 cwt. of sugar; duty 2*s.* 4*d.* per cwt.

(*d*) 2674 cwt. of crystallized fruit; duty 4*s.* 2*d.* per cwt.

(*e*) 189 tons 9 cwt. 14 lb. tea; duty 6*d.* per lb.

(*f*) 4 tons 6 cwt. 6 lb. cigars ; duty 6*s.* per lb.
(*g*) 36,894 lb. cigarettes ; duty 4*s.* 10*d.* per lb.
(*h*) 15 tons 17 cwt. tobacco ; duty 3*s.* 10*d.* per lb.
(*i*) 2 tons 13 cwt. 2 lb. snuff ; duty 4*s.* 4*d.* per lb.
(*j*) 178 tons 16 cwt. unmanufactured tobacco ; duty 3*s.* 3*d.* per lb.
(*k*) 9872 gal. sparkling wine ; duty 2*s.* 6*d.* per gal.

21. Find the excise duty on :
(*a*) 198,700 gal. beer ; duty 7*s.* 9*d.* per 36 gal. (to nearest 1*d.*).
(*b*) 783 tons glucose ; duty 2*s.* 9*d.* per cwt.
(*c*) 187 cwt. saccharine ; duty 1*s.* 3*d.* per ounce.

22. Income Tax being calculated according to the scheme below, find the net income, gross income, or tax, in the following cases :

PROPERTY AND INCOME TAX.

Schedule C, D and E, Income, 1*s.* in the £.

EXEMPTION AND ABATEMENTS.

Income not exceeding £160 Exempt.

Excd. £.	Not Excd. £.	£.
160	400	Abatement of 160
400	500	„ 150
500	600	„ 120
600	700	„ 70

	Gross Income.	Income Tax.	Net Income.
(*a*)	£450	?	?
(*b*)	?	£20	?
(*c*)	?	£31. 10*s.*	?
(*d*)	£1500	?	?
(*e*)	?	?	£760
(*f*)	?	?	£340. 10*s.*

23. Estate Duty is charged at the following rates:

Estate. £.			£.	Duty. per cent. £. s.
100 and does not exceed			500	1 . 0
500	„	„	1,000	2 . 0
1,000	„	„	10,000	3 . 0
10,000	„	„	25,000	4 . 0
25,000	„	„	50,000	4 . 10
50,000	„	„	75,000	5 . 0
75,000	„	„	100,000	5 . 10
100,000	„	„	150,000	6 . 0
150,000	„	„	250,000	6 . 10
250,000	„	„	500,000	7 . 0
500,000	„	„	1,000,000	7 . 10
1,000,000	„	„	—	8 . 0

Find the Estate Duty paid on the following (true to the nearest penny in all cases):

(*a*) £689. 12*s*. (*b*) £18526. (*c*) £3637.
(*d*) £189,764. (*e*) £689,124. (*f*) £532.
(*g*) £1896. (*h*) £7632. 10*s*. (*i*) £600,807.
(*j*) £178,981. (*k*) £631. (*l*) £2563.
(*m*) £7342. (*n*) £3,189,760.

24. Calculate the original value of an estate if, after paying Estate Duty, its value was as below:

(*a*) £396. (*b*) £6596. (*c*) £5432.
(*d*) £73,710. (*e*) £4850. (*f*) £7881. 5*s*.
(*g*) £16,320. (*h*) £518,000.

(Duty at the rates given in Ex. **23**.)

44. ERRORS IN DATA.

In numerous cases the **data**, that is, the given or observed facts or quantities available in a calculation, are not quite accurate; for example, a man wishes to calculate the area of a rectangular plate of metal; he has his foot rule for

measuring the length and the breadth, and with this he finds, true to the nearest *tenth* of an inch, that the plate is, say,

5·4 inches long.
4·5 „ broad.

The calculated area is therefore $5{\cdot}4 \times 4{\cdot}5$ sq. inches.

Multiply

$$\begin{array}{r} 5{\cdot}4 \\ 4{\cdot}5 \\ \hline 21{\cdot}6 \\ 2{\cdot}70 \\ \hline 24{\cdot}30 \end{array}$$

The area works out to be 24·30 square inches.

But when the length is taken as 5·4 in., all that is meant is that the length is nearer 5·4 in. than either 5·3 or 5·5, and similarly that the breadth is nearer 4·5 in. than either 4·4 in. or 4·6 in. The length might actually be either nearly 5·45 in. or only slightly over 5·35 in., while the breadth might actually be either nearly 4·55 in. or only slightly over 4·45 in. The difference between the greatest and lowest value for the area would, in the accompanying figure, be represented by the shaded figure BDCC′D′B′, which clearly has the area of

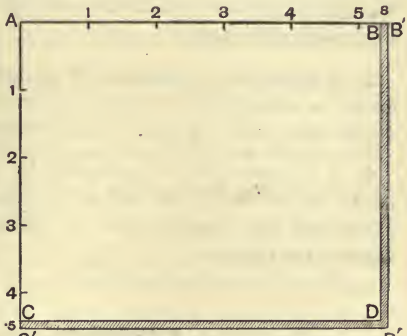


FIG. 36.

$(5{\cdot}35 \times 0{\cdot}1) + (4{\cdot}45 \times 0{\cdot}1) + 0{\cdot}01$, *i.e.* $9{\cdot}8 \times 0{\cdot}1 + 0{\cdot}01$
$= 0{\cdot}98 + 0{\cdot}01 = 0{\cdot}99$, or practically one sq. inch.

The estimated value might be therefore 0·5 sq. in. too large or too small.

EXERCISES LXV. a.

Graphic.

[*Apparatus: Squared Paper. Mathematical Instruments.*]

1. A rectangle is measured by a ruler reading only to 0·2 of an inch; its length is taken to be 3·6″ and its breadth 2·4″. Draw the rectangle according to these measurements; show also the smallest rectangle and the largest rectangle consistent with 3·6″ and 2·4″ as the nearest readings true to 0·2″; find the difference in the areas and the possible error in taking the area as being $3{\cdot}6'' \times 2{\cdot}4''$. Verify your work by calculation.

2. With an ordinary foot rule, divided into inches and eighths of an inch, the side of a square is measured and taken to be $4\frac{1}{4}''$; find graphically the *possible* error in taking $(4\frac{1}{4})^2$ sq. inches as the true value of the area.

3. A rectangle is measured by an ordinary millimetre scale; its length is estimated at 12·6 cm., its breadth 6·4 cm. What error might there be in the area as calculated from these data?

4. A circular disc is measured by a foot rule, graduated to tenths of an inch; its radius is taken to be 3·2 inches. Find graphically the possible error in working out the area from this value of the radius.

EXERCISES LXV. b.

Note.—In the following exercises it is assumed that there is no knowledge of the figures beyond those given, but that the figures cited are the most correct to the given degree of accuracy.

Thus, 3·7″ means something greater than 3·65″, and less than 3·75″, *i.e.* 3·7″ is nearer to the correct result than either 3·6″ or 3·8″.

Hence, 3·7″ would not mean exactly the same thing as 3·70″, 3·70″ meaning a length less than 3·705″, greater than 3·695″, *i.e.* 3·70″ is nearer the true length than either 3·71″ or 3·69″.

1. The sides of a rectangle are taken to be 6·8″ and 5·6″; find the difference between the least and greatest possible value of the area.

2. A rectangular block is measured, and the following values entered:

Length 12·2″,
Breadth 8·7″,
Thickness 6·0″.

What is the greatest possible volume of the block, and what its least volume?

3. The contents of a rectangular cistern are calculated from measurements of its dimensions; these are taken as

100·2 cm.
50·4 cm.
40·6 cm.

What error may exist when its contents are expressed in litres?

4. The dimensions of a rectangular grass plot are measured by a chain, and are found to be

10 ch. 12 links
8 ch. 18 links } (These can only be true to the nearest link.)

Find the difference between the least and greatest possible area as deduced from the measurements.

5. The sides of the following squares are measured with a plain scale graduated to tenths of an inch; find the least and greatest values of the areas:

(*a*) 3 ft. 4·5 inches. (*b*) 11·2 inches. (*c*) 1 ft. 6·8 inches.
(*d*) 2 ft. 0·1 inch. (*e*) 25·3 inches. (*f*) 5 ft. 0 inches.

6. The length of a rectangular block is measured and taken as 7·9 cm., its breadth 4·6 cm., thickness 3·4 cm. and specific gravity is estimated at 8·8. Find the limits to the possible weight of the block in grams.

(By **specific gravity** is meant the number obtained by dividing the weight of any volume of the substance by the weight of an equal volume of water.)

7. Evaluate in the same way the limits to the possible weights in grams of the following blocks, stating the answer to the units place only:

	Length.	Breadth.	Thickness.	Specific Gravity.
(*a*)	30·2... cm.	10·3... cm.	5·0... cm.	7·2... cm.
(*b*)	18·5	12·1	4·2	7·2
(*c*)	20·5	10·5	6·1	11·2
(*d*)	16·1	16·1	16·1	11·2
(*e*)	13·2	13·2	13·2	0·988
(*f*)	60·1	40·4	20·9	0·643

8. A sheet of metal is weighed and found to be 33·24... grams; its length is measured and taken as 20·2... cm. and breadth 10·1.. cm.; its specific gravity is taken as 7·2.... Find the limits to its thickness in millimetres.

9. Find the limits true to 0·01 cm. to the values of the thickness in the following blocks:

	Volume.	Length.	Breadth.
(*a*)	763·5 c.c.	18·1 cm.	7·1 cm.
(*b*)	1000·3 c.c.	20·0 cm.	10·0 cm.
(*c*)	1976·2 c.c.	35·9 cm.	8·3 cm.
(*d*)	968·0 c.c.	29·5 cm.	17·5 cm.
(*e*)	64·5 c.c.	8·1 cm.	3·2 cm.
(*f*)	118·6 c.c.	10·2 cm.	4·6 cm.

10. In order to calculate the length of wire in a coil, the following operations were performed:

(*a*) The coil was weighed.

(*b*) A piece of wire was cut off and its length determined.

(*c*) The piece of wire was weighed.

If the weight of the coil was 976·35... grams, the piece of wire 100·5... cm. long and 1·03 grams in weight, find the limits to the length of the wire in the coil in cm.

11. At a school sports, the time for the winner of the 100 yds. race was given as $11\frac{1}{5}$ secs. If this time was taken by a stop-watch reading to the nearest fifth of a second, find the limits to the speed of the boy in miles per hour. (Neglect errors in the length of the course, the personal errors of the time-keeper, etc.)

EXERCISES LXV. c.

Practical.

[*Apparatus: Blocks. Balance. Plates of Metal. Wire.*]

[When it is not convenient for the following operations to be performed by the class, they may be made by the teacher, and the results given to the class.]

1. Find the specific gravity of the given block from measurements of its weight, length, breadth and thickness. Show clearly the limits to the possible value of the specific gravity.

2. Find the weight of the given piece of metal in and out of water; find the loss of weight when weighed in water, also the ratio of the weight of metal to the loss of weight in water; indicate in your answer those figures the accuracy of which you can rely upon.

3. Find the volume of the given rectangular plate of metal by (*a*) finding the volume of water it displaces; (*b*) measuring its length, breadth and thickness; find the limits to the values according to the two methods, and point out which is the more accurate.

4. Measure the volume of the given wire by (*a*) finding the loss of weight in water; (*b*) finding its length and cross section. Estimate whether the difference in the values found by the two methods is greater or less than it ought to be.

45. APPROXIMATE OR CONTRACTED MULTIPLICATION.

It has been shown that not only is it unnecessary to carry calculations beyond a certain point, but that generally the data supplied are not absolutely correct. Special methods

are therefore usually employed for shortening processes of calculation. One of the most general and useful is that of contracted multiplication.

EXAMPLE.—*Find the volume of a block the length of which is* 18·23″, *breadth* 17·16″, *thickness* 14·12″ (*each true to* 0·01″).

Proceed by multiplying out the product of 18·23, 17·16, 14·12.

```
   18·23
     17·16
  ------
  182·3
  127·61
    1·823
    1·0938
  ------
  312·8268
        14·12
  ------
 3128·268
 1251·3072
   31·28268
    6·256536
 ------
 4417·114416
```

The volume would therefore be 4417·114416 cubic inches.

Now, not only would it probably be unnecessary to know the volume to its millionth part, but a large number of the figures in the multiplication are absolutely wrong.

For the length is not really 18·23 in. exactly; there must be figures after the 3. Call the 1st of such figures *a*. Similarly in the breadth there probably is an unknown digit *b* after the 6; and an unknown digit *c* in the expression 14·12 for the thickness.

Go through the multiplication again, marking those figures which are probably wrong with a light line drawn through them, and putting an asterisk for a figure (due to *a*, *b*, *c*, etc.), about which we know nothing.

```
  18·23a
    17·16b
 ---------
 182·3a....
 127·61*...   The 1 in the second decimal place is
   1·823a..   probably wrong, as when digit a is multiplied
              by 7 a figure would have to be carried over.
   1·0938*.   For a similar reason this 8 is wrong.
     *****    Figures exist here on account of digit b.
 ---------
 312·8268**   Note also that an error might exist in the 8 in
      14·12c  the first place of decimals.
 ---------
 3128·268**
 1251·3072**
   31·28268**
    6·256536**
     *********
 ---------
 4417·114416***
```

Clearly no reliance can be placed on any of the decimal figures, while the 7 in the units place is not to be relied upon as absolutely correct.

The work might therefore have been shortened by omitting figures for which no authority exists.

Thus:

```
                    18·23
                    17·16
                  -------
                   182·3
                   127·61
Here only 18·2 is multiplied.  1·82      No figures are set
    "    18     "              1·09      down beyond the 2nd
                  -------                decimal place; the 2nd
                   312·82                decimal place result, 2,
                     14·12               is not correct; neglect
                  -------                it therefore in evalu-
                   3128·                 ating the next product.
  Multiply only 312·8.        1251·3
     "     "    312.           31·3
     "     "    31.             6·3
                  -------
                   4416·9               This 9 is wrong, but
```

being certainly greater than 5 the result is taken as 4417.

Even if the given figures to be multiplied are absolutely correct, the product may only be required to a certain degree of accuracy. Thus:

To find the product of 313·4567 *and* 518·4374 *true to* 2 *decimal places.*

313·4567
518·4374

156728·35
3134·567
2507·6536
125·3827
9·4037
2·1942
·1254

162507·6766

Figures in the 3rd place of decimals will certainly affect the result in the 2nd place.

Figures in the 4th place may affect those in the 2nd.

Now strike out the 7 in the multiplicand, but since 4×7, *i.e.* 28, is nearer 30 than 20, carry a 3.

Now strike out the 6 in the multiplicand, but carry a 2, since 3×6 is nearer 20 than 10.

Now strike out the 5 in the multiplicand, but carry on a 4, since $7 \times 5 + \ldots$ is nearer 40 than 30.

The result is 162507·67, where 7 is the actual figure; or 162507·68, where 8 is the nearest figure in the 2nd decimal place.

More frequently it is required to evaluate results true to a certain number of *significant* figures, *e.g.* if the result reaches to millions, units may be of small significance, whereas if the result is very small millionths may be important.

EXAMPLE.—*Evaluate* $876208 \times 31{\cdot}6243$ *true to* 4 *significant figures.*

In most of these cases it is convenient to alter the form of the number as follows:

$$876208 = 8{\cdot}76208 \times 10^5.$$
$$31{\cdot}6243 = 3{\cdot}16243 \times 10^1.$$

The product is $8{\cdot}76208 \times 3{\cdot}16243 \times 10^6$, and if we work true to the 4th decimal figure our result will be certainly correct.

$$\begin{array}{r} 8{\cdot}76208 \\ 3{\cdot}16243 \\ \hline 26{\cdot}28624 \\ {\cdot}87621 \\ {\cdot}52572 \\ 1752 \\ 350 \\ 26 \\ \hline 27{\cdot}70945 \end{array}$$

The product is therefore $27{\cdot}70945 \times 10^6$, or, true to the nearest 4th significant figure, 27710000.

In most cases, and especially where the calculations are long, it is desirable to test *roughly* whether the answer found is a *likely one.* Such tests should be mental.

Thus $876208 \times 31{\cdot}6243$
$= 8{\cdot}76208 \times 3{\cdot}16243 \times 10^6.$
$8{\cdot}76208$ is slightly over $8\frac{3}{4}$.
Now, $8 \times 3{\cdot}16$ is slightly over 25.
$\frac{3}{4}$ of $3{\cdot}16$ (nearly $\frac{3}{4}$ of $3{\cdot}2$) is about $2{\cdot}4$.
The answer is near, therefore, to $(25 + 2{\cdot}4)$, *i.e.* $27{\cdot}4$ millions.
Or, it might have been reasoned that $0{\cdot}16245$ is *nearly* $\frac{1}{6}$.
3 times $8{\cdot}762$ is $26{\cdot}3\ldots$.
$\frac{1}{6}$,, $8{\cdot}76$,, $1{\cdot}4\ldots$.
The answer, therefore, is $27{\cdot}7$ millions 'about.'

EXERCISES LXVI. a.

Mental or Oral.

Evaluate the following to the degree of accuracy stated:

1. 981×99 to the nearest 1000.
2. 326×650 to 2 significant figures.
3. 801×420 to 3 „
4. $4{\cdot}26 \times 10^2 \times 2 \times 3{\cdot}141$ to 2 significant figures.
5. 958×167 to 2 significant figures.
6. 1836×2482 „ „
7. $0{\cdot}842 \times 4976$ „ „
8. $693{\cdot}2 \times 3358$ „ „
9. $0{\cdot}0628 \times 0{\cdot}07018$ „ „
10. 1764×996 „ „
11. 4018×7492 „ „
12. $2497 \times 0{\cdot}08653$ „ „
13. $38600 \times 0{\cdot}02496$ „ „
14. $36{\cdot}18 \times 110896$ „ „
15. $248 \times 4{\cdot}1247$ „ „
16. 7496×3987 „ „
17. 934×101 „ „
18. $696 \times 5{\cdot}167$ „ „
19. 333×3997 „ „
20. $876 \times 0{\cdot}0002004$ „ „
21. $119 \times 0{\cdot}374$ „ „
22. $128200 \times 0{\cdot}8748$ „ „
23. $3624 \times 0{\cdot}00498$ „ „
24. $998 \times 6{\cdot}049$ „ „
25. $397 \times 7{\cdot}332$ „ „

EXERCISES LXVI. b.

NOTE.—The figures given are approximate only.

1. Find the weight of a block of aluminium bronze 5643·2... cubic centimetres in volume, the weight of each cubic centimetre being 8·372... grams.

2. The following is an extract from a table of births and deaths for the year 1903; find, as accurately as you think the data allow (*i.e.* to the nearest unit, or ten, etc.), the actual number of births and deaths in each town:

	Cities, Boroughs, and Urban Districts.	Population.	Ratio per 1000.	
			Births.	Deaths.
(*a*)	London (the Metropolis) -	4,613,812	28·4	15·7
(*b*)	Aston Manor - - -	79,417	28·7	13·9
(*c*)	Birkenhead - - -	113,598	30·8	16·8
(*d*)	Birmingham - - -	533,039	31·8	17·8
(*e*)	Blackburn - - -	131,218	25·1	15·7
(*f*)	Bolton - - - -	173,401	27·0	17·5
(*g*)	Bootle - - - -	60,761	33·0	19·0
(*h*)	Bournemouth - - -	63,132	17·8	12·1
(*i*)	Bradford (Yorks) - -	283,412	23·3	16·4
(*j*)	Brighton - - - -	125,405	24·3	14·3
(*k*)	Bristol - - - -	338,895	27·4	14·3
(*l*)	Burnley - - - -	99,469	27·2	19·2
(*m*)	Cardiff - - - -	172,598	30·5	14·0

3. The chemical composition of dry air is as follows:

	By Volume.	By Weight.
Nitrogen - -	78·40 per cent.	75·95 per cent.
Oxygen - - -	20·94 „	23·10 „
Argon - - -	0·63 „	0·90 „
Carbon Dioxide -	0·03 „	0·05 „

Find the volume of nitrogen in a room 6·32... metres long, 5·46... metres broad and 3·27... metres high. State your answer true to the nearest cubic metre.

4. The length of a rail of iron is 24·321... ft. on a cold day; what will be its length on a day when each foot of the iron has expanded to a length of 1·00324... ft.?

5. What will be the yield in ounces of gold from 16,800 tons 4 cwt. (to the nearest cwt.) of ore, if each ton contains 3·426... oz. of gold?

6. The chemical changes which have taken place during the conversion of vegetable matter into the several varieties of coal may be seen by a comparison of their percentage analyses, as given in the following table:

	Carbon. %	Hydrogen. %	Oxygen and Nitrogen. %
Wood (Oak) - - - -	48·94	5·94	45·12
Peat (Irish) - - - -	55·62	6·88	37·50
Lignite (Bovey Tracey) - -	69·94	5·95	24·11
Bituminous Coal (Newcastle), -	88·42	5·61	5·97
Steam Coal (South Wales) -	92·10	5·28	2·62
Anthracite (South Wales) - -	94·05	3·38	2·57

Find the weight of carbon in the Bituminous coal brought up from a seam 5 acres in area and of average thickness 59·1... in., assuming that each cubic yard of coal in the mine yields 1 ton at the pit mouth.

7. The height of the barometer decreases as the elevation above the sea-level increases. Change the following table into one where the height of the barometer is in inches and the elevation in feet (assume that 1 metre = 39·37 in.):

Elevation in metres above sea-level.	0	100	200	300	400	500
Height of barometer in millimetres	760	751	742	733	724	716

8. What is the number of cubic metres of air in a room 6·426... metres long, 5·821... metres broad, 3·872... metres high? (State answer to the nearest trustworthy figure.)

9. The length of the stroke of the piston of a steam engine is 16″·00 and the number of strokes made per minute is 148. Find the actual travel of the piston per hour, expressed to the nearest mile.

10. Find to the nearest 1000, the number of bricks in a rectangular kiln the internal dimensions of which are 23 ft. long, 14 ft. wide and 11 ft. high, assuming each brick to occupy 100 c. in. and deducting a space of 0·6 per cent. of the capacity for setting the brick.

Evaluate the following to the degree of accuracy stated. (In the following examples the figures given are supposed to be exact.)

11.	$(625{\cdot}024)^2$	true to units' place.
12.	$872{\cdot}1256 \times 375{\cdot}2164$	„ 1st decimal place.
13.	$963{\cdot}34 \times 64{\cdot}234$	„ 1st „ „
14.	$(172{\cdot}131)^2$	„ nearest unit.
15.	$\{4{\cdot}18765\}^2$	„ 4th decimal place.
16.	$89{\cdot}006 \times 91{\cdot}083 \times 103{\cdot}025$	„ 1st „ „
17.	$2{\cdot}867 \times 3{\cdot}424 \times 1{\cdot}1$	„ 2nd „ „
18.	$103{\cdot}6 \times 102{\cdot}1 \times 92{\cdot}8$	„ units' place.
19.	$(1{\cdot}02)^3$	„ 4th decimal „
20.	$(2{\cdot}081)^3$	„ 2nd „ „
21.	$892{\cdot}562 \times 2{\cdot}018643$	true to 5 significant figures.
22.	$102{\cdot}345 \times 97{\cdot}655$	„ 5 „ „
23.	$324{\cdot}613 \times 133{\cdot}475$	„ 4 „ „
24.	$791{\cdot}82 \times 36325$	„ 5 „ „
25.	$36{\cdot}2345 \times 998{\cdot}75$	„ 4 „ „
26.	$897699 \times 19{\cdot}9891$	„ 6 „ „
27.	$7789{\cdot}13 \times 234{\cdot}56$	„ 4 „ „
28.	$3456{\cdot}2 \times 1903$	„ 5 „ „
29.	3876×10401	„ 3 „ „
30.	9969×8858	„ 4 „ „

46. CONTRACTED DIVISION.

For reasons similar to those described in the case of multiplication, contracted methods are often employed in division.

EXAMPLE A.—*To find the weight per foot of a cable when it is known that a piece estimated as* 235·24... *ft. long weighs* 928·34... *lbs.*

The fraction $\frac{928\cdot34\ldots}{235\cdot24\ldots}$ must be evaluated, and in all probability figures should follow the 4 in the numerator and also the 4 in the denominator. Suppose the figures following are *ab*... in the numerator, $\alpha\beta$... in the denominator.

$$\frac{928\cdot34ab\ldots}{235\cdot24\alpha\beta\ldots} = \frac{9\cdot2834ab\ldots}{2\cdot3524\alpha\beta\ldots}.$$

3·946 — Here a line is drawn through every figure which is not correct.

$2\cdot3524\alpha\beta \overline{)\ 9\cdot2834abc}$

7·057 2 — The 2 may not be correct, since a figure may be carried over from the $\alpha \times 3$.

2·226 2 *a* — The figure to be brought down is probably not a zero; it is represented as *a*.

2·11716

10906 *b*

94096

14944 *c*

141144

8296 — Without a single correct figure.

So far as the quotient is concerned, notice that the *correct* figures are the same as if the following process was adopted.

```
        3·946
2·3524 | 9·2834
       | 7·0572
2·352  | 2·2262      Instead of bringing down a zero, cut off
       | 2·1172      the last figure in the divisor, but carry on
2·35   |  1090       a 4 in multiplying by 9.
       |   941       So cut off the last figure 2 in the divisor,
2·3    |   149       carry a 1, since the 4 × 2 is nearer 10
       |   141       than 0.
                    Similarly cut off the 5.
```

It is not necessary to set down the divisor each time; the last digits might merely be crossed out lightly, while the procedure of the Italian method might also be adopted. The example appears thus:

3·946 = Quotient.

```
2·35̸2̸4̸ | 9·2834
        2·2262
          1090
            49
             8
```

A similar process is used when the divisor and dividend are supposed exact, but where the result is required true to a certain number of significant figures, or to a certain number of decimal places.

EXAMPLE B.—*Evaluate* 89356432 ÷ 234643 *true to* 4 *significant figures.*

89356432 ÷ 234643 = 893·56432 ÷ 2·34643.

Arrange as in Short Division,

$$2\cdot\ldots \;\big|\; \underline{893\cdot56432}$$
$$\overline{a\,b\,c\cdot d\,e}$$

The digit (a) of highest place value in the answer is a hundreds digit; the fourth (d) is in the first place of decimals, and the fifth (e) in the second place of decimals. This digit (e) is determined by the fifth significant figure (6) in the dividend, and the figures 432... beyond this might have been regarded as unknown, so we follow the method used in Ex. A.

(To know the result true to the 4th significant figure (d), it must be known whether the 5th significant figure (e) is greater or less than 5.)

EXAMPLE C.—*Evaluate* 29356432 ÷ 834643 *true to* 4 *significant figures.*

Arrange as in B, $\quad 8\cdot\ldots \;\big|\; \underline{293\cdot56432}$
$$\overline{a\,b\cdot c\,d\,e}$$

It is clear that *six* significant figures must be written down in the dividend. It will be safer to proceed thus in all cases: Write down the dividend to two significant figures beyond the number of significant figures required in the answer, and proceed as in Ex. A.

The work in B would therefore appear as below.

Thus $\qquad 380\cdot8 \quad = \text{Quotient.}$

$2\cdot3464\,2$	$\big\vert\; \underline{893\cdot564}$
$2\cdot346$	$189\cdot63$
$2\cdot34$	$1\cdot92$
$2\cdot3$	$1\cdot92$
2	4

Here, and always in future, make a *rough* approximation to the answer first. Thus:

$$89356432 \div 234643 = 893\cdot5\ldots \div 2\cdot34\ldots,$$

and the quotient clearly lies between 446 and 297 (taking as divisors 2 and 3), and it is nearly the same as

$$893 \div 2\tfrac{1}{3} = \text{about } \tfrac{1}{7} \text{ of } 2700$$
$$= 380.$$

Checks of this kind are only rough, and should be mental.

Consider another rough mental calculation.

$$345{\cdot}632 \times 24{\cdot}365 \div 143{\cdot}89$$
$$= 3{\cdot}456 \times 10^2 \times 2{\cdot}436 \times 10 \div (1{\cdot}43 \times 10^2)$$
$$= 10 \times 3{\cdot}456 \times 2{\cdot}436 \div 1{\cdot}43.$$

For a rough guess at the result, replace $3{\cdot}456$ by $3{\cdot}6$
$2{\cdot}436$ by $2{\cdot}4$
$1{\cdot}43$ by $1{\cdot}44$

The result will not differ much from $\dfrac{10 \times 3{\cdot}6 \times 2{\cdot}4}{1{\cdot}44}$

i.e. $345{\cdot}632 \times 24{\cdot}365 \div 143{\cdot}89 = 60$ approx.

If the answer be found very much smaller or very much larger than this, an error has probably taken place.

Later on, methods will be given for approximating still more accurately.

EXERCISES LXVII.

1. What is the specific gravity of a substance the weight of which in air is 819·324... grams and weight in water is 692·456... grams?

2. Find the number of gallons of water in a rectangular cistern measuring 12 ft. 5 in. long by 8 ft. 4 in. broad and 6 ft. 0 in. deep, assuming 277·264... c. in., to the imperial gallon. (State your answer to the nearest unit.)

3. What weight of steam will be used per minute in filling a cylinder the capacity of which is 1924 cubic inches, if the piston makes 50·0 strokes per minute, and if 4·33 cub. ft. of the steam weighs 1 lb.?

4. If a metre be 39·370 11... inches, convert
(*a*) 2394·622 inches into metres, etc.
(*b*) 8 metre, 2 dm. 3 cm. 4·2... mm. into ft. and inches.

5. Find the ratio of the weight of sediment carried to the amount of carrying water in the following rivers :

	Discharge in thousands of cubic ft. per sec.	Annual Discharge of Sediment in tons.	Ratio.
Danube	315	108 million	1 :
Nile	113	54 million	1 :
Rhine	65·8	36 million	1 :

State your answer in the form $1 : x$, where x is the weight of water for every unit of sediment carried. (Assume the weight of 1 cub. ft. of water to be 62·5 lb.)

6. Find in how many years the drainage area of the following rivers would be lowered by an average amount of 1 foot, on account of the action of the water :

		(1) Area of Basin in Square Miles.	(2) Annual Discharge of Sediment in cubic feet.
(*a*)	Mississippi - -	1,147,000	7,459,000,000
(*b*)	Danube - -	234,000	1,253,739,000
(*c*)	Upper Ganges -	143,000	6,368,077,000
(*d*)	Po - - -	30,000	1,510,137,000
(*e*)	Rhone - -	25,000	600,382,000

7. Find the circumference of a circle the radius of which is 89·324 ft. ($\pi = 3{\cdot}14159\ldots$)

8. Find the circumferences of the circles the diameters of which are

(*a*) 318·12... (*b*) 9·9521... (*c*) 8202·1...
(*d*) 0·26002... (*e*) 66211 (*f*) 33·914.

9. Find, as correctly as your data will allow, the radius of a wheel the circumference of which is 48·256 inches. ($\pi = 3{\cdot}14159\ldots$)

10. If a Norwegian mile be 11·299 kilometres or 7·021... English miles, express an English mile in kilometres.

47. SQUARE ROOT.

When 12 is multiplied by 12 the product = 144, or $12^2 = 144$.

144 is the **square** of 12, and 12 is called the **square root** of 144; represented symbolically, $12 = \sqrt{144}$.

Similarly, $3 = \sqrt{9}$, $0{\cdot}1 = \sqrt{0{\cdot}01}$, $\frac{1}{2} = \sqrt{\frac{1}{4}}$.

Extracting square roots by factorizing.

EXAMPLE.—*To obtain the square root of* 435600.

Split up 435600 into its prime factors;

$$3^2 \times 2^4 \times 5^2 \times 11^2$$

is obtained.

$$\therefore\ 435600 = (3 \times 2^2 \times 5 \times 11)(3 \times 2^2 \times 5 \times 11)$$

$$\therefore\ \sqrt{435600} = 3 \times 2^2 \times 5 \times 11 = 660.$$

EXERCISES LXVIII. a.

Mental or Oral.

1. State the numbers of which the following are the square roots: 1, 2, ..., 12, 13, 14, ... 20.

2. What must be the last digit of any number having an exact square root?

EXERCISES LXVIII. b.

Obtain, by factorizing, the square roots of

1. 1764.	**2.** 44100.	**3.** 5625.	**4.** 53361.
5. 7056.	**6.** 1296.	**7.** 2304.	**8.** 2371600.
9. 2916.	**10.** 3969.	**11.** 46656.	**12.** 4356.
13. 65536.	**14.** 1334025.	**15.** 36864.	

EXERCISES LXVIII. c.

Mental or Oral.

1. How many digits will there be in the squares of

(*a*) 8, (*b*) 13, (*c*) 386, (*d*) 89765, (*e*) 13692, (*f*) 4789862?

2. How many decimal places in the squares of

(*a*) 0·8, (*b*) 0·08, (*c*) 0.000636,
(*d*) 0·2062, (*e*) 0·5683, (*f*) 0·00073?

3. How many digits will there be in the integral and decimal portion of the squares of

(*a*) 2·6, (*b*) 3·006, (*c*) 563·0082, (*d*) 763·0063, (*e*) 23·86745?

4. How many digits will there be in the square roots of

(*a*) 64, (*b*) 121, (*c*) 169, (*d*) 289,
(*e*) 9, (*f*) 3969, (*g*) 46656, (*h*) 44100?

5. How many decimal places will there be in the square roots of the following (each of which has an exact square root):

(*a*) 0·64, (*b*) 0·0064, (*c*) 0·0324, (*d*) 0·046656?

6. State the number of figures in the integral and in the decimal portions of the square roots of

(*a*) 39·69, (*b*) 4·6656, (*c*) 29·8116, (*d*) 533·61, (*e*) 530·8416.

General Method.

Graphic Illustration. Suppose the square ABCD represents the number, and that it is known that A*bcd*, representing say n^2, is very nearly equal to it, so that n is a sort of first guess to the square root of the number.

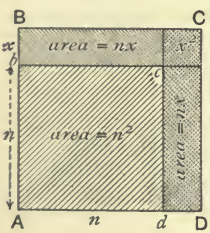


FIG. 37.

The problem is therefore to find the additional amount *b*B, representing say x, which has to be added to n to obtain the required square root.

From the figure it is clear that what is wanted is a number x, such that twice the product of x and n, together with the square of x, is equal to the difference between the square of n and the given number.

Further, that ABCD − A*bcd* is very nearly $2n \times x$, and therefore that x is *suggested by* (ABCD − A*bcd*) ÷ ($2n$).

This is the key of the Arithmetical Method of extracting square roots.

EXAMPLE.—*To evaluate* $\sqrt{4225}$.

Beginning with units and tens digits, divide up the number so far as possible into 'periods' of two digits.

		5 = Units } portion of $\sqrt{4225}$;
60	42|25	60 = Tens } $\because$ 4225 lies between 60^2 and 70^2.
	36 00	Set down 60 on both sides; multiply them as shown.
Twice 60 = 120	6 25	... = $4225 - 60^2$; $4225 - (60)^2 \div 120$ suggests 5 as units digit.
Add the 5	5	
125	6 25	Multiply 125 by 5.

Since there is no remainder $65 = \sqrt{4225}$.

EXAMPLE.—*To evaluate* $\sqrt{104976}$.

Divide into periods as in the preceding example, beginning with units and tens digits.

		4 = Units } portion of
		20 = Tens } $\sqrt{104975}$.
300	10|49|76	300 = Hundreds } $\because$ 104976 lies between 300^2 and 400^2.
	9 00 00	Set down on right and left side as shown; multiply.
Twice 300 = 600	1 49 76	... = $104976 - 300^2$; division by 2×300 suggests 20 as tens portion of $\sqrt{}$.
Add 20	20	
620	1 24 00	Multiply 620 by 20.
Twice 320 = 640	25 76	... = $104976 - 320^2$; division by 640 suggests 4 as units figure.
Add 4	4	
644	25 76	Multiply 644 by 4.

Since there is no remainder $\sqrt{104976} = 324$.

As in division and multiplication, when the processes are clear, the cyphers are omitted, and the work abbreviates into

```
      3  2  4 = Ans.
  3 | 10|49|76
    |  9
 62 |  1 49
    |  1 24
644 |    25 76
    |    25 76
```

Or:

```
      3  2  4
  3 | 10|49|76
 62 |  1 49
644 |    25 76
```

where, on the right, methods similar to the Italian methods in Division have been adopted.

There is no difference of procedure when finding the square root of a number partly integral and partly decimal. The following is an example worked out fully:

Dividing into periods on both sides of decimal pt.

```
                  7000 | 61|32|81|63|·31|29 | 7000 } = 7800 } = 7380 } = 7831  ·2 }
                       | 49 00 00 00        |  800 }       30 }       1 }          7831·2  }
          add { 14000  | 12|32|81|63|·31|29 |                                      ·03 }
              {   800  |                    |                                   7831·23
                 14800 | 11 84 00 00        |
twice 7800 = 15600 } add | 48|81|63|·31|92 |
                30 }     | 46 89 00         |
             15630       |  1|92|63|·31|92  |
twice 7830 = 15660 } add |   | 1 56 61      |
                 1 }     |   |              |
             15661       |   |36|02|·31|29  |
twice 7831 = 15662  } add |   |              |
                ·2 }      |   |31 32·44      |
twice          15662·2    |   |              |
7831·2 = 15662·4  } add   |   | 4|69·87|29  |
              ·03 }       |   |              |
          15662·43        |   | 4 69·87 29   |
```

And $7831 \cdot 23 = \sqrt{61328163 \cdot 3129}$.

Abbreviated, the procedure would be as follows:

```
          7  8  3  1 · 2  3
      7 | 61|32|81|63|·31|29 |
    148   12 32
   1563     48 81
  15661      1 92 63
  15662·2      36 02·31
  15662·43      4 69·87 29
```

EXERCISES LXVIII. d.

1. Find the sides of the squares the areas of which are :

(*a*) 169 sq. ft. (*b*) 289 sq. cm. (*c*) 529 sq. yards.
(*d*) 841 sq. cm. (*e*) 2209 sq. in. (*f*) 2809 sq. ft.

2. What numbers when multiplied by themselves yield

(*a*) 3364. (*b*) 4096. (*c*) 5476. (*d*) 7225.
(*e*) 7921. (*f*) 8649. (*g*) 9409. (*h*) 9801.

3. What length of fencing will be required to go round the following square plots :

(*a*) 15876 sq. yds. (*b*) 64009 sq. yds. (*c*) 94249 sq. metres.
(*d*) 191844 sq. ft. (*e*) 358801 sq. ft. ?

4. Evaluate

(*a*) $\sqrt{451584}$. (*b*) $\sqrt{502681}$. (*c*) $\sqrt{732736}$.
(*d*) $\sqrt{893025}$. (*e*) $\sqrt{4165681}$. (*f*) $\sqrt{14992384}$.
(*g*) $\sqrt{16144324}$. (*h*) $\sqrt{28153636}$. (*i*) $\sqrt{25847056}$.
(*j*) $\sqrt{5035747369}$.

5. Find the square root of

(*a*) 530·8416. (*b*) 4911·486724. (*c*) 0·853776.
(*d*) 241690·2244. (*e*) 43·1649. (*f*) 905·43819025.
(*g*) 0·00279841. (*h*) 0·0006441444 (*i*) 0·000079281216.
(*j*) 4096·896049.

6. A company of soldiers 864 strong is to be formed into the largest possible solid square. What is the number in the side of the square ; how many men if any are left over ?

7. What length of wire netting will be required to enclose a square lawn of 1 acre area ? (Answer to the nearest yd.)

8. From a rectangular piece of glass 19·4 cm. long and 14·8 cm. broad, the largest square piece is cut off. What is the area of the remaining rectangle, and what of the square ?

9. Three square flower beds, of 4 ft. 6 in., 5 ft. 9 in., and 7 ft 9 in. side are to be replaced by a large square one equal in area to the sum of the three. What must be the side of the square (true to the nearest inch) ?

10. A rectangular lawn measuring 196 ft. by 169 ft. is to be replaced by a square one equal in area; what is the length of the side of the latter? Compare the lengths of fencing necessary to enclose each.

11. The expense of a party during an outing was £1. 10*s*. 1*d*., and it was found that the expense in pence per head was equal to the number forming the party. How many formed the party, and what was the expense per head?

Square Roots of Vulgar Fractions.

The following methods may be adopted in different cases:

(1) Extract the square roots of numerator and denominator separately.

(2) Change into an equivalent fraction by multiplying numerator and denominator by the denominator (this may frequently be shortened).

(3) Change into a decimal and extract its square root.

Method (1) is of use when both numerator and denominator have exact roots.

EXERCISES LXVIII. c.

Evaluate the following:

(1) $\sqrt{\frac{576}{625}}$. (2) $\sqrt{\frac{1024}{961}}$. (3) $\sqrt{\frac{392204}{846081}}$.

(4) $\sqrt{\frac{86849}{1980025}}$. (5) $\sqrt{\frac{479961}{1435204}}$. (6) $\sqrt{\frac{5791121}{6785976}}$.

(7) $\sqrt{\frac{316}{125}}$ to two decimal places.

(8) $\sqrt{\frac{75}{128}}$ to two decimal places.

(9) $\sqrt{\frac{47}{63}}$ to two decimal places.

48. INTEREST.

When a sum of money is lent to a Bank (or placed on Deposit) or to a borrower, the money paid for the use of such sum is known as **Interest**; the money lent is frequently termed the **Principal**.

In the Post Office Savings Bank, the regulations as to Interest are governed by the following rule:

"X. Interest at the rate of £2. 10*s.* per cent. per annum is allowed on every *complete pound* deposited, and commences on the first day of the month next following the deposit, and is calculated to the 31st of December, when such interest is added to the principal and begins to carry interest as an ordinary deposit."

EXAMPLE.—*A person puts sums of money into the Post Office Savings Bank, as indicated by the following table:*

Date of Deposit or of Warrant, etc.	*Amount of Deposit in words or Number of Warrant in figures, etc.*	*Amount of Deposit.*
1901		£. *s. d.*
May 30	*Two pounds, ten shillings*	*2 10 0*
August 4	*Three pounds, twelve shillings*	*3 12 0*
October 17	*Seven pounds, fifteen shillings*	*7 15 0*

What does his interest and principal amount to at the end of the year?

Through June, July and August (3 months) the money bearing interest is £2 [the odd shillings do not bear interest, while the £3. 12*s.* 0*d.* will not begin to bear interest until September 1].

On £100 the interest for 12 months is £2. 10*s.* 0*d.*
∴ „ £100 „ „ 3 months is £0. 12*s.* 6*d.*
∴ „ £2 „ „ 3 months is $\frac{1}{50}$ (£0. 12*s.* 6*d.*) *i.e.* 3*d.*

After September 1, the person has £6. 2*s.* 0*d.* in the Bank, £6 of which bears interest, and until November 1 (2 months), he therefore receives interest on the £6, and the amount of this interest is determined as before,

i.e. $\frac{6}{100}$ ($\frac{2}{12}$ of £2. 10*s.* 0*d.*), *i.e.* $\frac{£2.\ 10s.\ 0d.}{100}$, or 6*d.*

From November 1 to the end of the year, the deposit bearing interest is £13. The interest on this for 2 months is

$\frac{13}{100}$ of $\frac{2}{12}$ of £2. 10*s.* 0*d.*, *i.e.* $\frac{13}{600}$ of £2. 10*s.* 0*d.* or 13*d.*

The total interest is therefore [3*d.* + 6*d.* + 13*d.*], *i.e.* 1*s.* 10*d.*

The amount to the person's credit at the end of the year is therefore

£2. 10*s.* 0*d.* + £3. 12*s.* 0*d.* + £7. 15*s.* 0*d.* + £0. 1*s.* 10*d.*,
i.e. £13. 18*s.* 10*d.*

Here, and in many similar cases, the interest can be calculated more quickly as follows:

The interest on £1 for 1 year is $\frac{2\frac{1}{2}}{100}$ of £1 or 6*d.*; the interest for 1 month is therefore $\frac{1}{2}$*d.*

Therefore the interest is (2 × 3 + 6 × 2 + 13 × 2) half-pennies or £0. 1*s.* 10*d.*

Similarly, if the rate of interest had been 5% per annum, each £ would bear 1*d.* interest per month.

EXERCISES LXIX. a.

Find the amounts due to depositors in the Post Office Savings Bank on Dec. 31st; in the following cases:

1.

Date of Deposit or of Warrant, etc.	Amount of Deposit in words or Number of Warrants in figures, etc.	Amount of Deposit.
1903		£ *s.* *d.*
Jan. 10	Eight guineas - - - -	8 8 0
Mar. 17	Eight guineas - - - -	8 8 0
May 24	Eight guineas - - - -	8 8 0
July 31	Ten guineas - - - -	10 10 0
Sept. 7	Ten guineas - - - -	10 10 0

2.

Date of Deposit or of Warrant, etc.	Amount of Deposit.
1902	£ s. d.
Feb. 3	5 12 0
Mar. 10	4 17 0
June 16	7 10 0
Nov. 22	9 9 0

3.

Date of Deposit or of Warrant, etc.	Amount of Deposit.
1903	£ s. d.
Feb. 17	9 11 0
Sept. 26	7 16 0
Nov. 13	8 18 0
Dec. 10	5 14 0

EXAMPLE.—*The books of the depositors below were not forwarded to be made up at the regular time. Find the amounts due to the depositors at the ends of the last years in which entries are quoted.*

Date of Deposit or of Warrant, etc.	*Amount of Deposit in Words or Number of Warrant in figures, etc.*	*Amount of Deposit.*
1902		£ s. d.
February 15	*Ten pounds only*	*10 0 0*
October 10	*Eight pounds, eighteen shillings*	*8 18 0*
1903		
January 7	*Eight pounds only*	*8 0 0*
May 6	*Six pounds, fifteen shillings*	*6 15 0*
July 16	*Nine pounds, twelve shillings*	*9 12 0*

In 1902 the depositor has £10 bearing interest for 10 months and £8 for 2 months.

The interest is, therefore, (100 + 16) halfpennies, or 58*d.*, *i.e.* 4*s.* 10*d.*

The amount to his credit on Jan. 1st, 1903, is £18. 18*s.* + 4*s.* 10*d.*, *i.e.* £19. 2*s.* 10*d.*

Of this sum £19 bears interest.

He therefore has as interest in 1903

$[19 \times 12 + 8 \times 11 + 6 \times 7 + 10 \times 5]$ halfpennies = 12*s.*

At the end of the year the amount which serves as principal for 1904 is

£19. 2*s.* 10*d.* + £8 + £6. 15*s.* + £9. 12*s.* + 12*s.*,

i.e. £44. 1*s.* 10*d.*

EXERCISES LXIX. b.

Find, as in the worked example on p. 281, the amounts due to the depositors at the end of the last years in which deposits are entered.

1.

Date of Deposit or of Warrant, etc.	Amount of Deposit.
	£ s. d.
1902	
Mar. 18	10 10 0
1903	
Aug. 16	8 10 0
Nov. 16	7 0 0

2.

Date of Deposit or of Warrant, etc.	Amount of Deposit.
	£ s. d.
1901	
Sept. 20	12 0 0
1902	
Aug. 10	14 0 0
Nov. 5	12 0 0

3.

Date of Deposit or of Warrant, etc.	Amount of Deposit.
	£ s. d.
1901	
June 3	10 0 0
1903	
June 2	20 0 0
July 3	10 0 0

EXERCISES LXIX. c.

Mental or Oral.

What interest will accrue to

1. £50 in 12 months at 2½ % payable yearly?
2. £1 „ 1 „ „ „
3. £40 „ 1 „ „ „
4. £50 „ 3 „ „ „
5. £80 „ 2 „ „ „
6. £80 „ 1 „ „ „

On what sum will the interest amount to

7. £1. in 1 year at 2½ % payable yearly?
8. ½*d.* „ 1 mo. „ „
9. 7*s.* „ 1 „ „ „
10. 10*s.*6*d.* „ 6 „ „ „
11. 12*s.* „ 8 „ „ „
12. 15*s.* „ 3 „ „ „

In how many months will the interest on

13. £1, at 2½ % payable quarterly, amount to ½*d.*?
14. £10, „ „ „ 3*s.* 9*d.*
15. £40, „ „ „ 10*s.*
16. £7, „ „ „ 3*s.* 6*d.*
17. £18, „ „ „ 3*s.*
18. £72, „ „ „ £1. 7*s.*

Simple Interest.

In most cases in which money is deposited in banks, the interest is added to the principal at fixed intervals, the sum obtained then serving as the new principal. Frequently other arrangements may be made, in which the interest-earning money remains unchanged; the interest charged or earned in such cases is known as **Simple Interest.**

The following worked examples will illustrate the method of procedure:

EXAMPLE.—*A person,* A, *lends another,* B, *the sum of* £350 *for* 4½ *years, at* 2½ *per cent. per annum Simple Interest. What interest does* A *receive during that time?*

This statement means that for every £100 lent to B, A receives £2½ yearly as interest. The interest, therefore, received by A is $\frac{350}{100} \times$ £2½ every year.

The interest received by A in 4½ years is $4\frac{1}{2} \times \frac{7}{2} \times 2\frac{1}{2}$

$$= \frac{9}{2} \times \frac{7}{2} \times \frac{5}{2}$$
$$= \frac{315}{8}$$
$$= £39.\ 7s.\ 6d.$$

EXAMPLE.—*What is the simple interest received on* £512. 17*s.* 6*d. in* 2 *years* 3 *months at* 3 % *per annum. State the answer true to the nearest farthing.*

Interest

= £512. 17*s.* 6*d.* $\times \frac{3}{100}$ in 1 year

= £512·875 $\times \frac{3}{100} \times \frac{9}{4}$ in 2 yrs. 3 mo.

= £34·619

= £34. 12*s.* 4½*d.*

512·875
3
1538·625
9
400 | 13847·625
34·619

EXAMPLE.—*At what rate per cent. per annum will the simple interest on £475 become £66. 10s. in 4 years?*

This is equivalent to asking what the interest would be on £100 in 1 year under the above conditions.

Interest on £475 in 4 years = £66·5.

" " 1 year = $\frac{1}{4}$ of £66·5,

and " £100 in 1 year = $\frac{100}{475}$ of $\frac{1}{4}$ of 66·5

$= \frac{6650}{1900}$

$= \frac{66 \cdot 5}{19} = 3\frac{1}{2}$ or £3. 10s.

Hence the rate per cent. per annum was $3\frac{1}{2}$.

EXAMPLE.—*In what time would a man have received £25. 10s. 0d. as interest, if he lent another £126. 5s. 0d. at 3% per annum Simple Interest?*

The interest on £126. 5s. 0d. in 1 year at 3% is $\frac{3}{100}$ of £126·25 = £3·7875.

Hence, the time required is $\frac{£25 \cdot 5}{3 \cdot 7875}$ years

= 6·733 years

= 6 yrs. 9 mo. nearly

= 6 years 267 days.

```
                6·733
3·7875 | 25·5...
         2·7750
            1238
             102
             365
               0·733
         ------
           255·5
            10·9
             1·1
         ------
             267
```

EXAMPLE.—*One man lent another a certain sum of money, receiving interest for the loan for 4 years at the rate of 3% per annum Simple Interest. In all, he receives £84. 12s. 6d. as interest. What was the sum of money lent?*

The interest on £100 for 4 years at 3% is £12.

If £12 is the interest on £100,

£84·625 is the interest on $£\frac{100 \times 84 \cdot 625}{12}$

= £705. 4s. 2d.

EXAMPLE.—*A person,* A, *borrows from* B *a sum of money, agreeing to pay* B *back at the end of* $2\frac{1}{2}$ *years a sum equivalent to both the principal and the interest accumulated during that time at the rate of* 5 % *per annum Simple Interest. He pays back the sum of* £405. *What money did he borrow?*

In $2\frac{1}{2}$ years, at 5 %, the interest on £100 would be £$12\frac{1}{2}$.
For £112·5 paid back, £100 was borrowed.

For £405 paid back, £$\frac{100 \times 405}{112 \cdot 5}$ was borrowed

$$= £\frac{405}{1 \cdot 125}$$
$$= £360.$$

A therefore borrowed £360 from B.

EXERCISES LXIX. d.

State answers in all cases true to the nearest 1*d.*

N.B.—In the Exs. 1-52, simple interest is charged; they must, however, not be regarded as illustrating practical transactions.

1. A man borrows £875 from a bank, paying back interest every year at the rate of 5 % per annum simple interest. How much will he have paid in interest at the end of three years?

Find the Simple Interest on the following:

2. £288. 10*s.* 0*d.* for 4 yrs. at 3 per cent. per annum.
3. £3569. 7*s.* 6*d.* for 3 yrs. at $3\frac{1}{3}$ per cent. per annum.
4. £192. 15*s.* 0*d.* for 4 yrs. at $2\frac{1}{2}$ per cent. per annum.
5. £693. 12*s.* 6*d.* for 4 yrs. at £3. 2*s.* 6*d.* per cent. per annum.
6. £1756. 8*s.* 9*d.* for $3\frac{1}{2}$ yrs. at $2\frac{1}{2}$ per cent. per annum.
7. £167. 15*s.* 0*d.* for 3 yrs. 4 mo. at 3 per cent. per annum.
8. £5016. 19*s.* 6*d.* for 5 yrs. at £3. 6*s.* 8*d.* per cent. per annum.
9. £159. 16*s.* 8*d.* for 5 mo. at 5 per cent. per annum.
10. £3584. 15*s.* 6*d.* for 7 mo. at $2\frac{3}{8}$ per cent. per annum.
11. £653. 18*s.* 9*d.* for 8 mo. at $4\frac{1}{2}$ per cent. per annum.
12. £1920 for 10 mo. at £2. 15*s.* 0*d.* per cent. per annum.

13. A man borrows £560, agreeing to pay interest at the rate of 5 % per annum, simple interest, the latter being charged on the number of weeks (a portion of a week being counted as a whole week, and reckoning 52 weeks to the year). If he pays back the money with interest at the end of 143 days, what does he pay?

14. Under similar conditions to the preceding, find the interest on

(*a*) £512. 17*s*. 6*d*. for 33 days at 3 % per annum, simple interest.

(*b*) £792 for 45 days at 5 % per annum, simple interest.

N.B.—In the following examples the interest is calculated as being *proportional strictly* to the time, although often the time is over a year.

Find the Simple Interest on

15. £852. 10*s*. for 146 days at 4 per cent. per annum.

16. £1349. 7*s*. 6*d*. for 4 yrs. 73 days at 3 per cent. per annum.

17. £9040 for 2 yrs. 292 days at 4 per cent. per annum.

18. £125 for 1 yr. 219 days at $3\frac{1}{2}$ per cent. per annum.

19. In what time will the simple interest on £732. 5*s*. at $2\frac{1}{2}$ per cent. per annum become £36. 12*s*. 3*d*.?

20. In what time will the simple interest on £193. 12*s*. 6*d*. amount to £19. 7*s*. 3*d*. at £3. 6*s*. 8*d*. per cent. per annum?

21. How long will the simple interest on £83. 15*s*. be in amounting to £12. 11*s*. 3*d*. at 3 per cent. per annum?

22. The simple interest on £491. 16*s*. 8*d*. at $2\frac{1}{2}$ per cent. per annum accumulated to £61. 9*s*. 7*d*.; how many years was the sum bearing interest?

23. Reckoning simple interest at £3. 6*s*. 8*d*. per cent. per annum, in what time will the interest on £2742 become £457?

24. In what time will the simple interest on £93. 8*s*. 9*d*. become £4. 13*s*. $5\frac{1}{4}$*d*., reckoning the rate to be at 2 per cent. per annum?

25. In what time will the simple interest on £452. 12*s*. 6*d*. amount to £56. 11*s*. $6\frac{3}{4}$*d*. at £$3\frac{1}{8}$ per cent. per annum?

26. Find the Simple Interest on :

(*a*) £95. 16*s*. 8*d*. for 300 days at 3 per cent. per annum.

(*b*) £165. 17*s*. 6*d*. for 250 days at 4 per cent. per annum.

(*c*) £759. 10*s*. for 3 yrs. 90 days at £2. 10*s*. per cent. per annum.

(*d*) £653. 7*s*. 6*d*. for 5 yrs. 200 days at £3 per cent. per annum.

27. Simple interest being charged at 3 per cent. per annum, in what time will the interest on £549. 7*s*. 6*d*. amount to £65. 18*s*. 6*d*.?

28. In what time will the simple interest on £5319. 15*s*. amount to £1994. 18*s*. $1\frac{1}{2}d$. at $2\frac{1}{2}$ per cent. per annum?

29. In what time will the simple interest on a sum of money be equal to the sum, reckoning interest at $3\frac{1}{3}$ per cent. per annum?

30. How much longer must £262. 10*s*. be invested, when the rate per cent. is $3\frac{1}{8}$ per annum, than when it is $3\frac{1}{3}$, the interest being £26. 5*s*. in each case?

31. At what rate per cent. per annum will the simple interest on £857. 12*s*. 6*d*. become £107. 4*s*. $0\frac{3}{4}d$. in 5 yrs.?

32. At what rate per cent. per annum will the simple interest on £1927. 14*s*. become £160. 12*s*. 10*d*. in $4\frac{1}{8}$ yrs.?

33. The simple interest on £176. 9*s*. 7*d*. in 3 yrs. amounted to £17. 12*s*. $11\frac{1}{2}d$.; what was the rate per cent.?

34. The simple interest on £200 in 4 yrs. amounted to 20 guineas. What was the rate per cent. per annum?

35. At what rate per cent. will the simple interest on £796. 18*s*. 4*d*. become £99. 12*s*. $3\frac{1}{2}d$. in 4 yrs. 2 months?

36. At what rate per cent. will £1957. 18*s*. 4*d*. bear £19. 11*s*. 7*d*. as interest in 146 days?

37. The sum £851. 15*s*. 0*d*. remained at interest for 15 yrs., when the interest had become £319. 8*s*. $1\frac{1}{2}d$. What rate was allowed?

38. The simple interest received on £9583. 15*s*. in 16 yrs. amounted to £3450. 3*s*. What was the rate per cent. per annum?

39. A banker charged a manufacturer £871. 17*s*. 6*d*. for the loan of £3875 for 5 yrs. What was the rate per cent.?

40. A money-lender charged a borrower 1*s.* for the loan of £1 for 1 month. What was the rate per cent. per annum?

41. The interest allowed on a deposit of £234. 8*s.* 9*d.* for 3 yrs. 9 mo. was £23. 8*s.* 10½*d.* What was the rate per cent.?

42. At what rate per cent. per annum will the interest on £587. 10*s.* in 11 months become £21. 10*s.* 10*d.*?

43. What sum of money will yield £23. 17*s.* 6*d.* as interest in 6 yrs. at 2½ per cent. per annum?

44. From what sum of money will £49. 5*s.* 6*d.* accrue as simple interest in 4 years at £3. 2*s.* 6*d.* per cent. per annum?

45. What sum of money remaining at simple interest for 5 years at 3⅓ per cent. per annum will amount to £512. 13*s.* 3*d.*?

46. A person borrowed a sum of money for 6 years paying 3¾ per cent. per annum as simple interest, and repaid the sum and interest thereon by the payment of £704. 7*s.* 6*d.* Find the sum borrowed.

47. The simple interest on a mortgage for 8 years at £4. 7*s.* 6*d.* per cent. per annum was £945. What was the amount of the mortgage?

48. A sum of money lent for 6 years at £3. 10*s.* 0*d.* per cent. increased to £290. 8*s.* 0*d.* What was the loan?

49. A borrowed from B the sum of £960 for 4 years, simple interest being charged at 3 per cent. per annum. It was to be repaid by one-fourth of the principal being paid at end of each of the first 3 years, and the last fourth with the interest to be paid at the end. What would the last payment be?

50. The simple interest every half year on a sum bearing interest at £3. 15*s.* 0*d.* per cent. per annum was £19. 15*s.* 0*d.* What was the sum invested?

51. A sum of £75 is annually deposited in a bank for 4 years; if it be supposed that simple interest is obtained on the amount, what amount will be standing to the credit of the depositor at the end of the fourth year interest being allowed at 2½ per cent. per annum?

52. From what principal will an interest of £52. 10*s.* 0*d.* accrue in 7 years, simple interest at 5 per cent. per annum being obtained?

53. In the following extracts from various advertisements, find at what rate per cent. per annum, simple interest, the purchaser is charged if he chooses the instalment system in preference to the cash:

(*a*) Sewing machine, price 49*s.* 6*d.*, or 11 monthly payments of 5*s.*

(*b*) 9-ct. gold chain, 30*s.*; 5*s.* monthly, or 27*s.* cash.

(*c*) Cash, £5. 5*s.*; or credit, 10*s.* 6*d.* down, and 10 monthly payments of 10*s.* 6*d.*

(*d*) Cash price, 55*s.*; or deposit 6*s.*, and nine monthly payments of 6*s.*

(*e*) Piano £32. cash, or 36 monthly payments of 21*s.*

(*f*) Billiard table, 5 ft. 4 in. by 2 ft. 10 in. Cash price, £5. 15*s.*, or 13 monthly payments of 10*s.*

54. At what rate per cent. per annum is simple interest being charged for credit in the following cases:

(*a*) "For students commencing their medical studies, entrance fee, 30 guineas. The entrance fee is payable by all students at the commencement of their studies, or may be paid in three instalments of twelve guineas at the beginning of the first, second, and third years."

(*b*) Pawnbroker's advertisement. "The use of £10 for one month costs 2*s.* 6*d.*"

REVISION PAPERS.

[Compiled mainly from Civil Service and other Examination Questions.]

A 1. Each of nine loaded trucks weighs 7 tons 12 cwt. What is their combined weight?

2. A sum of £11. 11*s.* is to be divided equally between 8 persons, how much will each receive?

3. How many centilitres are there in 37 litres 4 centilitres?

4. How many yards are there in 400 inches, and how many feet and inches over?

5. A jug which holds $\frac{3}{4}$ pint is filled from another containing $2\frac{1}{2}$ pints. How many pints are left in the latter?

6. Each of three measures will hold $2\frac{1}{2}$ lb. of butter. How many lb. of butter does it take to fill two of these and half fill the third?

7. I have four weights of 3·5, 2·75, 1·25, 0·05 kilograms. How many kilograms do they weigh together?

8. Express in pounds and decimals of a pound what is left after paying away £4·376 out of £6.

9. Find the area (in square feet) of a plank 6·35 ft. long and 0·66 ft. wide.

10. How many times can a bottle which holds 0·125 gallon be filled from a cask containing 8·5 gallons?

11. What fraction of a ton is 300 lbs. Give your answer in its lowest terms?

12. Express as a decimal the ratio of 21 pence to a £.

13. Express $\frac{2}{7}$ of a gallon in pints to the nearest pint.

14. What percentage of an hour is 3 minutes.

15. Find the value of 3 per cent. of £8. Neglect fractions of a penny.

16. Find in feet and inches the total length of 50 planks, each 11 ft. 9 in. long.

B 1. What is the least fraction which, added to the sum of $\frac{29}{12}$ and $\frac{15}{16}$, will make the result an integer?

2. It requires 1344 tiles, each 9 in. by $4\frac{1}{2}$ in., to cover a court-yard. What is its area?

3. A and B own a field in shares proportioned as 15 to 13. If A's share is $\frac{3}{8}$ of an acre, what is the size of the field in square yards?

4. What decimal of 11 cwt. is 3 qrs. 21 lbs.? *or* what decimal of 10 kilograms is 75 grams?

5. Find the product of 0·0119 and 2·967. Divide this product by 21·93.

6. What amount of interest does £1020 yield per annum, if invested at $2\frac{3}{4}$ per cent.?

7. Find the least common multiple of 78, 84 and 90.

8. Three cyclists who are riding together have machines with wheels, respectively, 78, 84 and 90 inches in circumference; what is the least distance in yards that they must travel in order that their wheels shall be simultaneously in the same position as at starting?

9. Find the decimal equivalent to the fractional expression

$$\frac{(1-\frac{5}{13})\times 6\frac{1}{2}\times(\frac{1}{3}\div 2)}{\frac{4}{9}\times 11\frac{1}{4}}$$

10. A box 4 feet long, 2 feet wide, and $1\frac{1}{2}$ feet deep (internal measurements) weighs 10 lbs. If it be filled with water weighing 1000 oz. for each cubic foot, what is the total weight of box and water?

C 1. A square field is 15 ac. 2 ro. 15 per. in area; find to the nearest foot the length of a path crossing it diagonally.

2. Given that a French metre is 39·37079 inches. Show that the difference between 5 miles and 8 kilometres is very nearly 51 yds.

3. Express in tons 203,549 ounces and 93 m. 660 yds. 2 ft. in inches.

4. A hogshead full of sugar weighs 13 cwt. 1 qr. 5 lb.; the hogshead itself weighs 2 qrs. 19 lb. How many packets of sugar each weighing $2\frac{1}{2}$ lb. can be made from the quantity, and what amount of sugar will be left?

5. I buy 500 oranges at two for three halfpence, and another 500 at two for three pence, after which 60 of the better sort are eaten. I then sell the remainder at five farthings each. How much do I gain or lose?

6. The cost of 5 cwt. 2 qrs. 14 lb. of coffee is £56. 13*s.* 9*d.* If it be sold at 1*s.* $11\frac{1}{2}$*d.* per lb., what is the gain or loss?

7. A merchant imports 2 tons of tea at £4. 10*s.* per cwt. He pays £10. 6*s.* 8*d.* for freight and dock charges and a duty of 4*d.* per lb. He sells the tea at £7. 5*s.* 9*d.* per cwt. Find his gain per cent.

8. A woman buys eggs at 5 for 3*d.* and sells them at 3 for 2*d.* How many eggs must she sell to gain 1*s.*?

9. By selling an article for £9. 12*s.* 6*d.*, a tradesman loses $3\frac{3}{4}$ per cent. At what price should he sell it to gain $2\frac{1}{2}$ per cent.?

D 1. A clock loses 8·5 secs. an hour when the fire is alight, and gains 5·1 secs. when the fire is out, but on the whole it neither loses nor gains. How long in the 24 hours is the fire burning?

2. If a farmer pays £33. 15*s.* as rent for 27 acres, how much should he pay as rent for 70 acres?

3. Find the value in minutes and seconds of 8 per cent. of 4 hrs. 20 min. 25 secs.

4. A farmer has a flock of 820 sheep; of these 123 die and the rest are sold. What percentage is sold?

5. How many sq. ft. of paper will be required for the four walls of a room 16 ft. 8 in. long, 12 ft. 6 in. wide and 12 ft. high?

6. If the wages of 12 men for 210 days be £514. 10*s.*, how much ought they to receive for 180 days' work?

7. Find the value of 15 per cent. of £96. 17*s.* 6*d.*

8. How much per cent. of 2 cwt. is 7 lb.?

9. What fraction added to the continued product of $\frac{31}{60}$, $2\frac{1}{7}$, $\frac{4}{93}$, $2\frac{6}{11}$ will give the continued product of $\frac{7}{8}$, 9, $\frac{2}{3}$, $1\frac{11}{21}$ and $\frac{3}{4}$?

10. A dealer purchases goods and sells them at a profit of $12\frac{1}{2}$ per cent. on the cost price; how much does he receive for each £ of his outlay?

11. Income tax being 8*d.* in the £, how much per cent. does this represent of a man's income?

E 1. Simplify $\dfrac{5\frac{1}{2} \text{ of } 3\frac{1}{3} \text{ of } 2\frac{1}{7} - 6\frac{1}{4} \text{ of } 3\frac{1}{5} \text{ of } 1\frac{1}{6}}{7\frac{4}{9} \text{ of } 3\frac{4}{7}}$.

2. Divide 3·425 by 0·002192.

3. What principal, if invested for 3 years at $2\frac{3}{4}$ per cent. per annum simple interest, will amount to £575. 10*s.* 7*d.*?

4. Find the value of $2\frac{1}{7}$ of $3\frac{1}{9}$ of 4 lbs. 8 oz. 10 dwt. 12 grs. Troy.

5. Reduce £4. 18*s.* $10\frac{1}{2}$*d.* to the decimal of 5 guineas.

6. Find the cost of 276 tons 16 cwt. at £3. 18*s.* $11\frac{1}{2}$*d.* per ton.

7. Find the least common multiple of 385, 231, 165, 105.

8. A room 21 ft. 4 in. long, 18 ft. 8 in. wide, and 15 ft. 6 in. high, is papered with paper 32 inches wide at one shilling a yard. What is the total cost?

9. If by selling a certain horse for £66 I should lose 28 per cent. of the cost at which I bought the animal, what is my loss?

10. Prove that the product of any two numbers which consist of three figures and four figures respectively must be a number consisting of not less than six or more than seven figures.

F 1. Find the cost of 49 cwt. 3 qrs. 14 lbs. at £13. 8*s.* 10*d.* per cwt.

2. Divide 5·4 by 0·00072 and the quotient by 1,470,568 to four places of decimals.

3. Find the greatest common measure of 27531 and 8740.

4. Add $\frac{1}{51}+\frac{2}{45}+\frac{3}{38}+\frac{4}{85}+\frac{5}{57}$.

5. Find (in its lowest terms) the difference between $\frac{1638}{2205}$ and $\frac{910}{1911}$.

6. The French metre is 3·281 English feet, the Rhenish foot is 12·356 English inches. Express 4·567 metres in terms of Rhenish feet to 3 places of decimals.

7. A sum of money put out at simple interest amounts to £688, when the rate is $2\frac{1}{2}$ per cent. and the time 3 years. What would the amount be if the rate were $3\frac{1}{4}$ per cent. and the time $2\frac{1}{5}$ years?

8. Show that if a square number ends in 6 the figure in the tens place is odd; if it ends in any other number, the tens figure is even.

9. If B's wages are $\frac{7}{10}$ of A's, for how many days will a sum which pays A's wages for 119 days pay the wages of both together?

10. If the manufacturer makes a profit of 25 per cent., the agent one of 8 per cent., and the shopkeeper one of 20 per cent., what is the cost to the manufacturer of an article which is sold in the shop for £32. 8*s*.?

11. A man buys an article and sells it at a profit of 10 per cent. If he had bought it at 10 per cent. less, and sold it for 6*d*. less, he would have made a profit of 20 per cent. Find the cost price.

12. To do a piece of work, a contractor can employ two classes of workmen, whose wages are in the ratio of 17 to 13. If he employs the higher paid men (who work the faster) he pays £148. 15*s*. in wages, being £10. 10*s*. less than the lower paid men would cost him. Compare their rates of work.

13. A milkman adulterates his milk as follows:—He takes a pint out of each gallon and replaces it with water. He then takes another pint out of the mixture and replaces that with water; and he repeats the operation a third time. If he then sells his adulterated milk at the price per gallon that the pure milk originally cost him, what is his gain per cent. on the selling price?

14. A tradesman makes up his receipts monthly. His receipts for the first two months average £3 less than those of the first month; those of the first three months average £4 more than the average of the first two; of the first four £5 more than that of the first three; of the first five £2 less than that of the first four; and of the first six £7 more than that of the first five. If the

average monthly receipts for the first six months are £611, what were the receipts for each month?

G 1. Find a decimal of 3 figures as nearly equal to $\frac{603}{1697}+\frac{601}{1760}$ as a decimal of 3 figures can be.

2. Find, to the nearest litre, the content of a tank 2·34 metres long, 1·23 metres broad, and 45 centimetres deep. A litre is a cubic decimetre.

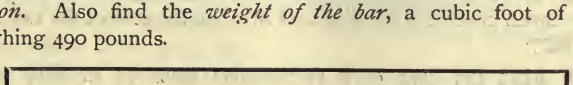
If these dimensions are only true to the nearest centimetre, find the greatest possible error in the result.

3. The population of the central area of London was 821,257 in 1891 and 753,260 in 1901. What is the percentage decrease during this period of 10 years, and, if the rate of decrease during the next 10 year period is the same, what will be the population in 1911?

4. In 1901 Ontario had 1,278,635 acres under wheat, which produced 21,515,780 bushels; Manitoba had 2,011,835 acres under wheat, which produced 50,502,085 bushels. Find out which province produced the heavier crop, and give for each (to the nearest whole number) the number of bushels to the acre.

5. A man borrows £100, to be repaid by instalments, and agrees to pay 4 % interest on what he owes at any time. He pays off £30 (including interest) at the end of each year, until a year comes when a less sum than £30 pays off the debt. What is this sum, and what is the total of the repayments?

6. A bar of iron 10 feet long has a rectangular section shown below. Measure the length and breadth, and find the *area of the section*. Also find the *weight of the bar*, a cubic foot of iron weighing 490 pounds.



H 1. A, B and C go into partnership, A investing £4000, B £3000, and C £2000. If the profits for the first year are £1530, how must they be divided?

2. Write down all the prime numbers between 50 and 100. Can you give any reasons why not more than two of these prime numbers are consecutive odd numbers?

3. A parliamentary grant is made at the rate of 5*s.* per head for all the children at elementary schools. If this grant is distributed at the rate of 5*s.* 9*d.* per child in town and 3*s.* 3*d.* per child in country schools, what percentage of the total number of children are in each class of school?

4. A batsman has a certain average of runs for 16 innings. In the 17th innings he makes a score of 85 runs, thereby increasing his average by 3. What is his average after the 17th innings? (There are no "not out" innings.)

5. A bicyclist rides up a hill 2 miles long in 9 min. 20 sec. and then down a hill of the same slope 3 miles long in 10 min. 30 sec. How long will it take him to ride back, assuming the speeds to be the same in each case, whether the hill be long or short?

6. How many postage stamps, each $\frac{15}{16}$ inch long and $\frac{3}{4}$ inch wide, would make a frieze a foot wide all round a room 16 ft. 6 in. long and 14 ft. 9 in. wide?

7. A cistern is full of water. A cylindrical brass piston 16″ long and 10″ in diameter is immersed in the cistern. How many cubic inches of water will run over, and how many gallons would the overflowed water fill?

8. There is a square lawn ABCD containing 1 ac. 785 sq. yds. How much shorter is the diagonal AC than the sum of the two sides AB, BC?

9. Find the area of a rectangular lawn 55 ft. long which measures 170 ft. all round.

10. A shallow rectangular dish is 0·36 metres long, 0·25 metres broad and 0·4 cms. deep. Express in grammes the weight of water it will hold.

PART II

49. MEASUREMENT OF LENGTH (*continued*).

EXERCISES on the use of the ordinary foot-rule and metre scale have been given already, and also on the use of calipers and other simple means of measurement; in the following exercises additional apparatus is necessary.

EXERCISES LXX.

Practical.

Use of Screw-gauge and Spherometer.

[*Apparatus: Micrometer Screw-gauge. Spherometer. Scale. Glass slips. Wires of different diameters. Glass rods. Balance.*]

1. Examine a Micrometer Screw-gauge (Fig. 38), note the distance the screw moves forward or backward for 1, 2, 3, etc., complete revolutions of the screw head.

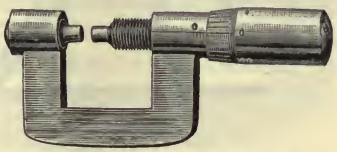


FIG. 38.—Micrometer Screw-gauge.

2. Find the reading when the screw is in contact with the lower jaw. Open the jaws, and introduce a thin glass slip between them; screw up the jaws and take a new reading.

3. Introduce a second slip between the jaws. Find the reading when the jaws are both in contact with the slip: open the instrument and re-introduce the slip used in Ex. **2** on the top of the second slip: find the new reading.

4. By experiments similar to Exx. **2** and **3** note that, when the screw head is turned through equal angles, the screw is moved forward or backward through equal distances, and that the methods of proportion may therefore be adopted in calculating the distance moved forward by the screw.

5. Measure the diameter of the given wires with a screw-gauge.

6. Check the measurements as follows: Wind the wire carefully round a lead pencil or glass tube, taking care that each turn of the wire touches the previous one at all points. Make 20 turns, and measure the distance from the outside of the first turn to the outside of the twentieth turn by means of a millimetre scale. Divide this measurement by 20, and so obtain the diameter of the wire.

Repeat the experiment with varying numbers of turns of wire.

7. Measure the thickness of the given rectangular glass slips.

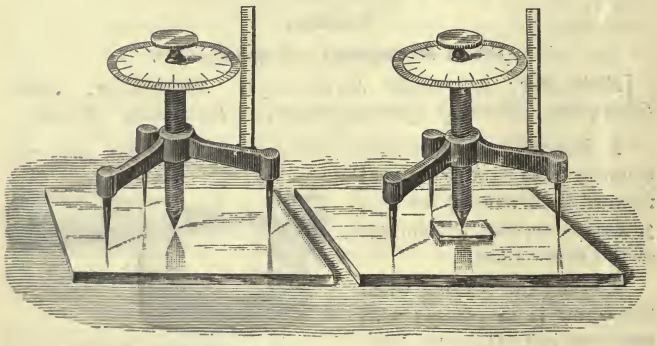


FIG. 39.—Spherometer.

8. Measure the length and the breadth of any one of the slips used in Ex. **7**; calculate its area. Next, find its volume by the balance or by displacement: divide the volume by the area, and so calculate the thickness. Compare your result with the result in Ex. **7**.

The principle of the spherometer (Fig. 39) is precisely the same as that of the micrometer screw-gauge.

9. Measure the thickness of the given plates by means of the spherometer (Fig. 39). Check your results by using the screw-gauge.

EXERCISES LXXI.

Graphic.

Use and Principle of Vernier.

1. On squared paper construct a scale of inches, mark it so as to show 10 inches, cut out another length similarly divided, and from the two make a single scale 20 inches long.

Mark off again a length of 9 inches (10 − 1) on another piece of squared paper, divide this up into 10 equal parts, number these 0, 1, 2, ... up to 10. This will serve as a **vernier.**

(*a*) Note the difference between 1 scale division and 1 vernier division, between 2 scale divisions and 2 vernier divisions, and so on.

(*b*) Slide the vernier until the division marked 1 on the vernier coincides with the division marked 1 on the scale, note the distance between the zero marks, slide the vernier until the divisions marked 2 coincide, again note the distances between the zero marks, and so on.

2. Set the vernier so that the distance in inches between the zero marks is

(*a*) 0·3, (*b*) 0·6, (*c*) 1·2, (*d*) 3·5, (*e*) 5·6, (*f*) 4·2.

3. Draw a scale with its main divisions inches, and the sub-divisions tenths of an inch ; construct a vernier attachment to read to 0·01 of an inch.

4. Construct a scale and vernier to read to 0·02 of a cm.

5. Draw a line 7·2 inches long ; divide it up into 8 equal parts ; alongside attach a vernier, in which a distance equal to the

scale has been divided up into 9 equal parts; what fractions of the whole line are there between corresponding divisions of the main scale and the vernier attachment, when the commencements of both scales are exactly opposite?

6. Attach to an ordinary scale of tenths of an inch, a vernier suitable for reading to the nearest 200th.

EXERCISES LXXII.

1. A screw has its pitch such that, on turning the screw 8 times round, its end moves through a distance of 1·84 in. How many times must it be turned round in order that its end may move through a distance of 0·8602 of an inch?

N.B.—The pitch of a screw is the distance between successive threads, or the distance the screw moves forwards or backwards during one complete revolution.

2. A spherometer (Fig. 39) consists of a screw, the end of which moves forward through a distance of 0·5 mm. for each complete revolution of the screw. What distance corresponds to a turning of the screw equal to 0·265 of a revolution?

3. In lifting weights by means of a windlass, the men have to move round through a distance of 25·12 ft. in order that the weight may be raised through a distance of 5·76 ft. What distance must the men move through, in order that the weight may be raised through a distance of 370·8 yd.?

4. A block of pulleys is used to raise a weight, and it is found that when the weight is lifted through 3·52 ft. an amount of rope 24·64 ft. in length has to be pulled in. What distance will the weight be lifted through when 61·25 ft. of rope are pulled in?

5. The graduated rim of the moving screw of a gauge (Fig. 38) is divided into 50 equal parts, and it is found that when the head is moved forward through a known distance of 0·542 cm. the screw has been turned through 10 complete revolutions, together with an additional 42 divisions. What is the pitch of the screw?

6. A boy made a rough spherometer as follows: He first cut a circle in cardboard, and divided the circumference of the circle

into 32 equal parts. He then attached it to the head of a screw the pitch of which was 0·05 in., and afterwards suitably mounted it. The thickness of a certain slab corresponded to 2 complete turns, and a further motion through 16 divisions. What was the thickness of the slab?

7. If the head of a screw moves forward through 0·24 in. when turned through 4·25 revolutions, what should it be turned through in order to move forward through a distance of 0·33 of an inch?

8. The pitch of a screw is known to be 0·08 in. What is the pitch of another screw which requires 6·24 revolutions to move forward through the same distance that the former screw moves through in 10·92 revolutions, and what is the distance?

9. The pitch of the screw of a spherometer is 0·5 mm., the diameter of the rim of the graduated circle is 4 cm. What distance will a point on the graduated rim be moved through in measuring a thickness of 2·36 cm.?

10. The diameter of the graduated rim of a spherometer is 2 in., and it is known that in measuring a distance of 0·18 in. any point on the rim has been turned through a distance of 22·608 in. What is the pitch of the screw of the spherometer?

11. A hockey stick has a cylindrical handle, which is wound as thickly as possible with thread. What is the diameter of the handle, and what is the diameter of the thread, if 450 turns of thread are required to cover the handle to a distance of 18 in., and if the length of thread used is 177 ft.?

12. When used in estimating a thickness of 1·6 mm., the reading of a spherometer is 8 on the scale, and 20 divisions on the graduated circular rim: when used in estimating a thickness of 2·46 mm., the reading is 12 on the scale and 45 on the rim. What is the thickness, if the reading is 13 on the scale and 30 on the rim? (The total number of divisions on the rim is 100 and the distance between the marks on the scale is equal to the pitch of the screw.)

13. Two similar millimetre scales are engraved on two flat rods A and B of different metal, both being at the temperature at which

ice melts; the rods are heated to the temperature at which water boils, and it is now seen that the scale divisions of one rod A are bigger than the scale divisions of B, and that 132 divisions of A seem to coincide with 133 of B. The divisions of B are 1·0018 times as long at the temperature of boiling water as they are at the freezing point. What is the new length of a portion of A which was 1 metre in length at the freezing point?

14. A line AB is divided up into 36 equal parts by 35 marks, numbered in succession 1, 2, 3, ... 35, while A is marked 0 and B marked 36. Another line *ab* has a length of 35 divisions, it also is divided into 36 equal parts and numbered similarly to AB. If the two 'scales' are placed alongside one another so that A and *a* coincide, what is the distance between the 29th marks on the two scales, when expressed as a fraction of the length of AB? If *ab* be moved along AB so that the 24th division of *ab* coincides with the 24th division on AB, what is the distance between A and *a*?

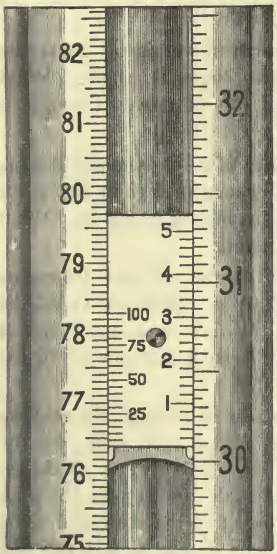


FIG. 40.

15. The right-hand side of Fig. 40 shows a portion of a barometer scale with vernier attachment, in which it will be noticed that each inch on the scale is divided up into 20 equal parts, and that 25 divisions of the vernier are equal to 24 of the scale. To what degree of accuracy can readings be taken?

Explain the vernier attachment shown on the left-hand side of Fig. 40.

16. Assuming that in dividing up a scale, it is not expedient to have divisions closer together than the twentieth part of an

inch, show how to divide up a scale of inches and a suitable vernier in order to read true to

(*a*) $\frac{1}{200}$ in. (*b*) $\frac{1}{250}$ in. (*c*) $\frac{1}{144}$ in. (*d*) $\frac{1}{256}$ in.
(*e*) $\frac{1}{240}$ in. (*f*) $\frac{1}{440}$ in.

17. A piece of metal (*a*) 0·9 gram, (*b*) 1·1 gram, is broken up into 10 equal pieces; explain how you would use them in conjunction with decigrams, to weigh to the nearest centigram.

18. The swinging pendulums of two clocks are viewed through a telescope, so that one appears to pass directly behind the other. The pendulums both appear vertical at 12 o'clock; they next appear vertical together at 8 minutes past 12 o'clock. If one clock beats seconds, find the time which elapses between the 250th passages since 12 o'clock; what also is the time of swing of the pendulum of the second clock?

19. A person observes that the pendulums of two clocks cross the vertical together every 5 minutes. If one clock be correct, what does the other gain or lose in one day?

50. THE USE OF THE CHAIN.

The ordinary chain used in surveying, and known as **Gunter's chain**, is 66 feet long. It is divided into 100 links, joined together by small rings. To every tenth link is attached (Fig. 41) a piece of brass (sometimes called a *finger*) in order to facilitate the counting of the links; and brass handles are attached to the ends of the chain for pulling it tight.

FIG. 41.—Gunter's chain.

Many small surveys can be made with a chain only; larger surveys require the assistance of the theodolite and other instruments.

EXAMPLE. *To survey the small field shown in Fig. 42.*

(*a*) Four main survey lines are obtained as near to the boundary of the field as possible; suppose these are AB, BC, CD, DA; the lengths of these lines are found by the chain.

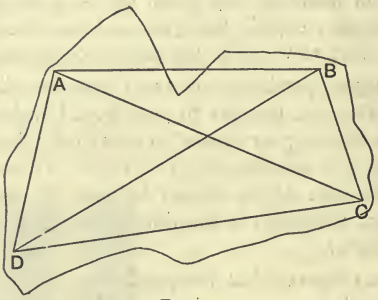


FIG. 42.

The lengths of the two diagonals AC, BD are also found (one diagonal is really sufficient, but it is generally a safer plan to measure both, one serving as a check for the whole survey). **Offsets** are then taken along these and probably other lines, thus:

Suppose P, Q, R, S, T (Fig. 43) are more or less important or salient points on the boundary in the neighbourhood of AB (line 1). Measurements are taken along AB from some convenient starting point O until a point p is reached, such that pP is at right angles to OAB, the distances Op and pP are noted, say 500 links and 250 links; similarly measurements of Oq, qQ; Or, rR; etc., are taken, and the readings set down in the 'field' book. The entry might be somewhat as follows:

Line 1

80	1500	
100	1200	
	1000	120
350	780	
250	500	
	O	↑

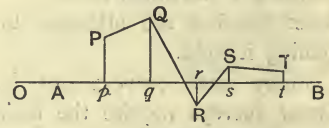


FIG. 43.

The readings mean that

$Op = 500$, $pP = 250$, and P is to the left of OAB.
$Oq = 780$, $qQ = 350$, and Q „ „
$Or = 1000$, $rR = 120$, and R is to the right of OAB.
$Os = 1200$, $sS = 100$, and S „ left „
$Ot = 1500$, $tT = 80$, and T „ „ „

The figures express the distances in links.

Similarly the form of the boundary in the neighbourhood of BC, CD or DA is found.

EXERCISES LXXIII.

Graphic.

1. Fig. 44 represents a railway, fence and stream, scale 1 chain to the inch. Make a field-book entry with reference to the line AB.

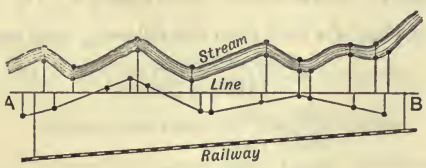


FIG. 44.

2. Trace the form of a portion of a fence from the following field-book entry :

	1000	o fence
	800	25
	700	20
fence o	500	
10	250	
15	180	
fence o	120	
	O	

3. Draw to scale the perimeter of a pond from the following readings, taken with reference to three lines forming a triangle within which the pond is situated.

End of Line 1.

2000	↖ Line 3
1500	150
1200	100
1000	30
700	30
300	250
O	↗ Line 2

End of Line 2.

Line 3 ↙	**2000**
40	1900
160	1500
170	1000
180	400
200	200
Line 1 ↖	O

End of Line 3.

Line 1 ↗	**2000**
30	1700
50	1500
60	1000
50	500
20	300
Line 2 ↘	O

4. Draw a plan of a field from the following field readings:

End of Line 1.

	1810 →Line 2
160	1800
160	1000
140	350
130	300
120	O ↙ Line 3

End of Line 2.

hedge—	**1800** ↖ Line 3
230	1750
220	1200
hedge〉	800
240	590
160	380
300	200
	O ↙ Line 1

End of Line 3.

200	**2000**	↖ Line 1
280	1500	
30	1300	
300	1000	
150	600	
50	500	
30	200	
	O	↙ Line 2

5. The following plans are drawn to scale; make field-book entries along the lines 1, 2, 3, 4, 5 for the points indicated.

(*a*) Fig. 45. (*b*) Fig. 46.

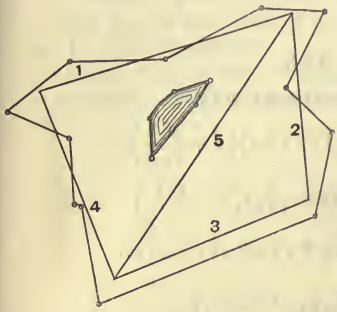


FIG. 45.

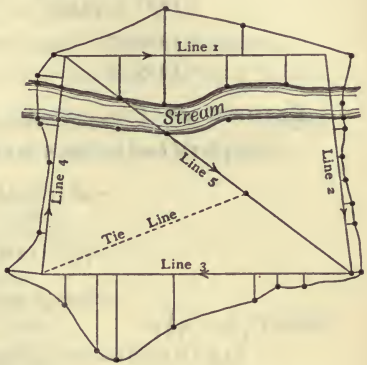


FIG. 46.

(Scale 1 chain to the centimetre.)

51. APPROXIMATIONS (*continued*).

Approximations in Multiplication.

In many cases calculations have to be performed in which the terms involved are *very nearly* convenient round numbers, but not quite.

Consider the following examples:

EXAMPLE 1.—*Evaluate* 127×96.

$$127 \times 96 = 125 \times 96 \text{ (approx.)} = 12000.$$

Accurately $127 \times 96 = 96 \times (125 + 2)$

$= (125 \times 96) + \frac{2}{125}(125 \times 96) = 125 \times 96 + \frac{16}{1000} \times 96 \times 125$

$= 125 \times 96$ increased by 16 in every 1000 or by 1·6 %

$\simeq 12000$ „ „ „ „

$= 12192.$

EXAMPLE 2.—*Evaluate* 253×484.

$253 \times 484 = 484 \times (250 + 3) = 250 \times 484$ increased by 3 in every 250, *i.e.* 1·2 % or 12 in each 1000

$= \frac{1000}{4} \times 484$ „ „ „

$= 121000$ „ „ „

$= 122452$.

EXAMPLE 3.—*Evaluate* $48{\cdot}1 \times 12{\cdot}6$.

$$(48{\cdot}1)(12{\cdot}6) = (48 + 0{\cdot}1) \times (12{\cdot}5 + 0{\cdot}1)$$

$$= 48\left(1 + \tfrac{1}{480}\right) \times (12{\cdot}5)\left(1 + \frac{0{\cdot}1}{12{\cdot}5}\right)$$

$$= (48)(12{\cdot}5)\left(1 + \tfrac{1}{480}\right)\left(1 + \frac{0{\cdot}1}{12{\cdot}5}\right)$$

$$= 600\left(1 + \tfrac{1}{480} + \tfrac{1}{125} + \tfrac{1}{125} \times \tfrac{1}{480}\right),$$

neglect $\frac{1}{480} \times \frac{1}{125}$.

$$\therefore\ (48{\cdot}1)(12{\cdot}6) = 600 + 600\left(\tfrac{1}{480}\right) + 600\left(\tfrac{1}{125}\right),$$

that is to say, since $\frac{1}{480}$ is roughly 0·2 %,

and $\frac{1}{125}$ is 0·8 %,

we increase 600 by (0·2 + 0·8) %, *i.e.* by 1 %, and an approximate value is 606.

Actually multiplying 48·1 by 12·6, 606·06 is obtained.

$$\begin{array}{r} 48{\cdot}1 \\ 12{\cdot}6 \\ \hline 481 \\ 96{\cdot}2 \\ 28{\cdot}86 \\ \hline 606{\cdot}06 \end{array}$$

A rough approximation should be *mental* or nearly so, and is chiefly of use when the numbers to be multiplied are large.

EXAMPLE 4.—*Evaluate* $497{\cdot}5 \times 600{\cdot}6$.

The terms are 2·5 in 500 less than 500, and 0·6 greater than 600, that is 0·5 % less than 500 and 0·1 % greater than 600.

A first approximation = $500 \times 600 = 300000$.

A second approximation increases this by $(0{\cdot}1 - 0{\cdot}5)\,\%$, *i.e.* decreases by $0{\cdot}4\,\%$, *i.e.* 4 in every 1000.

$\therefore$ second approximation = 298800 (accurately 298798·50).

EXAMPLE 5.—*Evaluate* $7493 \times 798{\cdot}2$.

$(7493)(798{\cdot}2) = 7500 \times 800$ nearly $= 6000000$ approx.

This has to be reduced by 7 in 7500 and also by 2 in 800, *i.e.* by $0{\cdot}26\,\%$ approx.

Hence $7493 \times 798{\cdot}2 = 6000000$ reduced by $0{\cdot}26\,\%$ approx. $= 5984400$. Multiplying accurately 5980912·6 is obtained.

$$\begin{array}{r} 7493 \\ 798{\cdot}2 \\ \hline 52451 \\ 67437 \\ 59944 \\ 1498{\cdot}6 \\ \hline 5980912{\cdot}6 \end{array}$$

The method is more particularly useful when several terms have to be multiplied together.

EXAMPLE 6.—*Find the weight of a block* 127·6 *cm. long*, 79·3 *cm. broad*, 60·9 *cm. thick and having a density of* 11·22 *gr. per c.c.*

The weight $= (127{\cdot}6) \times (79{\cdot}3) \times (60{\cdot}9) \times (11{\cdot}22)$ grams.

A first approximation is $125 \times 80 \times 60 \times 11 = 6600000$ grams or 6600 kilo.

A nearer approximation increases this by $(2 - 0{\cdot}9 + 1{\cdot}5 + 2)\,\%$, *i.e.* $4{\cdot}6\,\%$, or about 3 in 66, and a second approximation is 6900 kilo.

Approximations in Division.

It has been shown already (p. 45) that the area of a rectangle with sides $1 + x$ and $1 - x$ is equal to $1 - x^2$, that is,

$$(1 + x)(1 - x) = (1 - x^2);$$

$$\therefore\ 1-x=\frac{1-x^2}{1+x}\ \text{ and }\ 1+x=\frac{1-x^2}{1-x}.$$

Accordingly, if x be small, so that x^2 can be neglected,

$$\frac{1}{1+x}=1-x\ \text{ and }\ \frac{1}{1-x}=1+x.$$

Or, dividing by $(1+x)$ is practically equivalent to multiplying by $1-x$,

and dividing by $(1-x)$ is practically equivalent to multiplying by $(1+x)$.

EXAMPLE 1.—*Evaluate approximately* $\frac{789\cdot32}{99}$.

$$\frac{789\cdot32}{99}=\frac{789\cdot32}{100(1-\frac{1}{100})}$$

$$=\frac{789\cdot32}{100}(1+\tfrac{1}{100})\text{ approx.}$$

In other words, divide by 100 instead of 99 and increase by 1 %.

EXAMPLE 2.—*Evaluate* $\frac{9835\times792}{102\times297}$.

$$\frac{9835\times792}{102\times297}=\frac{10^4\times800}{100\times300}\text{ as a first approximation.}$$

But since 9835 is approx. 2 % less than 10000,

792 ,, 1 % ,, ,, 800,

102 ,, 2 % greater ,, 100,

297 ,, 1 % less ,, 300.

$$\frac{9835\times792}{102\times297}=\frac{10^4\times800}{100\times300}\text{ increased by }(-2\,\%-1\,\%-2\,\%+1\,\%)\text{ approx.}$$

$$=266\text{ increased by }(-4\,\%)$$

$$=256\text{ as a second approximation (actually}=257).$$

When generalized, the method used in the above example may be written as follows:

$$\frac{\mathrm{N}(1+x)(1+y)}{(1+z)(1+w)} = \mathrm{N}(1+x+y-z-w),$$

if x, y, z, w are *small* fractions, either positive or negative.

So also $$\mathrm{N}(1+x)^n = \mathrm{N}(1+nx),$$

$$\mathrm{N} \div (1+x)^n = \mathrm{N}(1-nx).$$

EXAMPLE 3.—*Evaluate* $\dfrac{(832)^3}{(612)^2}$.

$\dfrac{(832)^3}{(612)^2} = \dfrac{\{8\times(104)\}^3}{\{6\times(102)\}^2} = \dfrac{8^3\times 100^3}{6^2\times 100^2}$ increased by $(3\times 4\% - 2\times 2\%)$

$$= 10^2 \times \frac{128}{9} \quad \text{,,} \quad (8\%)$$

$$= 10^2 \times \frac{138}{9} \text{ approx.}$$

$$= 15{\cdot}3 \times 10^2.$$

By straightforward evaluation, $\dfrac{832^3}{612^2} = 15{\cdot}4 \times 10^2$.

EXERCISES LXXIV. a.

Mental or Oral.

Decimalize approximately:

1. $\frac{873}{101}$.
2. $\frac{962}{102}$.
3. $\frac{5636}{202}$.
4. $\frac{5978}{206}$.
5. $\frac{736}{99}$.
6. $\frac{816}{98}$.
7. $\frac{3624}{198}$.
8. $\frac{980}{102}$.
9. $\frac{1120}{808}$.
10. $\frac{2240}{792}$.
11. $\frac{144+3}{72+3}$.
12. $\frac{596}{152}$.
13. $\frac{756}{248}$.
14. $\frac{860}{(102)(103)}$.
15. $\frac{98\times 103}{101\times 97}$.

Decimalize approximately:

16. $\frac{868 \times 216}{430 \times 440}$. 17. $\frac{3{\cdot}2}{(7{\cdot}9)^2}$. 18. $\frac{1}{(1{\cdot}02)^3}$.

19. $\frac{97}{32} \times \frac{42}{256}$. 20. $\frac{89{\cdot}3}{29{\cdot}7}$.

EXERCISES LXXIV. b.

Fill up approximately the missing parts of the following tables. (Assume that the circumference of a circle = 3·14159 × diameter.)

		Diameter of circle.	Circumference of circle.
1.	(*a*)	9·8	?
	(*b*)	?	63·46
	(*c*)	25·1	?
	(*d*)	33·3	?
	(*e*)	50·5	?

		Mass in grams.	Vol. in c.c.	Density.
2.	(*a*)	258	64	?
	(*b*)	1936	241	?
	(*c*)	?	103	5·02
	(*d*)	1821	?	8·96
	(*e*)	344·6	?	8·53

		Amount of money.	Rate per cent.	Time in years.	Present value.
3.	(*a*)	£82. 6*s*.	2½	2	?
	(*b*)	£60	2	3	?
	(*c*)	£120	4	1	?
	(*d*)	£36	5	1	?

In the evaluation of the following exercises, it is found that the figures marked with an asterisk should have been changed as indicated. What difference approximately will this make in the answer?

	Examples.	Error in Statement.	Error in Result.
	Worked Examples—		
	(*a*) $98\overset{*}{9} \times 47 = 46483$	9 instead of 8	0·1 %, or 47 too large.
	(*b*) $2791{\cdot}490 \div 3{\cdot}9\overset{*}{8} = 708{\cdot}5$	8 „ 4	1 %, or 7 too small approx.
4.	$71656{\cdot}228 \div 9\overset{*}{5}{\cdot}06 = 753{\cdot}8$	5 instead of 4	?
5.	$71656{\cdot}228 \div \overset{*}{9}5{\cdot}06 = 753{\cdot}8$	5 „ 6	?
6.	$882{\cdot}54019 \div 9\overset{*}{0}70{\cdot}3 = 0{\cdot}0973$	7 „ 8	?
7.	$882{\cdot}54019 \div 9070{\cdot}\overset{*}{3} = 0{\cdot}00973$	3 „ 6	?
8.	$\overset{*}{5}94{\cdot}426842 \div 0{\cdot}0734 = 809{\cdot}63$	4 „ 7	?
9.	$150{\cdot}\overset{*}{3} \times 89{\cdot}7 = 13481{\cdot}91$	3 „ 9	?
10.	$21{\cdot}84 \div 4{\cdot}\overset{*}{1}6 = 5{\cdot}25$	6 „ 2	?
11.	$23{\cdot}6 \times 18{\cdot}5 \times 14{\cdot}\overset{*}{8} = 6461{\cdot}68$	8 „ 9	?
12.	$431{\cdot}25 \div (12{\cdot}\overset{*}{5} \times 7{\cdot}5) = 4{\cdot}6$	5 „ 6	?
13.	$70\overset{**}{5}6 \div 112 = 63$	56 „ 42	?
14.	$3{\cdot}142 \times 40{\cdot}\overset{*}{4} = 126{\cdot}9$ approx.	4 „ 6	?
15.	$189{\cdot}4 \div 3{\cdot}\overset{*}{1}42 = 60{\cdot}3$ „	4 „ 7	?
16.	$(3{\cdot}141)(61{\cdot}\overset{*}{2})^2 \div 4 = 2941$ „	2 „ 5	?
17.	$(3{\cdot}14159)(80{\cdot}\overset{*}{1})^2 \div 4 = 5039$ „	1 „ 9	?
18.	$\sqrt{656\overset{*}{1}} = 81$	1 „ 7	?

Approximations in the Extraction of Square Root.

EXAMPLE.—*Find the square root of* 98176352·876123..., *where the number is only approximately known, that is, where the decimal figures beyond the* 1, 2, 3 *are unknown.*

Write *a*, *b*, *c*, ... for the unknown digits, and proceed to extract the square root in the ordinary manner.

	9 9 0 8 · 3 9 8 0 9 8 3 8 7 3
9	98 | 17 | 63 | 52 | ·87 | 61 | 23 | *ab* | *cd* | *ef* | *gh* | *kl* | *mn* | *op*
	81
189	1717
	1701
19808	166352
	158464
19816·3	7888·87
	5944·89
19816·69	1943·9861
	1783·5021
19816·788	160·484023
	158·534304
19816·788	1·949719*ab*

Still follow the ordinary method, but indicate the figures in the remainder which are incorrect, and accent the figures on the left, when, on account of the unknown digits *a*, *b*, *c*, *d*, their effect is invalidated.

Up to the present all the figures have been supposed accurately known, but now *a*, *b*, ... represent figures known to exist, but the values of which are unknown.

No. of effective figures = 6. 19816·79609 | 1·949719*abcd* / 1 783511 6481

Next digit is 0. Period *c*, *d* is brought down.

Next digit is 9, but effect of last four digits is invalidated owing to *a*, *b*, *c*, *d* being unknown.

Digit is 8, but although one digit is added on left, effect of two will be invalidated owing to *e*, *f*.

" = 5. 19816·796188 | 166208×××× *ef* / 158534369504

" = 4. 19816·7961963 | 7674×××××× *kl* / 594503885889

" = 3. 19816·79619668 | 1729×××××××× *mn* / 15853436957244

" = 2. 19816·796196767 | 144×××××××××× *pq* / 138717573377369

" = 1. 19816·7961967743 | 6×××××××××××× *rs* / 59×× .....

The process cannot be carried any further as all the figures are now unknown, the answer being 9908·3980983873.

The last seven digits have been obtained without a knowledge of the figures to be brought down. Since ineffective figures are useless, the abbreviated process may be set down as below:

9	9	0	8 ·	3	9	8	0	9	8	3	8	7	3
9	98	17	63	52	·87	61	*23*	*a.b*	*c.d*	...	...	...	...	...
189	17	17												
19808	16	6352												
19816·3		7888·87												
19816·69		1943·9861												
19816·78		160·484023												

The digit 0 comes next; this should be added to the left, but two figures on left are invalidated owing to *a*, *b*; the 8 on the left is therefore struck out.

19816·7	1·949719
19816·	1·949719
1981	·166208
198	7674
19	1729
1	144
·	6
	·

The second 8 on the left is now struck out, the digit in the root being 9.

It will be noticed that all this abbreviated process is really that of *contracted division*; and that, having obtained any number of figures in the ordinary way, an equal number of figures can then be obtained by employing the methods of contracted division. Also, in finding the square root of any number, say $\sqrt{N}$, having had part of the answer, say n, the square root is approximately $n + \frac{N - n^2}{2n}$, that is, $n + \frac{\text{remainder}}{2n}$.

Surds.

Already it may have been noticed that many vulgar fractions can only be represented as decimals by using an infinitely large number of figures, thus $\frac{1}{3} = 0{\cdot}333\ldots$ and is represented by $0{\cdot}\dot{3}$.

There are other quantities which cannot be represented exactly either by a decimal or a fraction.

For example, consider $\sqrt{2}$.

$\because$ the square of 1 is 1

and „ „ 2 is 4,

clearly $\sqrt{2}$ is something lying between 1 and 2, that is, $1{\cdot}abc\ldots$, where a, b, c, ... are certain digits. These figures cannot terminate. But suppose they did terminate, and that c was the last digit. Go through the process of squaring $1{\cdot}abc$.

$$\begin{array}{l} 1{\cdot}abc \\ \quad 1{\cdot}abc \\ \hline 1{\cdot}abc \\ \quad \ldots\ldots\ldots\ldots \\ \quad\quad \ldots\ldots\ldots\ldots \\ \quad\quad\quad \ldots\ldots\ldots z \quad \text{(suppose } z \text{ the last digit)} \\ \hline 2{\cdot}000\ldots\ldots\ldots 00 \end{array}$$

The figure z must be a zero, and therefore $c \times c$ must give a number ending in 0, but the only possible values of c^2 are 1, 4, 9, 16, 25, 36, 49, 64, 81.

Hence the decimal does not terminate.

Again, $\sqrt{2}$ cannot be a vulgar fraction, say $\frac{p}{q}$, for then

$q\sqrt{2}$ would $= p$, where p is an integer,

and $2 \cdot q^2$ would $= p^2$.

In finding the square root of an exact square (p^2) we can find the factors in pairs; but on the left we have a certain number of pairs of factors, and an odd one;

$\therefore$ $\sqrt{2}$ cannot be a vulgar fraction.

A quantity of this kind, that is $\sqrt{2}$, is called a **Surd**. There is no difficulty, however, in evaluating surds to any required degree of accuracy.

EXERCISES LXXIV. c.

1. A body falling freely from rest describes a distance in feet equal to the product of the square of the time in seconds and a factor 16·1. Find the time in sec. taken by a body falling through a vertical distance of

(*a*) 28140 ft., (*b*) 580 ft., (*c*) 220 yd., (*d*) 400 yd.

2. In Ex. 1 what would be the factor corresponding to 16·1 in the metric system, where the distance is expressed in centimetres?

3. If the velocity in feet per second acquired by a falling body is numerically equal to the square root of the product of 64·44 into the height (in ft.) fallen through, find the velocity acquired in a fall of

(*a*) 120 ft. exactly, (*b*) 324 ft., (*c*) $\frac{1}{4}$ mile.

4. Two ships start from a lightship. One sails due North for 5 hours, at an average rate of 8·4 miles per hour; the other sails due East for 5 hours at the rate of 17·5 miles per hour. How far apart are they at the end of the 5th hour?

5. The length of an incline is 441 yards, and it has an average rise of 4 in. per yard. What is the height of the upper end over the lower?

6. The bottom of a mine is reached by two inclines, the first 234 ft. long, with a fall of 3 yd. for every 25 yd. along the incline; the second 516 ft. long, with a fall of 1 yd. in every 6 yd. along the incline. What would be the shortest distance between the top of the mine and the point where the lower incline meets the bottom?

7. Two houses are directly opposite one another, the road between them being 37·6 ft. wide; the bottom of a window in one house is 8 ft. higher than the bottom of a window opposite it in the other house. What is the distance between the two?

8. The area of a room, the length of which is half as much again as its width, is 216 sq. ft. What is its width?

9. A garden plot, the length of which is 8 times its breadth, has an area of 4418 sq. yd. What are its dimensions?

10. Find the side of the base of a square prism, the height of which is 11·2 in. and which contains 596·848 cub. in. of matter.

11. A cistern with a square base is 2 ft. 9 in. high, and has a capacity of 14 cub. ft. 1680 cub. in. What is the side of the base?

12. A water tank with a square base is 4 ft. 7 in. high; it can hold 1000 gals. Assuming a gallon to be equal to $277\frac{1}{4}$ cub. in., what is the side of the base, true to the nearest inch?

13. In the extraction of a square root the result obtained without any contraction is 862·5..., and the remainder is 62·03. State the next two figures in the square root.

14. (*a*) What is the length of the diagonal of a rectangle with sides 15 in. and 8 in.?

(*b*) What is the length of the diagonal of a square, the side of which is 43 ft.? (true to the nearest foot).

(*c*) How much shorter is the diagonal of a rectangle than the sum of its two sides, when the sides are 35 ft. and 69 ft.?

(*d*) The foot of a ladder 28 ft. long stands 9 ft. beyond a vertical wall of a house. What is the height to which the ladder will reach?

15. The area of a square field is 14 ac. 3 ro. 24 po. Find its side, true to the nearest link.

16. A window having 56 square panes of glass has a lighting area of 5378·24 sq. cm. Find the side of each pane.

17. A square lawn of 36 ft. side is surrounded by a gravel walk of a uniform width of 6 ft. What is the length of netting which will enclose the walk and lawn?

EXERCISES LXXIV. d.

Graphic Methods of finding Square Root.

1. Verify by measurement the correctness of the following construction for finding square roots:

Measure off a distance OA equal to unity; produce it to B so that OB represents the number N of which the root is required.

Describe a semi-circle on OB as diameter. Draw AC at right angles to OB, then OC represents $\sqrt{N}$.

2. Verify the construction in Ex. 1 in the following cases (1 cm. as unit):

$N=4$, $N=9$, $N=16$, $N=1{\cdot}69$, $N=3{\cdot}6$, $N=2{\cdot}25$, $N=6{\cdot}25$.

Show also that AC represents the square root of AB.

This construction may be extended in its application, thus:

3. (*a*) Measure off OA = 3 in.,
OB = 5 in.;

prove by measurement and calculation that OC represents the square root of the product of 3 and 5, *i.e.* of 15.

(*b*) Extract graphically the square roots of (i) 10, (ii) 14, (iii) 30, (iv) 22·5.

4. *Construction of square root curve.*

Mark off on squared paper two lines Ox, Oy, at right angles to one another. On the line xO mark off OL to the left, of length representing 4. Describe a number of circles, each with centre on LOx, passing through L, cutting Oy say at M, and Ox at N. Draw MP, NP at right angles to Oy and Ox respectively. The various positions of P will lie on a curve such that PN represents the square root of ON.

Use your curve to find (*a*) $\sqrt{3}$, (*b*) $\sqrt{22}$, (*c*) $\sqrt{19{\cdot}6}$, (*d*) $\sqrt{2{\cdot}89}$, each to one decimal place; verify by calculation.

EXERCISES LXXIV. e.

1. A boat starts at A and takes the following courses:

Moves 3 miles to the East, but at the same time 4 miles to the N.,
then „ 6 „ „ „ „ 6·3 „ „
then „ 4·5 „ „ „ „ 3·8 „ „

State its final position; also the distance between the initial position and the final.

2. In Ex. **1**, state the final position, also the distance between the initial and final positions, when the courses were as follows :

(*a*) (1) 2·21 m. to E., 1·08 m. to N.; then (2) 3·26 m. to E., 0·81 m. to N.; then (3) 1·85 m. to E., 0·21 m. to S.

(*b*) (1) 6·1 m. to W., 3·2 m. to N.; then (2) 1·8 m. to E.; then (3) 2·1 m. to E., 0·12 m. to S.

(*c*) (1) 5·6 m. to E., 4·1 m. to N.; then (2) 1·8 m. to E., 1·6 m. to S.; then (3) 4·2 m. to E., 1·8 m. to N.

3. Complete the accompanying table, showing the relative lengths of the three sides of a right-angled triangle in which the hypothenuse has different inclinations :

	No. of Degrees in base angle.	Horizontal Measure, Hypothenuse being 1.	Vertical Measure, Hypothenuse being 1.
(*a*)	10°	0·98481	?
(*b*)	20°	?	0·34202
(*c*)	30°	0·86603	?
(*d*)	40°	0·76604	?

Approximations in Square Root (*continued*).

Suppose a number, say 104, to be very nearly an exact square; the first two figures in the square root are 1 and 0, and $104 - 10^2 = 4$.

Two more figures are obtained by dividing 4 by twice 10, *i.e.* 0·20, and it is seen that whereas 104 differs from 100 by 4 %,

$\sqrt{104} = 10{\cdot}2$ approx. and differs from 10 by 2 %,

and more generally if a number N ,, n^2 ,, x %,

$\sqrt{N}$,, n ,, $\frac{x}{2}$ %.

This and the results on page 311 form the basis of a large number of examples of mental or approximate computations.

EXAMPLE 1.—*Evaluate* (*a*) $\sqrt{325}$, (*b*) $\sqrt{3636}$.

(*a*) $\sqrt{325} = \sqrt{324+1} = \sqrt{18^2+1}$
$= 18 + \frac{1}{36}$ approx.

(*b*) $\sqrt{3636}$; 3636 differs from 3600 by 1 %,
$\sqrt{3636}$ „ $\sqrt{3600}$ „ $\frac{1}{2}$ %
= 60 increased by $\frac{1}{2}$ %
= 60·3 approx.,

while the error in the approx. will be $\frac{1}{2}$ % of $\frac{1}{2}$ %, *i.e.* $\frac{1}{400}$ %.

EXAMPLE 2.—*A railway has a vertical rise of* 1 *in* 250 *horizontal. What is the length of rail corresponding to a rise of* 4 *ft. length?*

The length $= \sqrt{4^2 + 1000^2} = 1000 + \frac{16}{2000}$ approx. = 1000·008 ft.

EXERCISES LXXIV. f.

Mental or Oral.

1. On a railway incline, the vertical rise is 1 ft. for every 100 ft. horizontal. What is the length of the incline for each 100 ft. measured horizontally?

2. If in Ex. **1** the rise was 1 in 240, what was the length of the incline if the rise was 5 ft.?

3. In an incline a vertical rise of 3 ft. occurs for every 150 ft. horizontal. Find an approximation to the length of the incline corresponding to a rise of 6 ft.

4. State the number of decimal places to which the following square roots can be found, the number under the root being approximate:

(*a*) $\sqrt{100{\cdot}6\ldots}$. (*b*) $\sqrt{903\ldots}$. (*c*) $\sqrt{400{\cdot}04\ldots}$.
(*d*) $\sqrt{100000{\cdot}008\ldots}$. (*e*) $\sqrt{625{\cdot}35\ldots}$. (*f*) $\sqrt{4{\cdot}44\ldots}$.
(*g*) $\sqrt{64{\cdot}06\ldots}$.

Square Root (*continued*).

Subjoined is a table of squares of the first 60 numbers. It may be employed to write down approximations to the square roots of many numbers.

No.	Square.	No.	Square.	No.	Square.	No.	Square.	No.	Square.	No.	Square.
1	1	11	121	21	441	31	961	41	1681	51	2601
2	4	12	144	22	484	32	1024	42	1764	52	2704
3	9	13	169	23	529	33	1089	43	1849	53	2809
4	16	14	196	24	576	34	1156	44	1936	54	2916
5	25	15	225	25	625	35	1225	45	2025	55	3025
6	36	16	256	26	676	36	1296	46	2116	56	3136
7	49	17	289	27	729	37	1369	47	2209	57	3249
8	64	18	324	28	784	38	1444	48	2304	58	3364
9	81	19	361	29	841	39	1521	49	2401	59	3481
10	100	20	400	30	900	40	1600	50	2500	60	3600

EXAMPLE 1.—*Evaluate* $\sqrt{572}$.

$$\sqrt{572} = \sqrt{576 - 4} = 24 - \frac{4}{2 \times 24} = 24 - 0{\cdot}08 = 23{\cdot}92,$$

EXAMPLE 2.—*Evaluate* $\sqrt{2710}$.

$$\sqrt{2710} = \sqrt{2704 + 6} = 52 + \frac{6}{2 \times 52} = 52{\cdot}06.$$

EXERCISES LXXIV. g.

Use the above table to evaluate

1. $\sqrt{66}$. **2.** $\sqrt{630}$. **3.** $\sqrt{1640}$. **4.** $\sqrt{2412}$. **5.** $\sqrt{1040}$.

6. $\sqrt{3060}$. **7.** $\sqrt{236}$. **8.** $\sqrt{105}$. **9.** $\sqrt{1980}$. **10.** $\sqrt{1809}$.

52. PROPORTION (*continued*).

In questions of Simple Interest, several factors are involved. It has been seen (p. 278 *et seq.*) that the *interest* depends upon the

(1) *Principal*; and is, other things being the same, proportional to the principal;

(2) *Time*; and is, other things being the same, proportional to the time;

(3) *Rate per cent.*; and is, other things being the same, proportional to the rate.

Consider now more general problems in proportion, in which several factors are involved.

EXAMPLE 1.—*If* 18064 *lb. of biscuit formed the allowance to* 2016 *men for* 35 *days, what would be the equivalent allowance to* 1050 *men for* 42 *days?*

The factors involved are

(1) Time; and the weight of biscuits would be proportional to the time if the number of men remained the same;

(2) Number of men; and the weight of biscuits would be proportional to the number of men, if the time remained the same.

First, consider only the time.

If 18064 lb. be allowed to 2016 men for 35 days, then $18064 \times \frac{42}{35}$ lb. would be allowed 2016 men for 42 days.

Next, consider the number of men.

If $18064 \times \frac{42}{35}$ be allowed 2016 men for 42 days,

$18064 \times \frac{42}{35} \times \frac{1050}{2016}$ would be allowed to 1050 men for 42 days, and the required allowance

$$= 18064 \times \frac{42}{35} \times \frac{1050}{2016} \text{ lb.} = 11290 \text{ lb.}$$

EXAMPLE 2.—*If, in hiring goods, the cost is determined as being proportional to the value of the goods and to the time of hire; what would be the cost of hiring goods to the value*

of £15 for 5 days if the cost of hiring goods to the value of £20. 5s. for 12 days be £1. 10s. 0d.?

Here the hire is $£1.\ 10s. \times \frac{5}{12} \times \frac{£15}{£20.\ 5s.}$

$$= £1.\ 10s. \times \frac{5}{12} \times \frac{15}{20{\cdot}25}$$

$$= £1.\ 10s. \times \frac{25}{81}$$

$$= \frac{£37.\ 10s.}{81}$$

$$= 9s.\ 3d. \text{ to the nearest penny.}$$

EXAMPLE 3.—*One locomotive* A *is known to travel 52 ft. for every 35 ft. that another locomotive* B *travels, when drawing their customary trains. If* A *completes a journey of 25·84 miles in 56 min., how far will* B *go in 1 hr. 32 min.?*

The factors involved in computing the distance are the speed and the time.

The distance travelled by B

$$= \frac{\text{speed of B}}{\text{speed of A}} \times \frac{\text{time B travels}}{\text{time A travels}} \times \text{distance A travels}$$

$$= \frac{35}{52} \times \frac{92}{56} \times 25{\cdot}84 \text{ miles}$$

$$= \frac{\cancel{7} \times 5 \times \cancel{4} \times 23}{13 \times \cancel{4} \times \cancel{7} \times 8} \times 25{\cdot}84 \text{ miles}$$

$$= \frac{115 \times 25{\cdot}84}{104} \text{ miles.}$$

$$= 28{\cdot}57 \text{ miles, to 2 decimal places.}$$

EXERCISES LXXV.

1. The weight of the load which can be supported by a wire is proportional to the area of the cross-section of the wire, and depends also on the nature of the substance of the wire. If a steel wire can support a load of 1·30 tons for every 0·75 ton supported by a copper wire of the same cross-section, what load can be

supported by a wire of copper of 0·2565 sq. in. cross-section, if a steel wire, section 0·1350 sq. in. can support a load of 7·02 tons?

2. The extension of a wire when stretched is proportional to the product of the stretching force and the length of the wire. If a wire 15·6 ft. long increases in length by 0·24 in. when stretched by a weight of 18 lb., what increase will be produced in the length of 12 ft. of the same wire when the load is 26 lb. weight?

3. On a certain railway the passenger fares are computed as being proportional to the distance. If £581. 11*s.* 6*d.* be received from 2705 passengers, each having travelled an average distance of 51·6 miles, what fares would be received from 1384 passengers, each travelling an average distance of 62·5 miles?

4. The cost of lighting a house by electric light is based on the number of lamps alight, and the time during which they are alight. In two houses the consumption is as follows:

House A—25 sixteen-candle-power lamps, each alight on the average for 212 hrs.

House B—17 sixteen-candle-power lamps, each alight on the average for 180 hrs.

If the bill for electric light be £5. 10*s.* 5*d.* in house A, what is it in house B?

5. In silvering articles by electrolysis, the amount of silver deposited is proportional to the strength of the electric current in ampères, if the time remains the same, and to the time if the current remains constant.

If 1213·056 grams of silver are deposited by a current of 25 ampères in 12 hrs., how much silver will be deposited in 210 min. by a current of 36 ampères?

6. With the data of Question 5, find in what time 352·6 grams of silver will be deposited by a current of 36 ampères.

7. A steam engine employed in pumping water from a coal mine is found to consume an amount of coal proportional to the product of the amount of water pumped up and the depth from which the water is raised. If 112 lb. of coal are burnt in pumping 9600 gallons of water from a depth of 600 ft., how much coal will be burnt when 12,000 gallons are raised from a depth of 750 ft.?

8. The power (measured in horse-power) developed by a water wheel is proportional to the amount of water it receives per minute and to the depth of the fall.

If a water wheel receiving 9000 gallons of water per minute, with a fall of 12 ft., develops a power of 12·5 horse-power; find the horse-power of a similar wheel with a fall of 10·8 ft. and a feed of 8500 gallons of water per minute.

9. When the wind blows directly upon a surface, such as a wall, the force exerted by the wind is proportional to (*a*) the area of the wall, (*b*) the square of the velocity of the wind. If wind blowing at 15 miles per hour exert a force equal to the weight of 270 lb. on a wall 40 ft. long and 12 ft. high, what force would be exerted upon a wall 63 ft. long, 15 ft. high, by wind moving at the rate of 23 miles per hour? (State your answer to the nearest lb.)

10. The pitch of one screw A is 2·540 times as much as the pitch of another B; if the first, A, is turned through 260·5 revolutions in moving through a distance of 12·21 in., what distance will the second screw B move through in 452·5 revolutions? State your answer to second decimal place.

11. If the weight of a square bar of iron 2·5 in. broad and 12 ft. long be 241·44 lb., what is the weight of a square bar 4·75 in. broad and 7·5 ft. long?

12. The average rain fall per month at Scarborough was 2·253 in., and it was calculated that, during a certain period, the fall of rain on a certain field of 6·345 ac. amounted to 25·36 tons. What would have been the total weight of rain received during the same period on a field of 5·281 ac. at Londonderry, where the average rain fall was 3·375 in. per month?

13. Sheets of tin plate are generally sold in boxes, containing varying numbers of plates of different sizes. In one box there are 225 sheets, each $13\frac{3}{4}$ in. × 10 in., and of weight 1 cwt. 1 qr. 7 lb. What will be the weight of the sheets in another box of 200 sheets, each 15 in. by 11 in., the thickness of the sheets being the same in each case?

53. MEASUREMENT OF ANGLES.

Suppose OA and OB are two straight lines inclined to one another at any angle. With O as centre and any radius describe a circle cutting OA and OB at a and b respectively. If the length of the arc ab be measured (pp. 205, 206), and its length divided by the length of Oa the radius of the arc, then, AOB remaining the same, the result of the division will not depend on the particular length Oa chosen, and is known as the **circular measure** of the angle AOB.

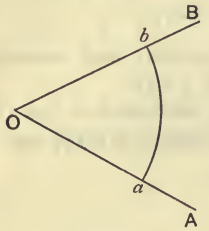


FIG. 47.

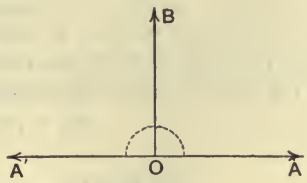


FIG. 48.

Suppose a line OA rotated about O, into a position OA′, in which the line points in a direction exactly opposite to OA.

The angle turned through is *two right angles*; a line such as OB making equal angles with OA and OA′ would form a right angle with either OA or OA′.

The circular measure of an angle of two right angles is the ratio of the length of the arc of a semicircle to the radius of the semicircle, and is denoted by the symbol π; its value is 3·14159...; but the approximation $\frac{22}{7}$ is often used.

The divisions and subdivisions of the right angle are given in the following table:

1 right angle = 90 **degrees** (90°).
1 degree = 60 **minutes** (60′).
1 minute = 60 **seconds** (60″).

EXAMPLE 1.—*Find the circular measure of an angle of* $50^\circ\ 13'\ 12''$.

$50^\circ\ 13'\ 12'' = 50^\circ\ 13{\cdot}2'$ since 60 seconds make one minute.

$50^\circ\ 13{\cdot}2' = 50{\cdot}22^\circ$ since 60 minutes „ degree.

But the circular measure of 180° is π.

$$\therefore \quad \text{„} \quad \text{„} \quad 1^\circ = \frac{\pi}{180},$$

$$\text{„} \quad \text{„} \quad 50{\cdot}22^\circ = \frac{50{\cdot}22\pi}{180} = 0{\cdot}279\pi$$

$$= 0{\cdot}279 \times 3{\cdot}14159\ldots$$

$$= 0{\cdot}8775\ldots .$$

EXAMPLE 2.—*Express in degrees, minutes and seconds an angle, the circular measure of which is* 1·234.

Since the angle of which the circular measure is π has 180°,

$\therefore$ the angle of which the circular measure is 1·234 has

$$\frac{180^\circ \times 1{\cdot}234}{\pi}$$

$$= \frac{180 \times 1{\cdot}234^\circ}{3{\cdot}14159} \text{ approx.}$$

$$= \frac{222{\cdot}12^\circ}{3{\cdot}14159}$$

$$= 70{\cdot}7030^\circ$$

$= 70^\circ\ 42{\cdot}18'$ (multiply the decimal part by 60 to reduce to minutes)

$= 70^\circ\ 42'\ 11''$ approx. (multiply the decimal part by 60 to reduce to seconds).

EXAMPLE 3.—*If an arc of* 6·2832 *in. subtend an angle of* 72° *at the centre of a circle of* 5*-inch radius, what will be the length of the arc of a* 3*-inch circle which subtends an angle of* 20°*?*

The arc is proportional directly to the radius, and directly to the angle subtended at the centre.

$$\therefore \text{ length of arc} = 6{\cdot}2832 \times \tfrac{20}{72} \times \tfrac{3}{5} \text{ in. approx.}$$

$$= 1{\cdot}0472 \text{ in. approx.}$$

In the practical measurement of angles in drawing, a protractor is usually employed. Figs. 49 and 50 show the method of marking off an angle, and also of measuring an angle.

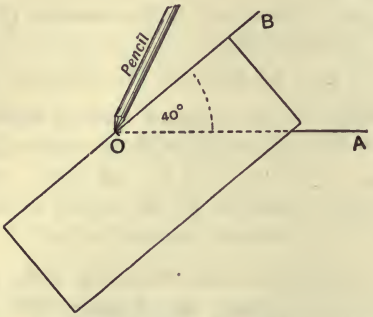


FIG. 49.—Marking off a line **OB** making an angle of 40° with a given line **OA**.

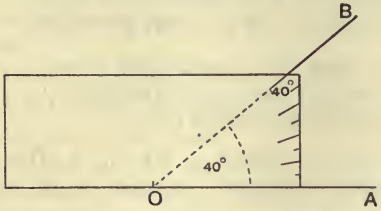


FIG. 50.—Measurement of an angle **AOB** (40°).

EXERCISES LXXVI. a.

Graphic.

1. Describe a circle of 5 cm. radius; on it mark off an arc also of 5 cm.; measure the angle the arc subtends at the centre and check your result by describing circles of other radii and setting off arcs equal in length to the respective radii.

2. Construct a graph for the conversion of degrees into circular measure.

(Set off degrees horizontally, scale 1° represented by a line 1 mm. in length; at a distance of 18 cm., set off a vertical 15·7 cm. long representing π on a scale where 5 cm. represents the unit of circular measure.)

Use your graph to find the circular measure of the following angles; verify by calculation:

(*a*) 30°, (*b*) 57°, (*c*) 60°, (*d*) 72°, (*e*) 112°, (*f*) 153°.

3. Describe any circle, and place within it chords, the lengths of which are the following multiples of the radius of the circle:

(*a*) 1·732, (*b*) 1·176, (*c*) 1, (*d*) 0·765, (*e*) 0·618.

Find the angles subtended by the chords at the centre, and express them as sub-multiples (approx.) of 360°.

4. Draw two straight lines O*x*, O*y* at right angles to one another; along O*x* measure any distance ON; at N erect a perpendicular NP. Join OP. If NP be n times ON, find the value of the angle PON when n has the following values:

(*a*) 2, (*b*) 3, (*c*) 4, (*d*) $\frac{1}{2}$, (*e*) $\frac{1}{3}$, (*f*) $\frac{1}{4}$.

5. Suppose PON is a right-angled triangle, with the right angle at N; measure the angle NOP when the rise NP is the following fractions of the distance OP:

(*a*) 0·1, (*b*) 0·2, (*c*) 0·3, (*d*) 0·4,
(*e*) 0·5, (*f*) 0·6, (*g*) 0·7, (*h*) 0·8.

EXERCISES LXXVI. b.

1. Calculate the circular measure of the following angles:

(*a*) 60° 20′ 30″, (*b*) 32° 5′ 17″, (*c*) 19° 12′ 50″, (*d*) 26° 0′ 10″,
(*e*) 15° 15′ 15″, (*f*) 34° 17′ 5″, (*g*) 12° 3′ 3″, (*h*) 83° 12′ 18″.

(Assume $\pi = 3{\cdot}14159$.)

2. Express in degrees, minutes, and seconds, the angles of which the circular measures are:

(*a*) 0·23457, (*b*) 0·823214, (*c*) 0·98728, (*d*) 3·11891.

3. Complete the following table :

	Length of Arc in cm.	Radius of Circle in cm.	Angle subtended by Arc at Centre of Circle.
(*a*)	1·834	3·12	? to nearest degree.
(*b*)	?	22·2	13°
(*c*)	25·8	19	? to nearest degree.
(*d*)	?	10^6	1′
(*e*)	85·0	?	29°
(*f*)	100	?	37°

54. AREAS OF CIRCLES.

EXERCISES LXXVII.

Graphic.

1. Describe on squared paper circles of the following radii, and find their areas :

(*a*) 1 cm., (*b*) 2 cm., (*c*) 3 cm., (*d*) 4 cm., (*e*) 5 cm., (*f*) 6 cm.

Draw a graph showing the relation between the areas of the circles and the squares of their radii.

Draw also a graph showing the relation between the circumferences of the circles and their diameters. Compare the two graphs and interpret your results.

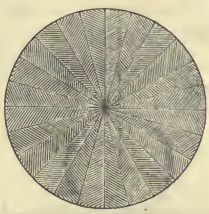


Fig. 51.

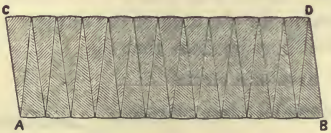


Fig. 52.

2. Cut out a circular disc of paper or cardboard, divide it into triangles as in Fig. 51 and arrange as in Fig. 52. Note that the

figure is practically a rectangle, and that one side of the rectangle is half the circumference of the circle, while the other side is equal to the radius of the circle. Hence deduce an expression for the area of a circle.

3. Cut out a sector of a circle of 6-inch radius, proceed as in Ex. **2**, and deduce an expression for the area of the sector in terms of the radius and the arc.

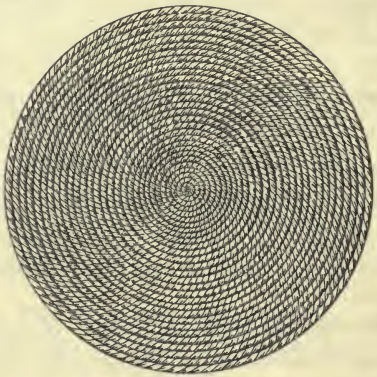


FIG. 53.

4. Suppose the space within a circle filled by circular concentric pieces of cord as in Fig. 53.

Suppose the cords all cut along a radius and straightened out as is the typical cord in Fig. 54.

What figure will result? Deduce an expression for the area of the circle.

It will now be clear that the

area of a circle

$= \frac{1}{2}$ **(radius)** $\times$ **(circumference)**

$= \pi r^2$.

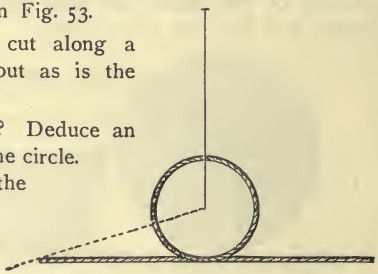


FIG. 54.

5. If a circle be divided up into a number of sectors all containing equal angles, it is clear that the areas of all the sectors are

equal, and therefore that the area of a sector containing $n°$ is equal to

$$\frac{n}{360}(\text{area of the circle}) = \frac{n}{360}(\pi r^2).$$

Describe sectors of a circle of 3-inch radius containing (*a*) 30°, (*b*) 50°. Calculate the areas of the sectors by means of the formula, and verify by the use of squared paper. Find the areas also by first measuring the lengths of the corresponding arcs and then multiplying by half the radius.

6. Fig. 55 represents graphically the relative distribution of land in Russia as (*a*) arable land, (*b*) meadows and pastures, (*c*) woods and forests and (*d*) infertile areas. Express the areas as percentages of the whole.

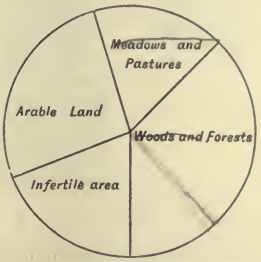


FIG. 55.

7. Fig. 56 represents the division of the population of the United States according to occupation. If the industrial class be 9·3%, find the percentage of the other classes shown.

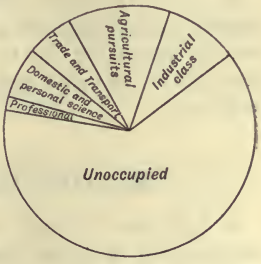


FIG. 56.

8. Construct four right-angled triangles, and in each case describe circles with the sides of the triangles as radii; measure the areas of the circles and find the relation existing between the areas of the circles corresponding to each triangle.

9. The circles (*a*), (*b*), (*c*) (Fig. 57) have areas proportional to the populations of China and its dependencies, the British possessions, and Russia with Finland respectively. If the population of

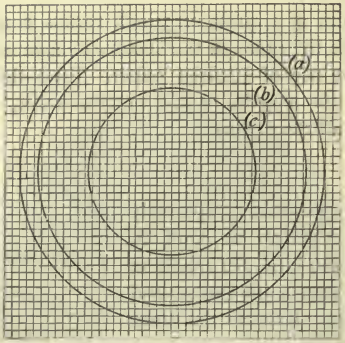


FIG. 57.

China and its dependencies be 434 millions, find the other populations represented.

10. Copy the given sector (Fig. 58) on squared paper, complete the circle, find the areas of the sector and the circle, calculate the angle of the sector and verify by using a protractor.

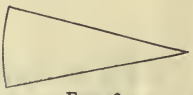


FIG. 58.

55. THE PRISM AND CYLINDER.

If a prism or cylinder be cut into a number of small prisms or cylinders of equal lengths by planes perpendicular to the length of axis, the pieces so formed will be clearly of equal volume and of equal lateral surface. Equal increases in the length of a prism or cylinder will therefore occasion equal increases in the volume and lateral surface; and, since the volume and lateral surface both vanish when the solids are indefinitely

short, the volume of a cylinder or a prism is proportional to the length of the cylinder or prism, and so also is the lateral surface.

But if the prism be 1 cm. long, it is clear that, corresponding to each square centimetre of the end, there is 1 cubic centimetre of volume. Hence, the volume is proportional to the area of the end of the prism, and the volume of a prism or cylinder = (area of end)(length), and

Fig. 59.

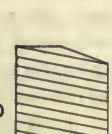


Fig. 60.

$$\therefore \text{ volume of cylinder} = (\pi r^2)(l),$$

where l is the length, r the radius of end.

Again, if a piece of paper be wrapped tightly round a cylinder or prism, it will be evident that, in both cases, the lateral surface = (perimeter of base)(length), and therefore for a cylinder the curved surface $= 2\pi rl$,

where r = the radius of end and l the length of the cylinder.

EXERCISES LXXVIII. a.

Practical.

[*Apparatus: Sheets of metal or cardboard. Tins or beakers. Callipers. Scale. Coil of wire. Cylinders. Burette. Balance. Squared paper. Screw gauge.*]

1. Weigh the given irregularly shaped sheet of metal or cardboard; cut out a rectangular piece, measure its length and breadth and then weigh it. Calculate the area of the irregular piece, check your result by tracing its outline on squared paper and counting the number of squares.

2. From a piece of cardboard cut out a circular disc and also a square, the radius of the circle being equal to the side of the

square. Weigh both and compare the area of the circular disc with the area of the square.

3. Pour water into a beaker or tin to a convenient depth; run in more water from a burette until the depth is increased by 1 cm.; note the volume run out from the burette. The volume in c.c. will equal the area of the section of the beaker in sq. cm. Find the ratio of this area to the square of the diameter of the section of the beaker.

4. Weigh the given coil of wire of known density, measure its diameter by means of a screw gauge, calculate the length of the wire.

5. Measure the length and diameter of the given cylinder and calculate its volume; check by finding the volume by displacement and also by finding the loss of weight in water.

6. Calibrate the given glass tube by introducing mercury into the bore and finding the length of the column at different places.

7. Calculate the volume of the given washer (*a*) by measurement, (*b*) by displacement.

8. You are given three pieces of wire of the same length and material; compare their cross-sections (*a*) by weighing, (*b*) by use of a screw gauge.

EXERCISES LXXVIII. b.

Mental or Oral.

In all the following exercises π may be taken as $\frac{22}{7}$ approximately.

1. What is the area of the end of a regular prism, having the volume 256 c.c. and length 32 cm.?

2. What is the internal volume of a hollow square prism of length 12 ft., outside edge of base 1 ft. and thickness 1 in.?

3. The curved surface of a cylinder is 44 sq. ft., the circumference 4 ft. What is the length?

4. The curved surface of a cylinder is 44 sq. ft., the length 1 ft. What is the radius?

5. What is the curved surface of a cylinder 3 ft. long, 7 ft. radius?

6. Find approximately the volume of a cylinder 4 ft. long and 7 ft. radius.

7. Find approximately the volume of a cylinder 14 cm. long and 35 cm. radius.

8. Find approximately the length of a cylinder 7 in. radius and volume 308 cub. in.

9. Give an approximate expression for the volume of the substance of a pipe 7 ft. long and 1 ft. internal radius, $\frac{1}{2}$ inch thick.

10. What weight of water could be contained in a cylindrical bucket 1 ft. deep and 1 ft. in diameter?

11. How many lb. does a steel cylinder weigh, 7 ft. long and 1 ft. radius, if 1 cub. ft. weighs 450 lb.?

12. What is the internal radius of a cylindrical tube, the internal volume of which is 198 c.c. and length 7 cm.?

EXERCISES LXXVIII. c.

1. A solid cylinder is 15 cm. long and 8 cm. in diameter. What is its volume?

2. A cylindrical boiler is 12 ft. long and 5·42 ft. in diameter. What is its contents?

3. A well is 6·23 ft. in diameter. How many cub. ft. are in the well when it contains water to a depth of 50 ft.?

4. A pump has a piston 9·32 in. in diameter, the length of the stroke is 25 in. What volume of water does the pump lift per stroke?

5. The internal surface of a pipe is 2704 sq. cm. and the contents 5822 c.c. What is the length of the pipe, also its internal diameter?

6. What is the volume of the material of a pipe per foot length, if the inside diameter is 0·892 ft. and outside diameter 0·922 ft.?

7. What is the weight of 20 ft. of an iron pipe, 9 in. outside diameter and 1·2 in. thickness, if a cubic foot of iron weighs 450 lb.?

8. If a solid rod, 6·62 in. diameter, weighs 17·2098 lb., what is the weight of a rod of the same metal 4·8 in. diameter, but 2·25 times as long?

9. Regarding an iron roller as a hollow cylinder, find the area it rolls per revolution, if its thickness be 1·8 in., its internal diameter 2 ft. 3 in., and its weight 1357 lb.

(1 cub. ft. of iron weighs 450 lb.)

10. Find the (i) volume and (ii) curved surfaces of the following cylinders:

	Diameter.	Length.		Diameter.	Length.
(*a*)	8·72 cm.	600 cm.	(*b*)	32·2 cm.	125 cm.
(*c*)	0·231 cm.	3 cm.	(*d*)	51·3 cm.	12 cm.

11. A piece of iron wire, 30 metres long, weighs 8·553 gr. in air and 1·1404 gr. less in water. What is its diameter?

12. In order to find the bore of a piece of thermometer tubing, mercury is let into it; and it is found that 0·134 gram of mercury occupies a length of 40 cm. of the tube. What is the mean diameter of the bore? (One c.c. of mercury weighs 13·6 gr. approx.)

13. Mercury is introduced into a piece of glass tubing; at one place A it occupies a length of 8·19 cm.; at B a length of 6·58 cm.; and at C a length of 8·64 cm. Express the diameters at A and B in terms of that at C.

14. If a certain piece of wire weighs 63·6174 grams, and has a diameter of 0·300 cm., what is the diameter of a piece of the same length and material, but which weighs 44·1787 gr.?

56. SIMILAR FIGURES.

EXERCISES LXXIX. a.

Mental.

1. A, B, C, D are four cubes, A of 1 cm. edge, B of 2 cm. edge, C of 3 cm. edge, D of 4 cm. edge. Write down their volumes.

2. A, B, C, D are rectangular blocks of the following dimensions:

	Length.	Breadth.	Thickness.
A	2 cm.	1 cm.	1 cm.
B	4 cm.	2 cm.	2 cm.
C	6 cm.	3 cm.	3 cm.
D	8 cm.	4 cm.	4 cm.

It will be noticed that all the blocks are of similar shape. Calculate their volumes, and mention anything you may notice about the result.

3. A, B, C, D are four blocks of the following dimensions:

	Length.	Breadth.	Thickness.
A	4·25 cm.	1·3 cm.	0·8 cm.
B	8·5 cm.	2·6 cm.	1·6 cm.
C	17·0 cm.	5·2 cm.	3·2 cm.
D	34 cm.	10·4 cm.	6·4 cm.

Fill up the following table with reference to the blocks B, C, D:

	Length of Block / Length of A	Breadth of Block / Breadth of A	Thickness of Block / Thickness of A	Volume of Block / Volume of A
B				
C				
D				

4. A, B, C, D are rectangular blocks having dimensions represented by the following:

A, length l in., breadth b in., thickness, t in.
B, „ $2l$ „ , $2b$ „ „ $2t$ „
C, „ $3l$ „ „ $3b$ „ „ $3t$ „
D, „ $4l$ „ „ $4b$ „ „ $4t$ „

Find the multiples that the volumes of B, C, D are respectively of that of A; and express the results in factor form.

Illustrate the result with respect to block B by a figure.

5. The volume of a block of length 36·2 cm., breadth 19 cm., thickness 11·1 cm., is 7634·58 c.c. Write down the volumes of the following blocks:

(*a*) length 72·4; breadth 38; thickness 22·2 cm.,
(*b*) „ 362; „ 190; „ 111 cm.,
(*c*) „ 18·1; „ 9·5; „ 5·55 cm.

6. A rectangular block of iron weighs 7 ton 4 cwt. What is the weight of another block of iron, half as long again as the first, half as broad again and half as thick again?

7. A leaden bullet is 1·2 mm. in diameter and weighs λ grams. What will be the weight of a bullet having the diameter 0·84 cm.?

8. How many bullets of 2·4 mm. diameter could be obtained from a large ball of 172·8 cm. radius?

9. A hollow shell has its outer diameter 12 in. and inner diameter 10 in.; if, instead of being hollow, it had been solid throughout, the weight would have been 243 lb. What was the weight of the actual shell? Express your answer in factor form.

EXERCISES LXXIX. b.

Graphic.

1. Copy Fig. 61 in your squared paper exercise book. Mark any point O within the figure. Join O to each corner or salient point *a*, *b*, *c*, ... Produce O*a* to A so that *a*A = O*a*, produce O*b* to B so that *b*B = O*b*... and so on. Join AB... and so obtain a figure AB... of the same shape as *ab*....

Find the ratio of

(i) AB : *ab*; BC : *bc*...,

(ii) the area of ABC... to that of *abc*....

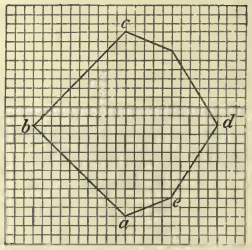


FIG. 61

2. Repeat Ex. **1**, but this time enlarge the figure so that OA is 1·4 times O*a*, OB is 1·4 times O*b* and so on.

3. The given plan (Fig. 62) is drawn to such a scale that 1 acre is represented by 1 square inch; (*a*) reduce it so that 4 acres are

now represented by 1 square inch, (*b*) enlarge it so that 1 rood is represented by 1 square inch.

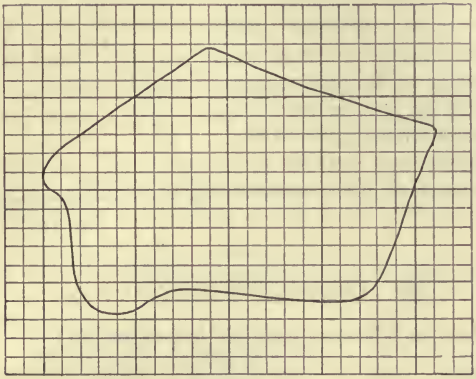


FIG. 62.

4. The areas of the circles marked A, B, C, D, E were employed on a chart to represent the population of the Colonial possessions and protectorates of England, France, the Netherlands, Belgium and Germany respectively. If the inhabitants of the Colonies, etc., of the British Empire number 400×10^6, what are the populations of the others?

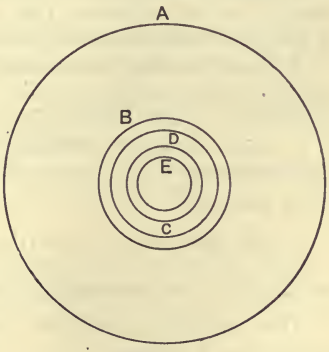


FIG. 63.

5. Draw a diagram showing, by the areas of circles, a comparison between the populations of Rhodesia (580000), Nigeria (467280), Bechuanaland (38000), North Central Africa Protectorate (330780), Cape Colony (277000), East Africa (200000).

6. Draw a triangle, with sides twice those of Fig. 64. Suppose that your triangle represents the number (160) of pupils per 1000 of inhabitants in the Netherlands, draw triangles of the same shape to represent the number per 1000 in each of the following countries: Great Britain 150, Austria 140, Belgium 124, Italy 84, Roumania 50.

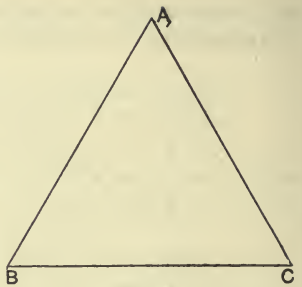


FIG. 64.

57. INVERSE PROPORTION.

Consider the following example in proportion:

EXAMPLE.—*One train* A *travels at* 30 *miles per hour, another train* B *travels at* 20 *miles per hour. If* A *completes a journey in* 3 *hr.* 20 *min., how long will* B *take to complete the same journey?*

If train A, travelling at 30 miles per hour, completes the journey in 200 minutes (3 hr. 20 min.), a train, travelling at 1 mile per hour, would complete the journey in 30×200 min.

A train, travelling at 20 miles per hour, would complete the journey in $\frac{30 \times 200}{20}$ minutes.

$\therefore$ train B would complete the journey in $\frac{30 \times 200}{20}$, *i.e.* 300 min. or 5 hr.

In this case

$$\text{the time taken by B} = (\text{time taken by A}) \frac{\text{speed of A}}{\text{speed of B}},$$

or, the times are **inversely** proportional to the speed.

The following cases are further examples in which the proportion is inverse:

(*a*) The base of a triangle is inversely proportional to its altitude, if the area remains the same.

(*b*) The densities of bodies are inversely proportional to their volumes, if their masses be the same.

(*c*) The time is inversely proportional to the principal if the interest and rate per cent. remain the same.

A great number of problems involving direct and inverse proportion are best done by use of equations, following out the method adopted in the following typical examples:

EXAMPLE 1.—*Two bodies are of the same mass. In the first, the volume is* 31·8 *c.c., and the density is* 8·82 *gm. per c.c. What is the volume of the second if the density is* 7·20 *gm. per c.c.?*

Mass of the first = mass of the second.

Vol. of second × density of second = vol. of first × density of first.

$$\text{Vol. of second} = \frac{\text{vol. of first} \times \text{density of first}}{\text{density of second}}$$

$$= \frac{31{\cdot}8 \times 8{\cdot}82}{7{\cdot}20}$$

$$= 38{\cdot}9 \text{ c.c.}$$

$$\begin{array}{r} 8{\cdot}82 \\ 31{\cdot}8 \\ \hline 264{\cdot}6 \\ 8{\cdot}8 \\ 7{\cdot}0 \\ \hline 7{\cdot}2 \;\big|\; 280{\cdot}4 \;(38{\cdot}9 \\ 64{\cdot}4 \\ \hline 6{\cdot}80 \end{array}$$

EXAMPLE 2.—*Two cylinders are of the same mass; the first* A *is* 12·6 *cm. long,* 3·4 *cm. in radius and each cubic centimetre of its substance weighs* 6·42 *grams. The second* B *is* 16·2 *cm. long and made of a substance, each centimetre of which weighs* 11·2 *grams. What is the radius of the second cylinder?*

Here, square of radius of B × length of B × density of B = square of radius of A × length of A × density of A.

∴ square of radius of B

$$= \frac{(\text{square of radius of A}) \times (\text{length of A}) \times (\text{density of A})}{(\text{length of B}) \times (\text{density of B})},$$

$$\text{radius of B} = (3{\cdot}4) \times \sqrt{\frac{12{\cdot}6 \times 6{\cdot}42}{11{\cdot}2 \times 16{\cdot}2}}.$$

On evaluating, the radius is found to be = 2·27 cm.

NOTE.—An *approximate* answer should always be found. This can frequently be done in several ways.

Thus, roughly, radius $= (3{\cdot}4) \times \sqrt{\frac{81}{180}} = (3{\cdot}4)\sqrt{\frac{9}{20}}$

$= 3{\cdot}4\sqrt{0{\cdot}45}$,

$\therefore$ lies between 6 and 7 times 0·34.

Again, radius $= (3{\cdot}4)\sqrt{\frac{9}{8} \times \frac{6{\cdot}42}{16{\cdot}2}}$

$= (3{\cdot}4)\sqrt{\frac{6{\cdot}42}{8 \times 1{\cdot}8}} = (3{\cdot}4)\sqrt{\frac{64{\cdot}2}{8 \times 18}}$

$= \frac{(3{\cdot}4)8}{12}$ approx.

$= \frac{6{\cdot}8}{3}$

$= 2{\cdot}27$ approx.

These approximations, however, should be generally mental.

EXERCISES LXXX. a.

Oral or Mental.

1. Two rectangles are of the same area: the length of the first is 6 times the length of the second, and its breadth is 12 ft.; what is the breadth of the second?

2. A rectangular leaden block, 1·6 ft. long, is melted down and recast into a block with a cross-section $\frac{1}{4}$ of its original amount: what is its new length?

3. If a field could be ploughed by a certain number of men in 14 days, in what time could it be ploughed by 7 times the number of men?

4. A certain field can be ploughed by a certain number of men in 6 days: in what time could a field of 3 times the extent be ploughed by twice the number of men?

5. Two masses are of equal weight: one has a density 7, the other a density 2; the volume of the one is 12 cub. in.; what is the volume of the second?

6. The bills for lighting two rooms were the same: in the first room there were 6 burners, each alight on the average 45 hours; there were 5 burners in the second room: how long were they each alight on the average?

7. The diameters of two wheels on a locomotive are 6 ft. and $3\frac{1}{2}$ ft. respectively; the first wheel makes 4200 revolutions in a certain journey: how many does the second wheel make?

8. A is 50% of B and 40% of C: what percentage is B of C?

9. A is 50% of B, B is 40% of C: what percentage is A of C?

10. If 100 ft. of a wall 8 ft. high can be painted for £10, what length of a 6 ft. wall could be painted for £12?

11. If a ream of paper 72 in. by 48 in. weighs 28 lb., what will be the weight of a ream of paper 24 in. by 16 in.?

12. The capital of a company A is $\frac{2}{5}$ths that of a company B: if company A declare a dividend of 10%, what dividend could be declared by B with twice the net profits?

13. The simple interest on a certain sum of money was £50 in 6 years: in what time would the interest on a sum half as large again reach the same amount, at the same rate of interest?

14. Two equal sums of money are put out at simple interest, and both obtain the same return: in one case the rate per cent. per annum was 8 and the time 6 months; in the second case the time was 9 months. What was the rate per cent.?

15. The number of swings made by a simple pendulum in a given time varies inversely as the square root of its length: if a pendulum 9 ft. long makes 60 swings in a certain time, how many more would be made in the same time by a pendulum only 4 ft. long?

16. If 10 men reap a field in 24 hours, in how many hours can 8 men reap it?

17. 5 men can mow 4 acres in 5 hours: how long would it take 3 men to mow 8 acres?

18. The carriage of 80 tons 4 cwt. for 50 miles is £11. 4*s.*: what would be the carriage of 401 tons for 20 miles at the same rate per ton per mile?

19. 8 gas burners in 32 hours use 1000 cub. ft. of gas: how long would the same quantity last with 6 burners using it?

20. A tank full of water is emptied by a pipe having a sectional area of 14·4 sq. cm. in 1·6 hours. In what time could it be emptied by a pipe having a sectional area of 18 sq. cm.?

Examination of Quantities for Proportion.

The following are examples in which tests are made with the object of finding out if one quantity is proportional, directly or inversely, to another.

EXAMPLE 1.—*In the table given below, discuss the connexion between the temperature at which water boils and the pressure of the air:*

Pressure in mm. - - - -	680	700	720	740
Temperature in degrees Centigrade -	96·92	97·72	98·49	99·26

Here both quantities increase together. If the temperature and the pressure were proportional, equal changes in pressure would mean equal changes in temperature. Plotting a table of differences, we find

Difference in pressure	20	20	20
,, temperature	0·80	0·77	0·77.

The last pair of differences might suggest however proportionality, but with an error in the first.

$\therefore$ evaluate $\frac{740}{99{\cdot}26}$, *i.e.* 7·45,

and $\frac{720}{98{\cdot}49}$, *i.e.* 7·31,

and $\frac{700}{97{\cdot}72}$, *i.e.* 7·16.

These results are not equal, and the quantities are therefore not proportional.

EXAMPLE 2.

	1	2	3	4	5	6
Diameter of wire in inches	0·500	0·400	0·300	0·040	0·020	0·0100
Resistance in ohms per 1000 ft.	0·040216	0·062837	0·11171	6·2837	25·135	100·541

Note columns (2) and (4). It is seen that decreasing the diameter to $\frac{1}{10}$th the original increases the resistance 100-fold.

Test, therefore, if the resistance is inversely proportional to the square of the diameter. Form a new row in which the number in row (2) above is multiplied by the square of the number in row (1). The result obtained is found to be

1	2	3	4	5	6
0·010054	0·01005392	0·0100539	0·0100539	0·0100540	0·0100541

From the equality of the products it is inferred that the resistance in ohms per 1000 ft. is inversely proportional to the square of the diameter in inches.

The example may be treated graphically. Plot the reciprocal of the resistance against the square of the diameter; if the quantities are proportional, the graph will be a straight line passing through the intersection of the reference lines.

EXERCISES LXXX. b.

Graphic.

1. The following table gives corresponding values of pressure and volume for a given quantity of air kept at constant temperature. Draw a graph showing how the reciprocal of the volume changes with the pressure. What deduction can you make from the graph?

Pressure in gr.wt. per sq. cm.	600	650	700	750	800	850	900	950
Volume in c.c.	37·26	34·40	31·94	29·81	27·94	26·30	24·84	23·50

Find also from your graph (*a*) the pressure when the volume is 30 c.c., (*b*) the volume when the pressure is 825.

2. The following table gives the weight, length, breadth and thickness of a number of blocks of metal ; draw a graph connecting the weight with the volumes ; state which of the blocks are probably made of the same substance :

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)
Weight in gr.	1382	634	1268	1760	518·4	880	3857
Length in cm.	8	6	12	10	4·5	20	14
Breadth in cm.	6	4	8	10	4	20	12
Thickness in cm.	4	3	6	8	4	1	2

	(*h*)	(*i*)	(*j*)	(*k*)	(*l*)	(*m*)
Weight in gr.	1351	1910	862	1980	1555	720
Length in cm.	8·5	12	7	15	9	10
Breadth in cm.	8·5	12	7	15	8	10
Thickness in cm.	8·5	6	2	4	3	1

3. By the use of a suitable graph, show that the average number of yards per lb. of bare copper wire is inversely proportional to the square of the diameter :

Diameter in inches - -	0·128	0·064	0·048	0·036	0·028
Average no. of yds. per lb.	6·7	26·9	47·9	84	140·5

4. The table below gives the average resistance in ohms per 1000 yds. of vertalin wire, and also the average number of yards per lb. Show that the two sets of numbers are proportional :

Average number of yds. per lb. - -	3·085	3·84	4·88	6·37	8·65
Approx. resistance in ohms per 1000 yds.	39·65	48·9	62·1	80·9	110·1

5. The following table gives the maximum current in ampères which can be obtained from accumulators having different numbers of plates, also the capacity in ampère hours.

Show that the maximum discharge and the capacity are proportional. What would the quantities be for cells of 9 plates?

No. of plates -	3	5	7	11	15	21
Max. discharge	1·3	2·6	4·0	6·5	9·0	13
Ampère hours	9	18	27	45	63	90

ampères.

6. The following table gives the length of some of the longest rivers in the world, together with the areas of their basins.

Draw graphs connecting the areas of the basins with (*a*) the lengths of the rivers, (*b*) the square of the length of the rivers.

Make whatever inference you can from your graphs:

Name of River.	Length of River in hundreds of miles.	Area of Basin in thousands of square miles.
Volga - - -	23	579
Danube - -	18	315
Yang-tse Kiang -	30	700
Yenisei - - -	29·5	970
Nile - - -	37	1120
Congo - - -	28·8	1428
Amazon (proper) -	35	2833
La Plata - -	24	1200

7. The electric resistance of a piece of wire is proportional to its length, inversely proportional to the square of the section, and depends also on the nature of the substance of the wire.

Find graphically which of the wires below are made of the same

material by plotting the resistance against the product of the length and the inverse square of the diameter:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Resistance in ohms	570	19·5	131	64·6	1140
Length in yards -	1000	500	1000	1000	500
Diameter in inches	0·0360	0·20	0·108	0·022	0·0180

	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
Resistance in ohms	79·5	546	450	203	942
Length in yards -	500	1000	2500	1000	1000
Diameter in inches	0·098	0·006	0·064	0·0124	0·028

8. The pressure exerted by a given gas is proportional to its absolute temperature directly, and to its volume inversely.

Complete the following table, connecting the pressure, volume and temperature of a certain amount of gas:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
Pressure - -	1000	?	?	900	600	?
Volume - -	50	80	90	100	?	200
Temperature (abs.)	300	400	500	?	700	800

9. The following is an extract from the catalogue of a dealer in scientific apparatus:

EVAPORATING BASINS.

Diameter - -	$5\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$ inches,
Capacity - -	18	30	45	70	90 ounces,
Price - -	2/3	3/-	4/-	5/-	5/6 each.

Show graphically that the capacity is nearly proportional to the cube of the diameter, but that the price is not.

What is (*a*) the capacity of an 8 in. basin, (*b*) the diameter of a basin of 50 oz.?

10. The following are particulars respecting some small dynamos :

	Dimensions in inches.	Weight in lb.
(*a*)	$4\frac{1}{2} \times 4\frac{1}{2} \times 5$	4
(*b*)	$8\frac{1}{2} \times 5 \times 5$	8
(*c*)	$9\frac{1}{2} \times 7 \times 7$	19
(*d*)	$11 \times 8 \times 9$	25
(*e*)	$12\frac{1}{2} \times 9 \times 9$	40

Show that in four of these the weight is proportional to the product of the dimensions.

EXERCISES LXXXI.

Proportion.

1. The cost of 4 tons 16 cwt. of rails is £18 : what will have to be paid for a truck load of rails weighing 12 tons when the cost per ton of the first to that of the second is as 3 : 4 ?

2. 15 men in 10 days of 9 hours dig a trench 30 yards long, 1·8 yards wide and 7·2 feet deep. In how many days of 8 hours each would 24 men dig a trench 40 yards long, 1·5 yards wide and 8 feet deep ?

3. The wages of 17 men for 12 days are £56. 12*s.* : what should be the wages of 24 men doing similar work for 9 days ?

4. The provisions in a besieged town of 4500 people are sufficient to last 54 days, but after 21 days 1000 more people take refuge in it. How long will the provisions now last, at the same rate as before ?

5. In supplying coal gas from a gas works, the number of cub. ft. of gas, forced per hour through a given length of pipe, is proportional to the square root of the pressure of the gas.

If 8873 cub. ft. of gas can be supplied in 60 min. by a pipe 5000 yds. long and 9 in. in diameter, when the pressure is estimated as 3·000 in. by a water gauge, what time would be taken in delivering 21,635 cub. ft. when the pressure is 2·000 in. by the water gauge ?

6. In what time will 323·2 gr. of silver be deposited by a current of 33 ampères, if 1213·056 gr. be deposited in 12 hrs. by a current of 36 ampères?

7. Two metal plates of the same substance are weighed and partly measured, with the following results:

First metal plate, 18·03 cm. long, 15·06 cm. broad, thickness 1·12 cm., weight 3060 grams.

Second metal plate, 9·24 cm. long, 7·12 cm. broad, thickness not observed, weight 1530 grams.

What is the thickness of the second plate? State your answer as accurately as the data allow.

8. A contractor undertakes to complete a contract in 42 weeks, by employing 64 men. At the end of 28 weeks he finds that only one half of the work is done. How many more men must be put on so as just to complete the contract in the specified time?

9. If 2000 soldiers have provisions for 95 days, how many men must be sent away in order to make the food last 100 days?

10. A ship with a crew of 32 sailors has food enough for 84 days, but at the end of 42 days 16 sailors are picked up: how long will the food now last, supposing the rations to continue unchanged?

11. If 12 cub. ft. of coal gas can be obtained from Cannel coal for every 10 cub. ft. contained from the same weight of Newcastle coal, how many tons of Cannel coal will be required to produce 1,050,000 cub. ft. of coal gas, if 15 tons of Newcastle coal are required to produce 135,000 cub. ft. of the gas?

12. Since the earth is nearly spherical, the top of the mast of a ship at sea can be seen before the lower part, when viewed from a point on the sea level. The distance at which the top of the mast can just be seen is proportional to the square root of the height of the mast.

If a mast 54 ft. high can be seen at a distance of 9 miles, find (*a*) the distance at which a mast 16·67 ft. high can be seen; (*b*) the height of the mast of a ship which can just be seen at a distance of 6 miles.

13. When 1800 men have provisions for 114 days, they are joined by 400 men. How long will the provisions now last?

14. Assuming that the cost of carriage is proportional to the weight, and also to the distance carried:

(*a*) What weight could be carried 2 miles 7 fur. at the same cost as 3 cwt. 1 qr. 27 lb. could be carried 48 miles 7 fur.?

(*b*) How many miles could 11 tons be carried for £5. 10*s*. if 3 tons be carried 54 miles at a cost of £2. 0*s*. 6*d*.?

(*c*) If the carriage of 3 cwt. 3 qr. 14 lb. for 104½ miles cost £1. 14*s*. 10*d*., what should be charged for the carriage of 5 tons for a distance of 93 miles?

15. 8 men can finish a piece of work in 75 days by working 10 hours a day. How many more men must be put on in order to finish the work in 40 days without increasing the number of hours?

16. If a lime kiln can burn on an average 2688 cub. ft. of lime in 12 days, find the least number of kilns to be employed if it be required to burn 97342 cub. ft. of lime in not more than 7 days.

17. On account of capillary attraction, the rise of water up a glass tube of narrow bore is inversely proportional to the diameter of the bore. The rise is 3·14 cm. when the diameter is 0·0513 cm.; what is (*a*) the rise when the diameter is 0·0273 cm., (*b*) the diameter if the rise is 4·27 cm.

18. The pressure required to compress a given quantity of air kept at constant temperature to a given volume varies inversely as the volume. If it requires a pressure of 786·4 grams weight per square centimetre when the volume is 38·90 c.c., find (*a*) the pressure when the volume is 25·89 c.c., (*b*) the volume when the pressure is 3 kilograms per square centimetre.

19. If 4 lb. of bread cost 5½*d*. when wheat is 44*s*. a bushel, what will 6 lb. of bread cost when wheat is 48*s*. a quarter, assuming the other factors in the cost to vary according to the price of the wheat?

20. If it cost as much to feed 3 men as 4 boys; and if for 3 boys the cost is 19*s*. 2¼*d*. a week, how much a week will it cost for 51 men?

21. When the cost of food for 84 men for 95 days amounts to £598. 10*s.*, what will be the cost of feeding 110 men for 33 days?

22. In India, where artificial irrigation is generally required, a field of rice at Mysore, 25·3 ac. in extent, requires the water from a source supplying 120·6 cub. ft. per sec.; at another field where the soil is different 35 cub. ft. are required for every 30 cub. ft. at Mysore. What flow per sec. will be required to irrigate the latter field, if its extent be 36·3 ac.? (State answer to nearest tenth of a cubic foot.)

23. The pressure a cylindrical boiler can sustain without bursting is directly proportional to the thickness of the boiler plates and inversely proportional to the diameter of the boiler. A boiler of 3·5 ft. diameter, with plates of $\frac{1}{2}$ in. thickness, can withstand a pressure of 573 lb. to the sq. inch:

(*a*) What is the diameter of a cylindrical boiler, plates $\frac{5}{8}$ in. thickness, which can just withstand a pressure of 418 lb. to the sq. inch?

(*b*) What pressure can a boiler, plates $\frac{3}{8}$ in., sustain if its diameter is 2 ft.? (State the answer to the nearest pound per sq. in.)

24. A ship's crew of 18 hands had food enough on Jan. 1st, 1905, to last 48 days, but on that day they picked up 6 shipwrecked sailors: how long will the food last?

25. When wheat is 15*s.* a bushel, 8 men can be fed for 12 days at a certain cost: for how many days can 6 men be fed for the same cost when wheat is 12*s.* a bushel?

26. A contractor had engaged to make a railway 58·5 miles long in 40 weeks, and began by employing 2160 men. At the end of 13 weeks, 19·5 miles were completed. How many men can be dismissed if he only just wishes to complete the work at the end of the 40 weeks?

27. The following table gives the velocity acquired by a body in falling from rest through different heights:

Fall	1	4	25	100	400	feet
Velocity	8	16	40	80	160	ft. per second

What is the connexion between the fall and the velocity acquired? Find also

(*a*) the velocity acquired during a fall of 375 ft.;

(*b*) the fall necessary in order that a velocity of 98 ft. per second may be acquired.

28. The speed, or velocity, with which waves move on the surface of the deep sea is proportional to the square root of the length of the wave. If the velocity of a wave 50 ft. in length be 12·85 ft. per sec., what will be the velocity of a wave 120 ft. in length? (State the answer true to 0·01 of a ft. per sec.)

29. With the assumption made as in Ex. 6, find the time in which a current of 16·32 ampères would deposit 10·83 grams of silver.

30. If a rod of larch can support a load of $10{\cdot}2 \times 10^2$ lb. weight per sq. in., what can it support in kilog. per sq. cm.?

31. The corrosion of iron and steel on account of 'rusting' is proportional to the surface. When cast iron is placed in clear sea water the amount of corrosion is 0·0635 lb. per year for every square foot of iron exposed, but in fresh water the amount is 0·0113 lb. per year for every square foot: find the amount of corrosion taking place in 4 years in a pipe, 12·62 cm. radius, lying in sea water, if a pipe of the same length, but 10 cm. diameter, lying in fresh water corrodes to the extent of 30·3 kilograms in 3 years.

32. Compare the thickness of slate, known as doubles, size 13 in. × 6 in., and 1000 of which weigh 15 cwt., with that of slate known as Countesses, size 20 in. × 10 in., 200 of which weigh 8 cwt.

33. In a non-manufacturing town the consumption of water is 17·5 gallons of water per person per day; the reservoir contains 200 days' supply, and receives the water from an area of 10,000 acres. What would be the number of days which a reservoir would supply a manufacturing town of the same population, but in which each person consumed 24 gallons daily, and in which the reservoir received the water from an area of 18,000 acres. (Assume the same conditions in both towns with respect to rainfall, evaporation, and so on.)

34. If 12·3 sq. yd. of a fine sand filter can filter 1000 gallons of water in 24 hours, how many sq. yd. of a similar filter would be necessary to filter 13,500 gallons of water in 15 hours?

35. Two pieces of copper wire are of the same length; one weighs 0·632 gram and its section is 1·12 mm. in diameter; the second weighs 0·835 gram: what is its diameter? (State your answer as accurately as the data permit.)

36. The following table gives the total load that a horse, moving at different speeds, can draw in a canal boat, along a canal:

Load	520	243	153	102	52	30	19	13	9	6·5 tons
Speed	2·5	3	3·5	4	5	6	7	8	9	10 mls. per hour

Show that the load is neither inversely proportional to the speed nor inversely proportional to the square of the speed.

37. The length of 1° on the meridian is 68·70 miles near latitude 0° and 59·23 mls. near latitude 60°. If a certain length, measured along the meridian near latitude 60°, be 0·08764 of a degree, what fraction of a degree would the same length be near the equator?

38. A commences business with a capital of £1875: at the end of 5 months B joins him with a capital of £1500, and at the end of 4 months more C enters with £1400. The profit at the year's end was £1116: how should this be divided between A, B and C?

39. Four people enter into a business, A with a capital of £868, B with £574, C with £794 and D with £764. A is to receive a fixed salary of £120 from the profits, and the remainder is to be divided among the four in the proportion of their subscribed capital. What amount does each receive from a profit of £270 on the year's working?

40. Two persons rent a grazing marsh for £10. 12*s.* a year. One puts in 420 sheep, but at the end of 7 months puts in 96 more for the remaining months of the year; the second puts in 760 sheep for 6 months, and then withdraws 320 for the rest of the year. How should the rent be apportioned?

41. Two workmen are engaged on a piece of work for which they are to receive 2 guineas. One workman works for 3 days of 9 hours a day, and the other 3 days of 8 hours a day; but the second does as much in 2 hours as the first does in 3 hours. How should the amount be divided between the two workmen?

42. The boarding expenses of a party of 14 persons for 3 weeks were £63. What would be the expenses of a party of 8 persons for 5 weeks?

43. The time taken in emptying a canal lock by a sluice is proportional to the square root of the head of water, and inversely proportional to the area of the sluice. If, in a certain lock, the time taken is 2 min. 20 sec., with a certain sluice and a head of 15 ft., find the time taken in a similar lock, with a sluice the area of which is half as much again, but in which the head is only 14 ft. (State the time to the nearest second.)

44. Two reels of copper wire are weighed: the first weighs 1·309 lb. including the reel, the wire is No. 13 wire gauge (diameter 0·092 in.); the second weighs 6·66 lb. and wire is No. 30 (diameter 0·0124 in.). If the length of wire on the first is known to be 46·52 ft., what is the length of wire on the second, the reels in each case weigh 0·13 lb.?

45. If a lath be supported at its ends, and loaded at the centre, it bends, and its centre is lowered by an amount proportional to the load, inversely proportional to the cube of the thickness, inversely proportional to the breadth, and directly proportional to the cube of the length of the lath.

If a load of 2053 grams produce a lowering of 5·21 mm. at the centre of a lath 100 cm. long and 0·420 cm. thick, find (*a*) the lowering at the centre when a lath of the same wood and breadth is used, but the length of which is 122·0 cm. and the thickness 0·260 cm., the load being 1600 grams; (*b*) the load necessary to produce a deflexion of 1·20 cm. in a lath similar to that used in (*a*), but 96·0 cm. in length instead of 122·0 cm. State your answer true to the accuracy possible from the given data.

46. The strength of a beam or girder is inversely proportional to its length, directly proportional to its width, and directly proportional to the square of its depth.

A wooden beam, of rectangular cross-section, is 4 ft. long, 2 in. wide, 3 in. deep, and it rests upon supports at its ends. The breaking load is 2000 lb. placed at its centre. What would be the breaking load of the same beam (*a*) when the supports are brought 2 in. each from the ends and (*b*) when the beam is placed so as to be 3 in. in width and 2 in. deep?

47. A standard beam of wood, 1 in. wide, 1 in. deep and 12 in. long, can safely carry 420 lb. at its centre. What must be the depth of a beam of the same wood, $2\frac{1}{2}$ in. wide and 5 ft. long, to carry safely a load of 1·5 ton? (See Ex. **46.**)

48. If wind blowing at 15 mls. per hour exerts a force equal to the weight of 270 lb. on a wall 40 ft. long and 12 ft. high, find the velocity of the wind if the force exerted on a wall 54 ft. long and 16 ft. high be 1728 lb., assuming that the force exerted is proportional to the square of the velocity.

49. When waves travel on the free surface of shallow water (the length of the wave being much greater than the depth of the water) the velocity of the wave is proportional to the square root of the depth. If a long wave take 4·5 sec. in describing a distance of 36 ft. when the depth is 2 ft.:

(*a*) How long will a wave take in describing a distance of 125 ft. when the depth is 5·12 ft.? (State the answer true to 2 significant figures.)

(*b*) What would be the depth if the wave moved with a velocity 4·62 metres per sec.?

58. AREAS OF RECTILINEAR FIGURES.

EXAMPLE 1.—*To find the area of a trapezium* ABCD.

Suppose in Fig. 65 AB and DC are at right angles to AD, and that the lengths of AB, DC, and AD are known.

Draw BB′, CC′ both parallel to AD meeting DC produced and AB at B′ and C′ respectively.

Then ADB′B is as much greater than ABCD as ABCD is greater than ADCC′; and therefore

the area ABCD is half the sum of the areas of AC′CD and ABB′D;

$$\therefore \text{ area ABCD} = \frac{\text{AB} + \text{DC}}{2} \times \text{AD}.$$

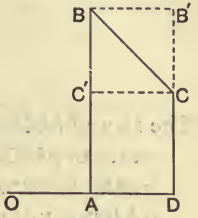


Fig. 65.

If AB $= y_1$, DC $= y_2$, and if A is at a distance x_1 from a certain point O along DA, while D is at a distance x_2; then

$$\text{the area ABCD} = \frac{y_1 + y_2}{2}(x_2 - x_1).$$

Example 2.—*To find the area of a small rectilinear plot from field-book readings taken with reference to a single base line.*

Suppose ABCDE (Fig. 66) represents the plot, O*abecd* the base line, and that

Case I.—*All the points* AB... *are on the same side of the base line.*

Let the field-book entry be as follows:

y_D	x_D
y_C	x_C
y_E	x_E
y_B	x_B
y_A	x_A
	O

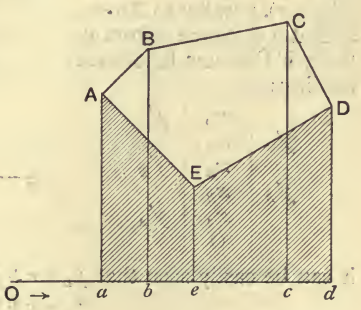


Fig. 66.

Alter the order of the readings to correspond to the case where the order is that in which the corners would be met by

a man going right round the field; and, say, starting at A, the entries would now be

y_E	x_E
y_D	x_D
y_C	x_C
y_B	x_B
y_A	x_A
	O

in which the order is ABCDEA.

The area of ABCDE

$$= \text{area of } a\text{ABCD}d - \text{area of } d\text{DEA}a$$

$$= a\text{AB}b + b\text{BC}c + c\text{CD}d - d\text{DE}e - e\text{EA}a$$

$$= \tfrac{1}{2}\{(x_B - x_A)(y_B + y_A) + (x_C - x_B)(y_C + y_B) + (x_D - x_C)(y_D + y_C) \cdot$$
$$- (x_D - x_E)(y_E + y_D) - (x_E - x_A)(y_E + y_A)\}$$

$$= \tfrac{1}{2}\{(x_B - x_A)(y_B + y_A) + (x_C - x_B)(y_C + y_B) + (x_D - x_C)(y_D + y_C)$$
$$+ (x_E - x_D)(y_E + y_D) + (x_A - x_E)(y_A + y_E)\}. \quad \ldots\ldots\ldots\ldots\text{(i)}$$

CASE II.—*The points observed are on different sides of the base line.*

Suppose in the preceding case that the field-book entry had been with reference to a line like O$a'b'e'c'd'$ (Fig. 67), parallel to O$abecd$, and at a distance r from it, then, if the new field-book readings are

	x_D	y'_D
y'_C	x_C	
	x_E	y'_E
y'_B	x_B	
	x_A	y'_A
	O	

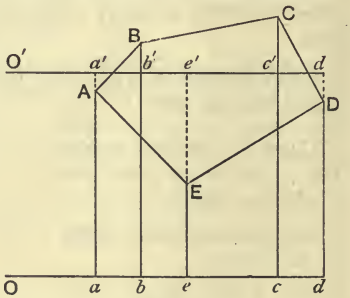


FIG. 67.

it can be easily seen that $y_B = r + y'_B$,

$y_C = r + y'_C$,

but that $y_A = r - y'_A$,

$y_D = r - y'_D$,

$y_E = r - y'_E$.

If these values be substituted in formula (i), it will be noticed that the terms in r all *cancel*, and therefore that the expression is just of the same form as in (i), only the distances from the base line are counted as *negative* if the points are to the right of the base line, and positive if to the left of the base line.

EXAMPLE 3.—*Find the area of a plot from the following field-book readings:*

	300	20
100	250	
60	200	
	150	80
30	120	
	0	

Here the right order is clearly obtained by first taking the points on the left going up the base line, and then the points on the right coming back along the base line; and the area expressed in sq. links

$$= \tfrac{1}{2}\{(200-120)(60+30)+(250-200)(100+60)$$
$$+(300-250)(-20+100)+(150-300)(-80-20)$$
$$+(120-150)(30-80)\}$$
$$= \tfrac{1}{2}\{80\times 90+50\times 160+50\times 80+150\times 100+30\times 50\}$$
$$= \tfrac{1}{2}\{7200+8000+4000+15000+1500\}$$
$$= \tfrac{1}{2}\{35700\}.$$

The area is therefore 0·1785 acre.

The expression

$$\tfrac{1}{2}\{(x_2-x_1)(y_2+y_1)+(x_3-x_2)(y_3+y_2)$$
$$+(x_4-x_3)(y_4+y_3)+\dots(x_1-x_n)(y_1+y_n)\}$$

may be written also in a form which is sometimes more convenient for calculation. Thus:

$$-\tfrac{1}{2}\{(x_1y_2-y_1x_2)+(x_2y_3-y_2x_3)$$
$$+(x_3y_4-y_3x_4)\dots+(x_ny_1-y_nx_1)\};$$

in this case the expression can be set down thus,

$$-\tfrac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix}$$

and the several components $(x_1y_2 - y_1x_2)$ are found by cross multiplication.

EXAMPLE 4.—*Find the area of Fig.* 68 *from the data given:* Oa = 10, aA = 10; Ob = 14, bB = 20; Oc = 30, cC = 20; Od = 23, dD = 10; Oe = 20, eE = 5 units of length.

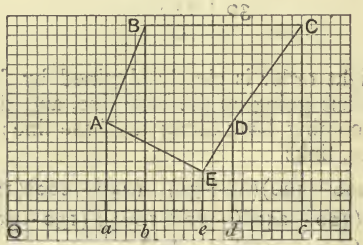


FIG. 68.

$$\text{Area} = -\tfrac{1}{2}\begin{vmatrix} 10 & 10 \\ 14 & 20 \\ 30 & 20 \\ 23 & 10 \\ 20 & 5 \\ 10 & 10 \end{vmatrix}$$

..... $(10 \times 20 - 10 \times 14) = 60$

..... -320

the cross multiplication.

..... -160

products are shown.

..... -85

to the right.

..... 150

$$\therefore \text{ area} = \tfrac{1}{2}\{-60 + 320 + 160 + 85 - 150\} = \frac{355}{2}$$

$$= 177{\cdot}5 \text{ units of area.}$$

Using the other form for the expression for the area,

$$\begin{aligned}\text{area} &= \tfrac{1}{2}\{(14-10)(20+10)+(30-14)(20+20)\\ &\qquad +(23-30)(10+20)+(20-23)(5+10)\\ &\qquad +(10-20)(10+5)\}\\ &= \tfrac{1}{2}\{120+640-210-45-150\}\\ &= \tfrac{1}{2}(355)\\ &= 177{\cdot}5 \text{ units of area.}\end{aligned}$$

EXERCISES LXXXII.

Find the areas of the plots given by the following field-book readings (expressed in links):

1.

100	500	
80	400	
60	200	

2.

	500	100
80	400	
60	200	

3.

20	1900	
10	1600	
30	1200	

4.

	1900	20
	1600	10
30	1200	

5.

	600	5
5	400	
20	300	
	200	10
10	100	

6.

	200	20
5	160	
	150	20
20	120	
10	80	

7.

50	250	
160	200	
40	150	
80	100	
10	80	

8.

	5000	10
10	4000	
	3000	5
10	2000	
8	1000	

9. Find the areas of the Figs. 69, (*a*) to (*f*), by choosing base lines and marking off readings. Check your results by (i) varying the base line, (ii) by the use of squared paper.*

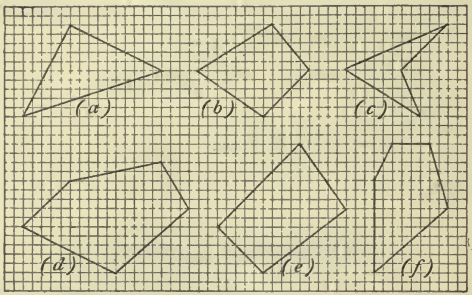


FIG. 69.

When the areas are not bounded by strictly rectilineal figures, the method can only give approximate results.

10. Find readings after the manner of field-book readings for the more salient points of Figs. 70, (*a*) to (*c*); calculate the areas from the readings and check by the use of squared paper.

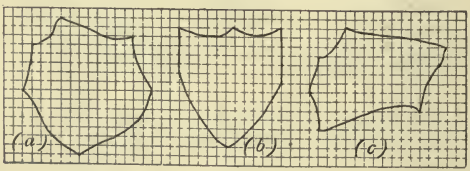


FIG. 70.

11. At equal distances apart draw a number (seven at least) of parallel ordinates across Fig. 71 (*a*), join the ends of the ordinates, calculate the area of the rectilineal figure so formed, note that this area must be nearly equal to that of the given figure and hence get an approximation to the area of the figure.

* In Exx. **9, 10, 11,** and **12** it may be desirable for the student to draw larger diagrams for himself.

12. Obtain an approximation to the area of Fig. 71 (*b*) after the method suggested in Ex. **11.**

13. Find the area of the triangle, of which the angular points are (0, 0), (x_B, y_B), $(b, 0)$. Prove from your result that the **area is one half the product of the base into the altitude.**

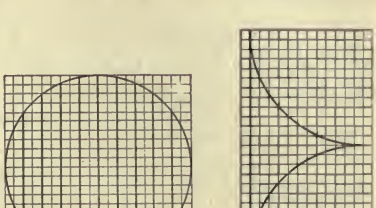


FIG. 71.

MISCELLANEOUS PROBLEMS LXXXIII.

1. The minute-hand of a clock passes over 60 divisions on the face of the clock for every 5 divisions that the hour-hand passes over, *i.e.* gains 55 divisions in one hour. The hands are together at 12 o'clock: find when next the minute-hand will be 15 divisions ahead of the hour-hand, *i.e.* when the minute-hand has gained 15 divisions.

2. Find when the hands of a clock are pointing in the same direction between (*a*) 5 o'clock and 6 o'clock (*i.e.* has gained 25 divisions since 5 o'clock), (*b*) 3 o'clock and 4 o'clock, (*c*) 10 o'clock and 11 o'clock.

3. Find when the hands of a clock point in opposite directions between the hours of (*a*) 2 and 3, (*b*) 4 and 5, (*c*) 9 and 10.

4. If a train, 80 yd. in length, and travelling at the rate of 60 miles an hour, overtakes another train, 70 yd. in length, moving at 35 miles an hour, it must do so at the rate of 25 miles an hour. Find how long such a train would take in passing the second completely, *i.e.* in gaining (80+70) yards on the second.

5. A train, 86 metres long, and moving at 40 kilometres an hour, overtakes another train, 78 metres in length, moving at 26 kilometres an hour: how long does it take completely to pass it?

6. If the trains in Question 5 had been moving in opposite directions, what time would have been taken in passing one another?

7. If the waste-pipe can empty a bath in 5 min., and the hot-water pipe can fill it in 15 min., and the cold-water pipe in 10 min., calculate how much water enters or leaves the bath in 1 min. when

(*a*) The hot-water pipe alone is open.

(*b*) The hot and cold-water pipes are open.

(*c*) The three pipes are open.

Find also (*d*) the time taken in emptying the bath if all the pipes are open.

8. Two pipes are such that a cistern fed by them could be filled by each separately in 7 min. and 11 min. respectively: in what time would the cistern be filled by the two taps together?

9. A can give B 40 yd. start in a quarter-mile race; B can give C 40 yd. start in a similar race. If A can run the quarter-mile in 58 sec., how long would C take to run it, assuming B and C to run the full quarter at the same rate as they ran the portions mentioned?

10. A can just give B 10 yd. start in a 100 yd. race; B can just give C 15 yd. start: what is the utmost start A can give C?

11. One person starts from a town A to meet another person coming from a town B 12 miles off. If the first person walks at 3 miles an hour, the second at 4 miles an hour, and if they start at the same time (5 p.m.), at what rate do they approach one another, and when and where will they meet?

12. The period of revolution of the earth round the sun is 365 days, that of Mars (in the same direction) is 687 days. Show that, if the motion was uniform, the apparent year of Mars with regard to the earth would be 770 days (*i.e.* show that this would be the time in which the earth would gain one revolution with respect to Mars, so that the two planets would again be in the same straight line with the sun).

13. The apparent year of Venus as seen from the earth is 584 days: show that the actual year (on the same supposition as in the preceding question) is about 225 days.

14. Find also the apparent year of Venus, if observed from Mars.

15. A person mixes or blends two kinds of tea; one kind is worth 1*s.* 4*d.* per lb., the other worth 2*s.* per lb.; the mixture is worth 1*s.* 7*d.* per lb.; so that for every lb. of tea of the first kind there is a gain of 3*d.*, but for every lb. of the second kind there is a loss of 5*d.* Show that 5 lb. of the first kind must be mixed with every 3 lb. of the second kind. Find the amount of tea of the second kind to be mixed with 1 ton 5 cwt. of the first kind, and what gain per cent. on the cost price would result if the blend be sold at £2. 10*s.* per quarter.

16. How would you blend two teas worth respectively 1*s.* $3\frac{1}{2}d.$ and 2*s.* 1*d.* per lb. so that the mixture is worth 1*s.* $7\frac{1}{2}d.$ per lb.?

17. In what proportion must a merchant mix one kind of tea at 3*s.* with another at 1*s.* 6*d.* in order that by selling the mixture at 2*s.* 8*d.* he may make a profit of 25 per cent.?

18. A train leaves London for Brighton at 9 a.m., travelling at a uniform rate of 15 miles an hour. An express train leaves Brighton for London at 10 a.m. and travels at a rate of 40 miles an hour. How far apart are the trains when the train for London starts, at what rate do they approach each other, at what time will they pass each other and at what distance from London, the distance from London to Brighton being 50 miles?

19. A hare sees a hound 176 yards away from her, and scuds off in the opposite direction at a speed of 12 miles an hour; thirty seconds later the hound perceives her and gives chase at a speed of 18 miles an hour. How soon will he overtake the hare, and at what distance from the spot whence the hare took flight?

20. A starts from London for Epsom, distant 14 miles, walking at the rate of 3 miles an hour. B starts from London on the same road $1\frac{3}{4}$ hours later, driving 8 miles an hour. At what distance from Epsom will B overtake A?

21. The first of a series of cogged wheels, working into each other in a straight line, has a certain number of teeth; the number of teeth in the second is to that of the first as 6 : 7; of the third to the second as 5 : 6; and of the fourth to the second as 2 : 3. If

the wheels are set in motion, how many revolutions must each wheel make before they are simultaneously in their original positions?

22. By selling goods for a certain sum a man gains 5 per cent. on the cost price. If he had sold them for 3*s*. more he would have gained 6 per cent. Find their cost price.

23. A cistern which can be filled by one tap in 28 hours is emptied by four others. The first of these alone would empty it in 10 hours, the second in 12 hours, the third in 15 hours and the fourth in 21 hours. Supposing the cistern to be full, in what time would it be emptied if all the five taps were set running together?

24. There is a well containing 750 gal. of water; two pumps raising 20 and 30 gal. per minute respectively are employed to empty it, while it is constantly supplied by a spring which can refill it in half an hour. The two pumps work together for 15 minutes, when that of larger capacity ceases work for 10 minutes; the two pumps then work together until the well is empty. How long will each pump have been employed?

25. Two boats start for a race at 3 o'clock. The race is over at $6\frac{2}{5}$ minutes after 3, the losing boat being 40 yards behind at the finish: at 4 minutes past 3 this boat was 700 yards from the winning post. Find the length of the course, and the speed (supposed uniform) of each boat in miles per hour.

26. A tricycle, going at the rate of 5 miles an hour, passes a milestone, and 14 minutes afterwards a bicycle, going in the same direction at the rate of 12 miles an hour, passes the same milestone: find when and where the bicycle will overtake the tricycle.

27. Two clocks are together at 12 o'clock: one loses 7 sec. and the other gains 8 sec. in 12 hours: when will the faster be half-an-hour before the other, and what o'clock will it then show?

28. Two watches are together at 12 o'clock noon. One gains 75 sec. per hour and the other loses 45 sec. When will they be together again at 12 noon?

29. If A, B and C could reap a field in 18 days; B, C and D in 20 days; C, D and A in 24 days; and D, A and B in 27 days: in what time would it be reaped by them altogether?

30. If 4 men can do a piece of work in 7 days, which 5 women can do in 8 days, or 7 boys can do in 10 days: how long will it take 3 men, 2 women and 3 boys to do the same if they work together?

31. A can do a piece of work in 11 days, B in 20 days and C in 55 days: how soon can the work be done if A is assisted by B and C on alternate days?

32. If 6 women and 3 boys weed $\frac{7}{12}$ of a field in 4 days, how long will it take 4 women and 1 boy to weed the rest of it, the work of 4 women being equal to that of 5 boys?

33. A certain field could be reaped by 7 men in a certain time, and 5 boys could do as much as 2 men. Find how many boys would be required, in addition to 30 men, for the reaping of a field of twice the size, in a third part of the time.

34. An astronomical clock has its dial divided into 24 divisions instead of 12, and the small hand goes round once in 24 hours, the large hand going round once every hour. The 24th hour is noon. Find when the hands are at right angles to one another between 24 and 1, and find also the interval between two successive meetings of the hands.

35. Strong spirit is mixed with inferior spirit, valued at 5*s.* per gallon, in the proportion of 6 to 1. The mixture is then worth 9*s.* the gallon. Find the value of a gallon of the strong spirit.

36. A certain turnip-field can be hoed by 10 women in 4 days, or by 6 boys in 10 days, or by 2 men in 12 days. One man, three boys and three women are employed: what is the total cost of hoeing the field, a man's daily pay being 2*s.* 8*d.*, a woman's 1*s.* 8*d.*, and that of a boy 1*s.*?

37. A milkman pays 1*s.* 1*d.* per gallon for his milk; he adds water and sells the mixture at 2*d.* per pint, thereby making altogether 40 per cent. profit. Calculate the proportion of water to milk his customers receive.

38. A certain piece of work was done by A, B and C, all working together, with the exception that B left off half an hour before the completion of the work. A alone could have done the whole in

$7\frac{1}{2}$ hours, B in $8\frac{1}{3}$ hours, C in 10 hours. In what time was the work done?

39. I row against a stream flowing $1\frac{1}{2}$ miles an hour to a certain point, and then turn back, stopping two miles short of the place whence I originally started. If the whole time occupied in rowing be 2 hr. 10 min., and my uniform speed in still water be $4\frac{1}{2}$ miles an hour, find how far upstream I went.

40. A and B start from the same point to run in opposite directions round a circular race-course, 9755 feet in circumference, A not starting until B has run 105 feet. They pass each other when A has run 4850 feet. Which will first come round again to the starting point (their speeds being uniform throughout) and what distance will they then be apart?

41. A and B start at the same time from London to Blisworth. A walking 4 miles an hour, B riding 9 miles an hour. B reaches Blisworth in 4 hours, and immediately rides back to London. After 3 hours' rest he starts again for Blisworth at the same rate. How far from London will he overtake A, who has in the meantime rested for 6 hours?

42. With what two quantities of spirits, that cost me 14*s.* 6*d.* and 17*s.* 6*d.* a gallon respectively, did I make a blend of 12 gallons, i by selling the mixture at 16*s.* 3*d.* a gallon I should have incurred a loss of $2\frac{1}{2}$ per cent.?

43. A man rode a bicycle from A to B, 54 miles, at an average rate of 8 miles an hour; another man started from A on horseback half an hour after the bicyclist and arrived at B 15 minutes before him. Find the ratio of their speeds.

REVISION EXERCISES LXXXIV.

Miscellaneous. (Mainly from examination papers.)

A 1. Divide £720. 11*s.* 3*d.* by £2. 4*s.* $5\frac{3}{4}d.$

2. A man bought 10 tons 3 cwt. of sugar at £18 a ton and sold it at $2\frac{1}{4}d.$ a pound. What profit did he make?

3. (*a*) Express each of the following numbers as the product of prime factors: 468, 561, 1547 and 2431.

(*b*) Find the least square number which is divisible by 7, 8 and 9.

4. Simplify $\dfrac{2\frac{1}{3}+\frac{3}{4}}{15\frac{1}{2}-1\frac{5}{8}}+1\frac{2}{3}$ of $2\frac{4}{15}$.

5. (*a*) Multiply 1·183 by 2·145 and divide the result by 0·0845.

(*b*) How much butter at 2 francs 25 centimes a kilogram would cost 9 francs 90 centimes?

6. A rectangular plot of building land with a frontage of 34 feet and a depth of 125 feet was let on a building lease at the rate of £60 per acre. Find the rent of the plot to the nearest penny.

7. Find to three places of decimals the square root of 1314·151. Find a whole number between 1000 and 2000 which is a perfect square and which is divisible by 13.

8. The rateable value of a certain town is £74360. Find (to a farthing) the smallest rate per pound which must be levied in order to raise a sum of not less than £3115.

9. Find as accurately as the data allow, the weight of a cubical block of metal, the edge of which is 3 yards, 2 feet, 3·1 inches long (true to the nearest hundredth of an inch), while a cubic foot of the metal weighs 8·8764 cwt. (true to the fourth decimal figure).

In your answer show up all the figures which are accurate but no more.

10. The price of a standard troy ounce of silver on 1st January in each of the ten years (1891-1900) was (in pence)

45, 40, 36, 29, 30, 31, 28, 27, 27, 28.

Draw a curve showing its value approximately at any time during these ten years.

B 1. Find the value of 0·03125 of £3. 4*s*.

2. Extract the square root of 7 to 3 decimal places.

3. If the cost of keeping 25 horses for 64 days is £96, what will be the cost of keeping 75 horses for 28 days at the same rate?

4. The rainfall on a flat roof 20 ft. long and 12 feet wide is collected in a covered rectangular cistern 5 ft. long, 4 ft. wide and 3 ft. deep; what depth of water will be found in the cistern after a fall of 2 inches of rain?

5. If numbers are formed by arranging all the digits 2, 3, 4, 5, 6 in any order, show that each of these numbers when divided by 9 will have a remainder 2.

6. A tradesman bought 5 cwt. of tea for £37. 6*s.* 8*d.*, at what price per lb. should he retail it in order to make a profit of 25 per cent.?

7. What rate of interest do I receive if I get £82. 10*s.* by lending £500 for $5\frac{1}{2}$ years, simple interest?

8. Express in decagrams the weight of 44·7 litres at 19 grains per cubic centimetre, taking 1 gram = 15 grains. (1 litre is 1 cubic decimetre.)

9. Find, to the nearest millimetre, the value of

$$\frac{0{\cdot}1453 \times 3{\cdot}14159}{18} \text{ of } 2{\cdot}51 \text{ metres.}$$

10. A rectangular piece of paper is measured in inches and in centimetres, with the following results:

Length, 8·73 inches; 22·17 centimetres.
Breadth, 5·57 inches; 14·15 centimetres.

Find the area of the paper in square inches and in square centimetres, and hence find the number of square centimetres in a square inch. If the measurements are correct to the nearest figure in the second place of decimals, to how many places may your results be regarded as accurate?

C 1. The internal measurements of a rectangular cistern are—length 2·40 metres, breadth 1·22 metres and height 1 metre. How many litres of water will it hold?

2. On what sum will the simple interest for $1\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum be £47. 1*s.* 6*d.*?

3. Find the cost of papering a room 19 feet long, 16 feet broad and 10 feet high (one-fourth of the wall space being taken up by the fireplace, the doors and the windows) with paper which is 21 inches wide and costs $2\frac{1}{2}d.$ a yard.

4. The Navy Estimates for the year 1903 were £35,837,000, and for 1904 they were £38,328,000. Find, to the nearest tenth, the percentage increase.

5. If 36 horses can plough 54 acres of land in 15 days, how many days will 26 horses take to plough 117 acres at the same rate?

6. A plan is to be enlarged so that a plot of ground which occupied 1·5 sq. in. in the original shall occupy at least 1 sq. foot in the enlarged plan. If the original measured 6 in. by 4 in., what must the least dimensions of the enlarged plan be?

7. A centimetre being 0·3937 inch, find to four significant figures how many acres are in a square kilometre.

8. The return railway ticket from a suburb to London costs 2*s.* 2*d.*, and the annual ticket £12. 5*s.* 0*d.* How many days in the year must a man travel to save money by taking an annual ticket instead of paying for each day?

9. (*a*) One degree of longitude at the equator measuring 365,000 feet, determine the equatorial circumference of the earth in miles.

(*b*) Express as the decimal of 5 furlongs the difference in height between Mount Everest (29,002 feet) and Mont Blanc (15,732 feet).

10. How many parcels each containing 1 franc and 2 marks can be obtained for £1063. 10*s.*, when £1 is worth $25\frac{1}{5}$ francs *or* $20\frac{1}{2}$ marks?

D 1. Divide 54076539 by 63, using the method of factors and short division. Give a reason for the method by which you find the remainder.

2. A man has £108. 13*s.* 4*d.* Find how many things each costing £5. 4*s.* $2\frac{1}{2}d.$ he can buy, and how much money will remain over.

3. Find the values of

(i) $13\frac{1}{16} + 17\frac{16}{21} - 26\frac{17}{24} - 2\frac{3}{56}$,

(ii) $230{\cdot}69 \times 0{\cdot}0018 \div 0{\cdot}059$.

4. Find the values of (i) $\frac{5}{18}$ of 10 tons 7 cwt., (ii) 3·825 of 3 miles. Express the latter result in miles, furlongs and yards.

5. Find, to the nearest centime, the price of a carpet 6 metres 24 centimetres long, and 4 metres 35 centimetres broad, at 8 francs 5 centimes per square metre.

6. If I pay a cabdriver one penny per minute, and he drives $3\frac{1}{2}$ miles in half-an-hour, at what rate do I pay him per mile?

7. Find the simple interest on £3757 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. per annum.

8. Find the value of $35{\cdot}705682 \times 581{\cdot}35823 \div 82{\cdot}05972$ to three places of decimals, using contracted methods.

9. The Navy expenditure is given below for certain years. Plot these amounts on a diagram and draw a broken line through the points. Draw also a straight line to lie as near this broken line as possible, and taking this line to indicate what the expenditure would be in 1905-6, and in 1906-7, if there was no change of plans, estimate the expenditure for each of these years. If the population may be estimated as 44 millions in 1906-7, what would the cost per head be in that year?

	£
1897-8 - - - -	20,848,863
1898-9 - - - -	23,880,875
1899-00 - - - -	25,731,220
1900-1 - - - -	29,998,529
1901-2 - - - -	30,981,315
1902-3 - - - -	31,003,977
1903-4 - - - -	35,709,477
1904-5 - - - -	36,889,500

E 1. (*a*) Express 3 tons 12 cwt. 3 qr. as the decimal of 5 tons.

(*b*) Find the value of 4 ac. 3 ro. 35 sq. po. at £5. 8*s*. 6*d*. per acre.

2. Evaluate true to two decimal places :

(*a*) The square root of 87321·7342592.

(*b*) The number of cubic inches in a cubical block of 23·572 inches side.

3. The crop on 17 acres is worth £140. 5*s.* at 45*s.* a quarter. What number of acres will give a crop worth £154 at 42*s.* a quarter?

4. What two figures placed to the right of 785634 will give a number divisible by 97?

5. Find the greatest common measure and the least common multiple of the numbers 15496 and 12665, and show, by general reasoning, that these four numbers form a proportion.

6. The interest of £25 for 3½ years at simple interest was found to be £3. 18*s.* 9*d.*; what was the rate per cent.?

7. An embankment, 15 feet high and 60 feet long, is built by 30 men of equal strength in 60 days; another embankment, 20 feet high and 80 feet long, is built by 45 men, also of equal strength, in 64 days. What is the ratio of the strength of the two classes of men?

8. On 16th December the average price of wheat was 28*s.* 5*d.* per 480 lb., of barley 24*s.* 5*d.* per 400 lb., of oats 17*s.* 11*d.* per 312 lb. Find, correct to the nearest penny, the price of each per cwt.

9. The following table gives the mileage and annual cost of certain urban main roads in the County of Bedfordshire :

	Miles.	Fur.	Ch.	Cost.
Bedford	9	5	6	£3000
Biggleswade	10	6	5	£1575
Dunstable	2	0	1	£353
Leighton Buzzard	8	1	4	£1225
Luton	16	2	6	£3300

Calculate the average cost per mile in each case. You may assume that the distances are correct to the nearest chain. A chain = 22 yards.

F 1. Find the value of a farm 125 acres 3 roods 32 sq. poles in extent at £32. 6*s*. 8*d*. per acre.

2. How many bricks 9 inches long, 4 inches wide and 3 inches thick will be required to build a wall 108 ft. long, 8 ft. high and 15 inches wide, if $\frac{1}{6}$ of the volume of the wall is occupied by mortar?

3. A man bought certain goods of which he sold $\frac{1}{3}$ at a profit of 14 per cent., $\frac{2}{5}$ at a profit of $17\frac{1}{2}$ per cent., and the remainder at a profit of 20 per cent. What was his profit per cent. on the whole?

4. Given that the mean thickness and diameter of a farthing are respectively $\frac{27}{40}$ and $\frac{2}{3}$ of those measurements of a penny, find how many farthings will weigh the same as 48 pennies.

5. Explain the difference between *a multiple*, *a common multiple* and *the least common multiple.*

Find the two numbers of which 317 is the G.C.M., 10,461 the L.C.M.

6. If a cubic foot of water weighs 62·426 lb. and a linear foot is equal to 30·48 centimetres, find the number of pounds in a kilogram, correct to the first decimal place.

7. Explain any method with which you are familiar for shortening the process of obtaining the square root of a number. Illustrate by applying your method to the following example:

Find the number of feet in a metre given that

1 hectare = 107642·993419... sq. ft.

8. Express the difference between $3\frac{1}{7}$ and $\frac{355}{113}$ as a vulgar fraction.

If you were to calculate the area of a circle, using in turn for π each of these approximations, show that the radius must be as much as 28 cm. for the difference in the results to amount to 1 sq. cm.

9. If an eight-oared boat is rowed $4\frac{1}{4}$ miles in 21 minutes 36 seconds, find the time, to the nearest tenth of a second, in which it is rowed its own length of 59 feet.

10. One tradesman states his profit as 15 per cent. on the buying price; another as 15 per cent. on the selling price. Find the difference in their actual profits on goods sold at £3875. State what you consider the relative advantages and disadvantages of the two methods of computing the profits.

G 1. A room is 14 feet 6 inches long, 11 feet wide and 9 feet high. It has a window 8 feet by 4 feet and a door 7 feet by 2 feet 6 inches. Find the area of its walls and the quantity of brown paper required to cover them. The paper is in sheets of 45 inches by 29 inches.

2. A swimming bath, 50 feet long and 25 feet wide, is 7 feet deep at one end and 3 feet 6 inches at the other. Find the number of cubic feet and the number of gallons of water required to fill it to within 6 inches of the top. A gallon weighs 10 pounds and a cubic foot of water may be taken to weigh 1000 ounces.

3. Prove that the least common multiple of two numbers is the product of the numbers divided by their greatest common measure.

Find the least common multiple of 1404, 3042, 663.

4. If 20 men, working 6 hours a day, can in 11 days build a wall $\frac{1}{4}$ mile long; in how many days of $4\frac{1}{2}$ hours each can a party consisting of 7 men and 10 boys build a similar wall 156 yards long, supposing that 3 men can in a given time do as much work as 5 boys?

5. If the charge for conveying a party of tourists is two shillings per mile, express this charge in francs per kilometre, supposing that a kilometre is 1093·6 yards and that 25·08 francs are equivalent to £1.

6. In the following advertisement, find the rate per cent. per annum paid by a customer adopting "credit" terms: "Cuckoo-clock offered at 12*s.* 6*d.* for cash, or on receipt of 5*s.* and your promise to pay 2*s.* 6*d.* monthly until 15*s.* is paid."

7. In 1892, Jan. 1 was a Friday. When did this occur again in that century, and when in the next, 1900 not being a leap year? Give all the steps of your reasoning.

8. If 675 metres of silk cost 4921 francs 25 centimes, what is the equivalent price per yard in English money? (1 metre = 39·37 inches and £1 = 25 francs.)

9. A table is formed showing the average number of runs per innings of a certain batsman. Compare the effect, in increasing or diminishing the preceding average, of any given score in the 7th innings with that of the same score in the 42nd, every innings having been completed and the average after the 41st being known to be the same as after the 6th.

What is that average, if a score of 88 in the 7th innings adds $6\frac{2}{3}$ more to it than the same score in the 42nd?

10. Find the product of 186·438 and 2·374 true to three places of decimals.

If the two numbers given above represent in metres the length and breadth respectively of a rectangular area, true to the nearest figure in the third decimal place, to how many places of decimals can you rely on the product as representing the area, when expressed in square metres?

H 1. A number of books will fill an exact number of shelves with 36, 45, 54 or 60 books on each shelf. What is the least number of books for which this is possible?

If the difference between the greatest and least number of shelves that may be used is 42, what is the number of books?

2. Simplify $\dfrac{\frac{1}{50}-\frac{3}{50^2}+\frac{3^2}{50^3}-\frac{1}{53}}{\frac{1}{47}-\frac{3}{50^2}-\frac{3^2}{50^3}-\frac{1}{50}}$.

3. What decimal of 365 days, correct to *ten* places of decimals, is the difference between $\frac{97}{400}$ of a day and 5 hr. 48 min. 45 sec.?

4. There were 5852276 names of scholars on the registers in 20513 schools, and the average number of those scholars attending daily was 4890237. Determine (i) what percentage, correct to *two* places of decimals, the average daily attendance was of the total number on the registers; and (ii) at the average rate, what should

have been the average daily number of absentees, to the nearest unit, from schools containing 87 and 753 scholars respectively.

5. Find by the contracted method the square root of 3121, correct to within a hundred-thousandth; and thereby obtain $(55)^2$, $(5586)^2$, and the first seven figures of the square of 5586591. Show that $\sqrt{10}$ is less than $\frac{721}{228}$ and greater than $\frac{4443}{1405}$; and that it differs from the latter by a quantity less than $\frac{1}{2 \times 228 \times 1405}$.

6. What rate per cent. per annum does a customer pay, who adopts credit terms in buying the watch referred to in the following advertisement: "6*d.* deposit. Watch 6*s.* 6*d.* cash. Government stamped case, . . . three-quarter plate jewelled, timed and seven years' guarantee. For advertisement only on receipt of 6*d.* deposit and payment of 6*d.* per week, making a total of 7*s.* 6*d.* in all."

7. Assuming that the square of the speed of a falling body is proportional to the height fallen through, and that after falling through a height of 1 foot the speed is 8·025 feet per second, find, to within one-hundredth of a foot per second, what the speed will be after falling through 873·4 feet.

8. If radium costs £700 per gram, and 110 milligrams of radium are required to illustrate a lecture, the charge for admission to which is four shillings, how many people would have to be present in order to pay the cost of providing the radium?

9. The three dimensions of a rectangular block of stone are $12\frac{11}{16}$, $8\frac{1}{8}$ and $9\frac{19}{32}$ inches. Give its volume in cubic inches to the nearest integer, first converting the given dimensions to decimals to the necessary number of places.

10. What will it cost to cover the walls of a room 22 ft. 6 in. long, 18 ft. wide and 14 ft. high, with paper 2 ft. 3 in. wide, the price of which is 3*s.* 6*d.* per piece of 12 yards?

---

11. What is the difference in value between a thousand guineas and a million farthings?

2. Define a *measure* of a number. Write down *all* the measures of 363, and the measures which are common to that number and to 231.

3. Express as vulgar fractions in their lowest terms:

$$\text{(i)}\ \frac{1\frac{43}{48}}{11\frac{2}{3}}; \qquad \text{(ii)}\ \frac{\frac{5}{16}-(\frac{7}{8}\text{ of }\frac{1}{2}\frac{7}{1}-\frac{2}{3})}{3\frac{1}{2}-2\frac{7}{11}+1\frac{1}{3}}.$$

4. A man stands by a basket, and ten potatoes are placed in a straight line upon the ground at intervals of 10 yards, the first potato being 10 yards from the man. If he takes them up singly and drops them into the basket, how far must he go?

If there are 100 potatoes similarly placed, but at intervals of 1 yard, can you find a short way of calculating how far the man will have to go?

5. Find the squares of 2, $2\frac{1}{4}$, $2\frac{4}{17}$ and $2\frac{7}{72}$, as decimals to the nearest thousandth. For each square state its difference from 5, in excess or defect.

6. Find, to the nearest square metre, the area of a garden 26·71 metres long and 18·37 metres broad.

If the length and breadth were given to the nearest centimetre, to what degree of accuracy could the area be calculated?

7. What percentages (to the nearest integer) of the United Kingdom do England, Wales, Scotland and Ireland form?

Area of England	- - -	32,346,000 acres.
„ Wales -	- - -	4,774,000 „
„ Scotland	- - -	19,456,000 „
„ Ireland	- - -	20,334,000 „

8. A rate of 2*s.* 4½*d.* in the pound is levied on a district whose rateable value is £369,782. What sum does this rate produce?

9. £1000 is advanced to a man who agrees to repay the debt by three equal instalments to be paid at the end of one, two and three years from the date of the advance. The total value of the instalments at the time when the last one is paid, including interest at 4 % per annum allowed on each instalment from the time when

it is paid, is to be equal to the amount of the loan with interest at the same rate for three years. What is the amount of each instalment?

10. Find the acreage of a field ABCD having four straight boundary lines. E and F are points in AB so that AE = 2·3 chains, EF = 3·9 chains, FB = 2·2 chains, ED = 2 chains and FC = 2·3 chains. ED and FC are both at right angles to AB.

(1 chain equals 22 yards. The area of a right-angled triangle is half the product of the sides about the right angle.)

J 1. Taking a year to contain 365 days 5 hr. 48 min., calculate the yearly income which averages 1 farthing per minute.

2. If a tradesman bought 1526 hats at 16*s.* 6*d.* each and sold them for cash at a guinea apiece, how many had he left when the cash taken for those sold was equal to the cost price of all the hats?

3. Find to the nearest inch the length of paper which would be used in papering the four walls of a room 21 feet long by 13 feet broad by 8 feet 6 inches high, if no allowance were made for windows, doors, wastage or overlapping; the breadth of the paper used being $20\frac{1}{2}$ inches.

4. Simplify $2\frac{1}{6}$ of $\frac{3}{16} + \frac{4\frac{1}{3}}{3\frac{7}{15}} - \frac{3\frac{2}{9}}{4\frac{8}{27}}$ of $\frac{7}{8}$.

5. Statistics show that of 696419 boys who reach the age of 15 years, 30785 may be expected to reach the age of 85 years. Hence, of 729 boys 15 years old, determine, to the nearest unit, how many may be expected to reach the age of 85 years.

6. Divide 0·1817 by 4·6, and express 17 grams 3 centigrams as the decimal of $2\frac{1}{2}$ kilograms.

7. Find the square root of $\frac{5}{19}$, correct to the nearest figure in the third decimal place.

8. How much will 465 miles 5 fur. 165 yd. of wire cost at £15. 16*s.* 8*d.* per mile?

9. What must be the yield per acre of wheat grown on certain land when wheat is worth 29*s.* 9*d.* per quarter in order that the value of the crop may be $12\frac{1}{2}$ per cent. greater than when the yield was $31\frac{1}{2}$ bushels to the acre and wheat was worth 34*s.* per quarter?

10. Calculate the simple interest on £15637. 10*s.* 6*d.* for the days *between* 3rd June and 31st October of the same year of 365 days, at $3\frac{1}{2}$ per cent. per annum, correct to the nearest penny.

11. One lot of articles cost £246. 15*s.* and another lot of similar articles cost £283. 10*s.* If each article cost the same sum, what is the least total number of articles that can have been bought?

K 1. What fraction added to the continued product of $\frac{31}{60}$, $2\frac{1}{7}$, $\frac{4}{93}$, $2\frac{6}{11}$ will give the continued product of $\frac{7}{8}$, 9, $\frac{2}{3}$, $1\frac{11}{21}$ and $\frac{3}{4}$?

2. Express with a common *numerator* the fractions

$$2\tfrac{1}{24},\ \tfrac{3}{5},\ \frac{\frac{7}{12}}{\frac{3}{40}},\ \tfrac{1}{7},\ \tfrac{13}{14}.$$

3. The solar year contains 365·242218 mean solar days. Find, to the nearest minute, the error produced in 2000 years by our calendar, which, by means of Leap years, intercalates 97 days in 400 years.

4. A tank one metre deep has a capacity of 35·1 litres. Another tank of the same depth, but having its length and breadth each 1 centimetre longer, is found to hold 39·2 kilograms of water. Find the length and breadth of the first tank.

(A litre is a cubic decimetre ; a litre of water weighs a kilogram.)

5. If it takes 2 days 1 hour to do 60 miles walking 7 hours each day, how long will it take to do 42 miles walking one hour more each day, but half a mile an hour less?

6. A square field containing 10 acres is surrounded by a ditch 3 feet deep, 2 feet wide at the bottom and 6 feet wide at the top. How much land is taken up by the ditch, and how many cubic yards of earth had to be dug out to make it? (An acre=4840 square yards.)

7. In a chemical experiment I require to dissolve one gram of copper in acid. I have a reel of copper wire 100 metres long, which weighs $3\frac{1}{2}$ kilograms. What length of wire must I cut off for the purpose? Mark off the length in question in your answer book, writing your numerical answer below the measured length, calculated to the degree of approximation attainable in measuring.

8. Knowing the number of pounds in a cubic inch of a substance, you can find the number of kilograms in a cubic centimetre by multiplying by $0{\cdot}4536 \div (2{\cdot}54)^3$. Express this multiplier as a decimal to three places.

If steel weighs 488 lb. per cubic foot, how many kilograms per cubic centimetre does it weigh?

9. Find the smallest number that when divided by 21, 24 and 28 leaves as remainder 9, 12 and 16 respectively.

10. A passenger steamer started from Liverpool for New York and maintained a uniform speed for 40 hours, when her engines broke down, causing a delay of 6 hours. After temporary repairs she proceeded on her journey at a rate less than her former rate by $3\frac{1}{2}$ miles per hour, and in consequence arrived at New York 26 hours late. If she had left Liverpool at the reduced rate of speed, and proceeded uniformly without mishap, she would have arrived two hours later than she did. Find the distance between the two ports.

L 1. A cubic foot of Canadian elm weighs 0·725 as much as a cubic foot of water; and a cubic foot of water weighs 1000 oz.: what will be the weight of a beam of Canadian elm 12 feet 6 in. long, 1 ft. 6 in. deep and 1 ft. 3 in. thick?

2. A man buys a parcel of coffee and re-sells it, losing 3 per cent. on the transaction. If he had obtained £14 more, he would have gained 4 per cent. What was the original sum paid for the coffee?

3. A person gave £75 for 20 casks of oil, each containing 30 gallons. He sold 5 casks at three shillings per gallon, one cask was stove in and the whole of its contents lost, and 15 gallons were also lost by ordinary leakage. He then sold the remainder

at a price per gallon which made his gain amount to 20 per cent. on the whole transaction. What was his selling price per gallon at the second sale?

4. What principal will amount to £10,672. 9*s*. 3*d*. in 7 years and 3 months at $5\frac{1}{2}$ per cent. per annum simple interest?

5. Define greatest common measure and least common multiple. Resolve 2310, 6552 and 12165 into their prime factors, and thence deduce their L.C.M.

6. For 9 months it costs £315 to feed 15 horses; how long can 24 horses be fed for £448?

7. Given that a quadrant of the earth = 10,000,000 metres and that the earth's radius = 3956 miles, find the value of the metre in English measure, correct to the nearest hundredth of an inch, given that the circumference of a circle = 3·1416 times the diameter.

8. On a certain map a road 1320 yd. long is represented by a length of $18\frac{3}{4}$ inches. Determine the scale of the map. What area on the map would represent an area of $\frac{1}{10}$ sq. mile?

9. Work out to three significant figures the value of

$$\frac{(93{\cdot}7)\sqrt{275{\cdot}311}}{0{\cdot}0002938}$$

10. The temperature taken every two hours one day showed:

Midnight	41°·0	2 p.m.	51°·2
2 a.m.	40°·8	4 p.m.	53°·0
4 a.m.	40°·7	6 p.m.	46°·5
6 a.m.	39°·5	8 p.m.	46°·3
8 a.m.	40°·8	10 p.m.	46°·7
10 a.m.	44°·5	Midnight	47°·4
Noon	48°·0		

Draw a curve to show the variation of temperature throughout the day, and estimate the temperature at 3 p.m.

M 1. 1 square metre = 10·7643 square feet. Find the length of 1 decimetre in inches to three decimal places.

2. Working in decimals and by contracted methods, find the price per cwt. (to the nearest penny) when 23 cwt. 3 qr. 24 lb. of goods cost £452. 11*s.* 3½*d.*

3. Find the number of hours and the number of Thursdays from noon on April 12th, 1794, to noon on April 12th, 1894.

4. If a pipe of 9-inch bore discharges a certain quantity of water in 6 hours, how long would 4 pipes of 6 inches bore take to discharge three times the quantity? (The rates of discharge are as the squares of the diameters of the pipes.)

5. The length, breadth and height of a rectangular room are l, b and h yards respectively. Find formulae for (i) V, the volume of the room; (ii) A, the total area of walls, ceiling and floor; (iii) D, the diagonal of the room. Evaluate these when $l=8$, $b=6$, $h=4$.

If v, a and d denote the volume, area and diagonal of a similar room, each dimension of which is half the corresponding dimension of the larger room, state the relations between v and V, between a and A and between d and D.

6. If 7 rix dollars are worth 2 ducats and 9 ducats worth 4 moidores and 20 moidores worth £27, how many rix dollars are there in £72?

7. In the following give your reasons, a "yes" or "no" alone being of no value:

(*a*) 15 tons of coal cost £16. 10*s.*, and 9 tons cost £9. 18*s.* Is the price proportional to the quantity?

(*b*) The base of one equilateral triangle is 2 in. and its area 0·866 sq. in., the base of another is 4 in. and its area 3·464 sq. in. Are the areas proportional to the bases?

(*c*) £50 put out at interest amounts in two years to £54, in three years to £56 and in five years to £60. Is the amount proportional to the time? Is the interest proportional to the time?

8. A swimming bath holds 37,500 gallons. Find the cost of filling it at 9*d.* per thousand gallons.

If, however, the charge is based on the reading of a water meter

which may be wrong by 10 per cent. of the true value, how much too great or too little may the charge be?

9. Find to within 1 mm. the length and the breadth of a rectangular table whose area is 5·5 square metres and which is twice as long as it is wide.

N 1. Prove that the least common multiple of two numbers is obtained by dividing the product of the numbers by their greatest common measure.

The least common multiple of two numbers is 16830 and their greatest common measure is 198; find the numbers. Show that there are two answers.

2. The average number of thousands of depositors in the Post Office Savings Bank for the years 1891, 1892, 1893, 1894 was 5606 (thousands). The number of depositors in the year 1896 was 6·6 per cent. higher than in 1895, while the average for the six years 1891 to 1896 was 5945. Find the numbers of thousands of depositors in each of the years 1895 and 1896.

3. The net prices of goods described in a catalogue are 15 per cent. less than the catalogue prices, while a further discount of 5 per cent. on the net prices is given when payment is made within a month. A customer, however, obtains 20 per cent. off the catalogue price, and finds that by so doing he has gained £4. 10*s.* 6*d.* What was the catalogue value of the goods he bought?

4. The outer dimensions of the concrete foundations of the walls of a building are: length, 10 metres 25 centimetres; breadth, 9 metres 38 centimetres; and depth, 85 centimetres. The inner dimensions are: length, 9 metres 45 centimetres; breadth, 8 metres 58 centimetres; and depth, 85 centimetres. Express in steres the amount of concrete required.

5. The dimensions of a floor are 42 ft. 9 in. and 35 ft. 9 in. What would be the error in the calculated area if the dimensions were taken

(i) as 42 ft. and 35 ft. respectively,

(ii) as 43 ft. and 36 ft. respectively?

6. Express the difference between the Julian year ($365\frac{1}{4}$ days) and the Gregorian year (365 days 5 hours 48 minutes 47·6352 seconds) as the decimal of 1 day.

7. A ladder is 2 ft. 6 in. wide at the base and 1 ft. 4 in. at the top. If there be 23 rungs, and if the middle rung be equi-distant from the base and top of the ladder, find the total length of wood which has to be used for the rungs.

8. I wish to fix the size of a circular enclosure which I may not enter; I therefore note the following points: It is surrounded by a footpath 10 feet wide, and four lamp posts stand along the outer boundary of the footpath at the corners of a square. Each side of this square cuts from the enclosure a strip whose greatest width is equal to the width of the path.

Find the diameter of the enclosure by calculation and also by drawing. For your drawing take a circle of radius 4 inches for the outer boundary of the footpath.

9. Find the rate per cent. per annum, simple interest, that a borrower pays, according to the following scheme: "Loan Fund Association. Money promptly advanced on personal and other security. £20, 12 monthly repayments of £1. 17*s.* 8*d.*"

10. A rectangular casket (with a lid) 11 inches long, 7 inches wide and 6 inches deep, outside measurements, is made of metal of a uniform thickness of $\frac{3}{8}$ths of an inch, and plated inside and out with silver $\frac{1}{150}$th of an inch thick. Find the weight of silver on the casket. (A cubic inch of silver weighs 4·5456 ounces troy.)

59. LOGARITHMS.

Introductory.

A logarithm is defined formally as follows:

The logarithm of a number to a given base is the power to which the base must be raised to equal the number.

Thus, since $2^3 = 8$,

3 is the logarithm of 8 to the base 2,

or, more briefly, $3 = \log_2 8$.

Similarly, $5 = \log_3 243$, for $243 = 3^5$.

The application of logarithms will be seen later. By the use of logarithms, processes of multiplication become processes of addition, processes of division become processes of subtraction and processes of evolution and root extraction become processes of multiplication and division respectively.

Clearly $3^3 \times 3^4 = (3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^7$.

Also, $x^3 \times x^4 =$ three x's all multiplied together, and the answer multiplied by the product of four more x's

$= x^7$, where x stands for any whole number.

The result can be extended, and

$$x^m \times x^n = x^{m+n},$$

where the **powers** or **indices** m, and n, are any whole numbers.

Frequently fractional indices have to be employed; thus:

$x^{\frac{1}{2}}$ has the same meaning as the square root of x,

$y^{\frac{1}{3}}$,, ,, ,, cube ,, y,

$z^{\frac{1}{5}}$,, ,, ,, fifth ,, z, and so on.

$a^{\frac{3}{2}}$ can be regarded either as the square root of the cube of a, or the cube of the square root of a.

Some of the following exercises are designed to show that the relation $x^m \times x^n = x^{m+n}$ is true when m and n are fractional.

Also that $x^m \div x^n = x^{m-n}$.

EXAMPLE 1.

$$256^{\frac{1}{2}} \times 256^{\frac{1}{4}} = 2^{\frac{8}{2}} \times 2^{\frac{8}{4}} = 2^4 \times 2^2 = 2^6 = 256^{\frac{6}{8}}.$$

EXAMPLE 2.

$$15625^{\frac{1}{3}} \times 15625^{\frac{1}{2}} = 5^{\frac{6}{3}} \times 5^{\frac{6}{2}} = 5^2 \times 5^3 = 2^5 = 15625^{\frac{5}{6}}.$$

EXERCISES LXXXV. a.

1. Find the products of the following, expressing the results as powers of the numbers given :

(*a*) $3^2 \times 3^3$; (*b*) $4^2 \times 4^4$; (*c*) $10^3 \times 10^8$; (*d*) $4^3 \times 4^{10}$;

(*e*) $100^3 \times 100^2$; (*f*) $25^{31} \times 25^{32}$; (*g*) $\left(\frac{1}{6}\right)^{10} \times \left(\frac{1}{6}\right)^{11}$;

(*h*) $x^4 \times x^{11}$; (*i*) $a^x \times a^y$; (*j*) $\left(\frac{1}{b}\right)^{2x} \times \left(\frac{1}{b}\right)^{2y}$.

Fill in the missing parts in the following equations, working on the lines of examples 1 and 2 on p. 388 :

2. $1024^{\frac{1}{2}} \times 1024^{\frac{1}{5}} = 1024^{(\cdots\cdot)}$.

3. $1024^{\frac{1}{2}} \times 1024^{\frac{1}{10}} = 1024^{(\cdots\cdot)}$.

4. $1024^{\frac{3}{10}} \times 1024^{\frac{4}{10}} = 1024^{(\cdots\cdot)}$.

5. $\left(\frac{64}{729}\right)^{\frac{1}{2}} \times \left(\frac{64}{729}\right)^{\frac{5}{6}} = \left(\frac{64}{729}\right)^{(\cdots\cdot)}$.

6. $(10^{12})^{\frac{1}{3}} \times (10^{12})^{\frac{1}{4}} = (10^{12})^{(\cdots\cdot)}$.

7. $(78125)^{\frac{4}{5}} \div (78125)^{\frac{3}{5}} = 78125^{(\cdots\cdot)}$.

8. $(100000000)^{\frac{7}{8}} \div (100000000)^{\frac{3}{8}} = (100000000)^{(\cdots\cdot)}$.

Since $2^3 \times 2^5 = 2^8$,

therefore $2^{\frac{8}{11}} \times 2^{\frac{5}{11}} = 8^{\frac{8}{11}}$.

Obtain relations similar to the above in the following cases :

9.	$3^5 \times 3^{11} = 3^{16}$.	Extract the	12^{th} root.
10.	$10^7 \times 10^9 = 10^{16}$.	„	11^{th} „
11.	$(0{\cdot}6)^{13} \times (0{\cdot}6)^{15} = (0{\cdot}6)^{28}$.	„	47^{th} „
12.	$\left(\frac{3}{4}\right)^{100} \times \left(\frac{3}{4}\right)^{101} = \left(\frac{3}{4}\right)^{201}$.	„	30^{th} „
13.	$8^{13} \div 8^5 = 8^8$.	„	17^{th} „
14.	$13^{30} \div 13^{17} = 10^{13}$.	„	40^{th} „
15.	$10^{12} \div 10^9 = 10^3$.	„	50^{th} „
16.	$n^3 \times n^9 = n^{12}$	„	11^{th} „
17.	$n^a \times n^b = n^{a+b}$.	„	c^{th} „
18.	$x^a \times x^b = x^{a+b}$.	„	d^{th} „

Since $n^a \div n^b = n^{a-b}$,

it is clear that $n^{a-b} = \frac{n^a}{n^b}$.

EXERCISES LXXXV. b.

Oral.

Change into or from a fractional form :

1. 10^{13-3}.
2. $10^{\frac{5}{8}-\frac{7}{8}}$.
3. $\frac{11^{\frac{3}{8}}}{11^{\frac{3}{10}}}$.
4. N^{2x-b}.
5. n^{x-y}.
6. $\frac{3^x}{3^y}$.
7. $(100)^{12-0\cdot125}$.
8. N^{10-a}
9. $\frac{a^x}{a^5}$.
10. $\frac{a^{\frac{p}{q}}}{a^{\frac{c}{d}}}$.
11. $\frac{a^x}{a^n}$.
12. $\frac{b^2}{b^2}$.
13. $\frac{c^{100}}{c^{100}}$.
14. $\frac{(0\cdot01)^{\frac{3}{4}}}{(0\cdot01)^{\frac{3}{4}}}$.
15. x^{y-y}.

It has probably been noticed that

$n^x \div n^x = n^{x-x} = n^0$ by following out the rule,

whence $n^0 = 1$.

This is of great importance.

Hence also $\frac{1}{n^a} = \frac{n^0}{n^a} = n^{0-a} = n^{-a}$.

EXERCISES LXXXV. c.

Oral or otherwise.

Express

1. $\frac{1}{4}$ as a power of 2.
2. (*a*) 0·001, (*b*) 100, (*c*) 0·01, (*d*) $\frac{1}{10000}$ as powers of 10.
3. (*a*) $\frac{1}{625}$, (*b*) 125, (*c*) 0·008, (*d*) 0·04, (*e*) $\frac{1}{15625}$ as powers of 5.
4. (*a*) 0·25, (*b*) 0·125, (*c*) $\frac{1}{1024}$, (*d*) $\frac{1}{256}$ as powers of 2.
5. (*a*) $\frac{1}{3}$, (*b*) $\frac{1}{27}$, (*c*) $\frac{1}{729}$ as powers of 3.

Between what two consecutive powers (positive or negative) of 10 do the following lie:

6. 782.	**7.** 0·063.	**8.** 0·0007.	**9.** 973·2.
10. 108932.	**11.** 0·000006.	**12.** 699999.	**13.** 82·0063.
14. $\frac{5}{8}$.	**15.** $\frac{11}{13}$.	**16.** $\frac{136}{9}$.	**17.** $\frac{18932}{193}$.
18. $\frac{11}{1936}$.	**19.** 1·0683.	**20.** $\frac{1}{9356}$.	**21.** $\frac{3286}{17}$.

22. 0·000007. **23.** $\frac{32}{10^8}$.

EXERCISES LXXXV. d.

Find the logarithms or

1. (*a*) 4, (*b*) $\frac{1}{2}$, (*c*) 0·0625, (*d*) 256, (*e*) 64, (*f*) $\frac{1}{32}$, (*g*) 1024, (*h*) 0·25 to the base 2; *i.e.* find the power to which 2 must be raised to yield the given number.

2. (*a*) 3, (*b*) 3^3, (*c*) $\frac{1}{3}$, (*d*) 243, (*e*) $\frac{1}{81}$, (*f*) $\frac{1}{27}$, (*g*) 27 to the base 3.

3. (*a*) 2, (*b*) 8, (*c*) $\frac{1}{8}$, (*d*) $\frac{1}{64}$ to the base 64.

4. (*a*) 5, (*b*) 0·2, (*c*) 0·04, (*d*) 125, (*e*) 625, (*f*) $\frac{1}{3125}$ to the base 5.

5. (*a*) $\frac{1}{7}$, (*b*) 49, (*c*) 343, (*d*) $\frac{1}{343}$, (*e*) $\frac{1}{49}$ to the base 7.

6. (*a*) 5, (*b*) 125, (*c*) $\frac{1}{625}$, (*d*) $\frac{1}{3125}$ to the base 25.

7. (*a*) 3, (*b*) 27, (*c*) 9, (*d*) $\frac{1}{9}$, (*e*) $\frac{1}{243}$ to the base 81.

8. (*a*) 0·1, (*b*) 0·01, (*c*) 1000, (*d*) 100, (*e*) 10, (*f*) 10000, (*g*) 0·00001 to the base 10.

9. Express the following relations otherwise than in logarithmic form, proving the results where not obvious:

(*a*) $\log_{10}0{\cdot}1 = -1$, (*b*) $\log_{10}100 = 2$, (*c*) $\log_{10}100000 = 5$,

(*d*) $\log_3 81 = 4$, (*e*) $\log_4 0{\cdot}125 = -1{\cdot}5$, (*f*) $\log_2 1024 = 10$,

(*g*) $\log_{10}0{\cdot}001 = -3$, (*h*) $\log_5 0{\cdot}008 = -3$, (*i*) $\log_x x^2 = 2$,

(*j*) $\log_4 8 = 1{\cdot}5$, (*k*) $\log_{16}8 = \frac{3}{4}$, (*l*) $\log_8 16 = \frac{4}{3}$.

Proofs of Fundamental Processes.

Proofs will now be given of the fundamental processes in the application of logarithms.

$$(a)\ \log_b m + \log_b n = \log_b (m \times n),$$

$$(b)\ \log_b m - \log_b n = \log_b (m \div n),$$

$$(c)\ \log_b \sqrt[n]{m} = \frac{1}{n} \log_b m.$$

If $x = \log_b m$, then $m = b^x$,

$y = \log_b n$, then $n = b^y$,

and hence $m \times n = b^x \times b^y = b^{x+y}$;

$$\therefore \log_b mn = x + y = \log m + \log n.$$

Similarly, $m \div n = b^x \div b^y = b^{x-y}$;

$$\therefore \log_b \frac{m}{n} = x - y = \log_b m - \log_b n.$$

Again, $\sqrt[n]{m} = (m)^{\frac{1}{n}}$

$$= (b^x)^{\frac{1}{n}}$$

$$= b^{\frac{x}{n}}.$$

$$\log_b \sqrt[n]{m} = \frac{x}{n} = \frac{1}{n} \log_b m.$$

As a particular case of multiplication or of root extraction it will be clear that

$$\log_b m^n = n \log_b m.$$

In practical applications the base is always 10; it will therefore be assumed in future that the base is 10, if no specific mention is made of the base. Thus, by log 5 will be meant the logarithm of 5 to the base 10.

Tables of logarithms are published, and a table is given at the end of the book, but it is interesting to obtain the logarithms in certain cases, even if the methods employed are not general methods and are only applicable in special examples.

The logarithm of any power of 10 can be obtained by inspection.

Thus: $\log_{10} 1 = 0$ for $10^0 = 1$,

$\log_{10} 10 = 1$ „ $10^1 = 10$,

$\log_{10} 100 = 2$ „ $10^2 = 100$,

$\log_{10} 1000 = 3$ „ $10^3 = 1000$, and so on.

Also $\log 0{\cdot}1 = -1$ „ $10^{-1} = \frac{1}{10} = 0{\cdot}1$,

$\log 0{\cdot}01 = -2$ „ $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0{\cdot}01$.

The logarithm of any number, pure or mixed, between 1 and 10 lies between 0 and 1; that of any number between 10 and 100 lies between 1 and 2, and so on.

EXERCISES LXXXV. e.

Oral.

State the integers (positive or negative) between which lie the logarithms of the following numbers:

1. (*a*) 15, (*b*) 225, (*c*) 7, (*d*) 18625, (*e*) 989256, (*f*) 7·9234, (*g*) 79·234.
2. (*a*) 0·002, (*b*) 0·003, (*c*) 0·04, (*d*) 0·00006.
3. (*a*) 0·5632, (*b*) 0·05632, (*c*) 1983, (*d*) 198.
4. (*a*) 3892, (*b*) 0·3892.

Methods of obtaining Approximations to log 2, log 3, log 7.

EXAMPLE 1.—*To find an approximation to log 2.*

2 lies between 10^0 and 10^1;

$\therefore$ $\log 2$ lies between 0 and 1.

2^2 lies between 10^0 and 10^1;

$\therefore$ $2 \log 2$ lies between 0 and 1,

and $\log 2$ lies between 0 and 0·5.

2^3 lies between 10^0 and 10^1;

$\therefore$ $3 \log 2$ lies between 0 and 1,

and $\log 2$ lies between 0 and 0·333....

Again, since $2^{10} = 1024$, and is greater than 1000,

$\therefore$ $\log 2^{10} = \log 1024$, and is greater than $\log 10^3$, *i.e.* > 3;

whence $\log 2 > 0{\cdot}300$ and $< 0{\cdot}333$.

EXERCISES LXXXV. f.

1. By considering the logarithms of the numbers in the first column, supply answers to the queries in the accompanying table as in Example (1) above:

(*a*)	2^3	$\log 2$	*is greater than*		0	*and less than*		0·333
(*b*)	2^{10}	,,	,,	,,	0·300	,,	,,	0·4
(*c*)	2^{13}	,,	,,	,,	?	,,	,,	?
(*d*)	2^{33}	,,	,,	,,	?	,,	,,	?
(*e*)	3^2	$\log 3$	,,	,,	?	,,	,,	?
(*f*)	3^{15}	,,	,,	,,	?	,,	,,	?
(*g*)	3^{23}	,,	,,	,,	?	,,	,,	?
(*h*)	3^{17}	,,	,,	,,	?	,,	,,	?
(*i*)	7^6	$\log 7$	,,	,,	?	,,	,,	?
(*j*)	7^{13}	,,	,,	,,	?	,,	,,	?

Write down as accurately as you can the value of $\log 2$, $\log 3$, $\log 7$, obtained from the above table.

(More accurately $\log 2 = 0{\cdot}30103...$,
$\log 3 = 0{\cdot}47712...$,
$\log 7 = 0{\cdot}8451....$)

2. Hence obtain (*a*) $\log 4$, (*b*) $\log 5$, (*c*) $\log 6$, (*d*) $\log 8$, (*e*) $\log 9$.

EXERCISES LXXXV. g.

Graphic.

1. Draw a graph in which horizontal distances represent the numbers 1 to 10, and vertical distances the corresponding logarithms.

(Take the lengths representing units to be in the ratio of 1 : 10; for example,

$\log 2 = 0{\cdot}30103\ldots$; the vertical distance would be $10 \times 0{\cdot}30103$ and the horizontal distance 2.)

2. Find by your graph the logarithms of 3·6, 4·2, 1·9, 7·3. Also the numbers the logarithms of which are 0·32, 0·45, 0·78.

3. The table below gives the logarithms of the numbers quoted. Draw suitable graphs showing how the logarithm changes with the change in the number.

(*a*)

No.	600	601	602	603	604
log	2·7782	2·7789	2·7796	2·7803	2·7810
No.	605	606	607	608	609
log	2·7818	2·7825	2·7832	2·7839	2·7846

(*b*)

No.	8·00	8·01	8·02	8·03	8·04
log	0·9031	0·9036	0·9042	0·9047	0·9053
No.	8·05	8·06	8·07	8·08	8·09
log	0·9058	0·9063	0·9069	0·9074	0·9079

Make what deduction you can from your result.

4. The table below gives the difference in the logarithms for the same difference in the number. Plot graphs showing how the

differences in the logarithms vary (*a*) with the number, (*b*) with the reciprocal of the number:

Numbers.	Approximate difference in logarithms.
100 and 101	0·0043
200 „ 201	0·0021
300 „ 301	0·0015
400 „ 401	0·0010
500 „ 501	0·0008
600 „ 601	0·0007
700 „ 701	0·0006
800 „ 801	0·0005
900 „ 901	0·0005

60. LOGARITHMIC TABLES AND INSTRUMENTS.

To find the Logarithm of a Number.

EXAMPLE 1.—*To find* $\log 575{\cdot}3$.

Since $575{\cdot}3 = 10^2 \times 5{\cdot}753$, $\log 575{\cdot}3 = \log 10^2 + \log 5{\cdot}753$,

but $\log 10^2 = 2$ and $5{\cdot}753 > 1 < 10$; $\therefore\ 5{\cdot}753 > 10^0 < 10^1$.

Hence $\log 5{\cdot}753$ lies between 0 and 1, and $\therefore$ $\log 5{\cdot}753$ is a pure decimal.

The whole number, that is 2, is called the **characteristic**,
the pure decimal „ „ **mantissa.**

The mantissa alone is given in the tables, and *without* the decimal point; the characteristic is seen on inspection.

In the table of logarithms, begin by looking down the first column for 57.

The corresponding line is as follows:

No.	0	1	2	3	4	5	6	7	8	9	Difference columns.							
											1	2	3	4	5	6	7	8	9
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7

The meaning of this line is that

$\log 5{\cdot}70 = 0{\cdot}7559$, $\log 5{\cdot}71 = 0{\cdot}7566$, $\log 5{\cdot}72 = 0{\cdot}7574$ and so on.

To obtain $\log 5{\cdot}701$, or $5{\cdot}711$, etc., } we add on the corresponding figure in the difference column: here 1;

whence $\log 5{\cdot}701 = 0{\cdot}7559 + 0{\cdot}0001 = 0{\cdot}7560$,

so $\log 5{\cdot}707 = 0{\cdot}7559 + 0{\cdot}0005 = 0{\cdot}7564$,

and in particular

$$\log 5{\cdot}753 = 0{\cdot}7597 + 0{\cdot}0002 = 0{\cdot}7599;$$

hence $\log 575{\cdot}3 = 2{\cdot}7599$.

EXAMPLE 2.—*To find* $\log 0{\cdot}005754$.

$$0{\cdot}005754 = 5{\cdot}754 \div 1000 \qquad = 5{\cdot}754 \times 10^{-3};$$

$$\therefore \log 0{\cdot}005754 = \log 5{\cdot}754 + \log 10^{-3} = \log 5{\cdot}754 - 3;$$

but $\log 5{\cdot}754 = \log 5{\cdot}75 +$ difference for 4

$$= 0{\cdot}7597 + 0{\cdot}0003 = 0{\cdot}7600$$

and $\log 0{\cdot}005754 = 0{\cdot}7600 - 3$.

This is written, however, in the form $\bar{3}{\cdot}7600$.

To find the Number corresponding to a given Logarithm.

EXAMPLE.—*Find the number, the logarithm of which is* 3·8828.

By inspection it is seen that the logarithm is that of a number between 10^3 and 10^4 and $= 10^3$ times the number the logarithm of which is 0·8828.

Looking through the table of logarithms, the following line is found:

No.	0	1	2	3	4	5	6	7	8	9	Difference columns.							
											1	2	3	4	5	6	7	8	9
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	**3**	**3**	4	5	5

0·8828 lies between 0·8825 and 0·8831, and therefore between $\log 7{\cdot}63$ and $\log 7{\cdot}64$. But $0{\cdot}8828 = 0{\cdot}8825 + 0{\cdot}0003$.

Look at the difference column; it is seen that 0·0003 is the difference corresponding to a **5** or **6**;

$$\therefore 0{\cdot}8828 = \log 7{\cdot}635 \text{ or } \log 7{\cdot}636.$$

The ambiguity, that is, whether the third decimal figure is 5 or 6, is due to the fact that the logarithms are approximate only.

Multiplication by Logarithms.

EXAMPLE.—*Evaluate* $(0{\cdot}19 \times 132)$ *by logarithms.*

$$\begin{aligned}\log(0{\cdot}19 \times 132) &= \log 0{\cdot}19 + \log 132\\ &= \bar{1}{\cdot}2788 + 2{\cdot}1206\\ &= 1{\cdot}3994\\ &= \log 25{\cdot}08\,;\end{aligned}$$

$$\therefore\ 0{\cdot}19 \times 132 = 25{\cdot}08.$$

Division by Logarithms.

EXAMPLE.—*Find the density of a substance,* 973·2 *c.c. of which have a mass of* 9983 *grams.* (*Density = mass ÷ volume.*)

$$\begin{aligned}\text{Density (D)} &= 9983 \div 973{\cdot}2.\\ \log \mathrm{D} &= \log 9983 - \log 973{\cdot}2\\ &= (3{\cdot}9991 + 0{\cdot}0001)\\ &\quad - (2{\cdot}9881 + 0{\cdot}0001)\\ &= 1{\cdot}0110\\ &= 1{\cdot}0086 + 0{\cdot}0024\\ &= \log 10{\cdot}26\,;\end{aligned}$$

From tables,
log 9·98 = 0·9991,
diff^{e} for 0003 = 0·0001;
log 9·732 = 0·9881,
diff^{e} for 2 = 0·0001.

From tables,
log 1·02 = 0·0086.
24 = difference for 6 nearly.

$\therefore$ density = 10·26 gr. c.c. units.

Involution by Logarithms.

EXAMPLE 1.—*Evaluate* $(13{\cdot}27)^3$ *by logarithms.*

$$\begin{aligned}\log(13{\cdot}27)^3 &= 3\log 13{\cdot}27\\ &= 3(1{\cdot}1206 + 0{\cdot}0023)\\ &= 3{\cdot}3687\\ &= 3{\cdot}3674 + 0{\cdot}0013\\ &= \log 2337\,;\end{aligned}$$

$$\therefore\ (13{\cdot}27)^3 = 2337.$$

EXAMPLE 2.—*Evaluate* $(0{\cdot}0864)^4$ *by logarithms.*

$$\log (0{\cdot}0864)^4 = 4 \log (0.0864)$$
$$= 4(\bar{2}{\cdot}9365)$$
$$= \bar{5}{\cdot}7460$$
$$= 5{\cdot}7459 + 0{\cdot}0001$$
$$= \log 0{\cdot}00005571\,;$$
$$\therefore\ 0{\cdot}0864 = 0{\cdot}00005571.$$

(NOTE. $\bar{2}{\cdot}9365$ means $+0{\cdot}9365 - 2$, and 4 times this
$= 3{\cdot}7460 - 8$
$= 0{\cdot}7460 + 3 - 8$
$= 0{\cdot}7460 - 5$.)

Evolution by Logarithms.

EXAMPLE 1.—*Evaluate* $\sqrt[3]{897600}$.

If X stands for the required answer,

$$\log X = \tfrac{1}{3} \log 897600$$
$$= \tfrac{1}{3}(5{\cdot}9528 + 0{\cdot}0003)$$
$$= 1{\cdot}9844$$
$$= 1{\cdot}9841 + 0{\cdot}0003 = \log 96{\cdot}45\,;$$
$$\therefore\ \sqrt[3]{897600} = 96{\cdot}45.$$

EXAMPLE 2.—*Evaluate* $\sqrt[3]{0{\cdot}00008976}$.

If X stands for the required answer,

$$\log X = \tfrac{1}{3} \log 0{\cdot}00008976$$
$$= \tfrac{1}{3}(\bar{5}{\cdot}9531)$$
$$= \tfrac{1}{3}(\bar{6} + 1{\cdot}9531)$$
$$= \bar{2}{\cdot}6510\,;$$
$$\therefore\ X = 0{\cdot}04477.$$

Note carefully the alteration of $\bar{5}$ to $\bar{6}+1$, so that the negative characteristic can be divided exactly by 3.

EXERCISES LXXXVI. a.

NOTE.—Wherever possible make at the beginning a mental approximation to the first, or the first two, significant figures.

1. Calculate by logarithms the weights of the following :

(*a*) Leaden block, length 98·34 cm., width 34·89 cm., thickness 16·03 cm. (density of lead 11·36).

(*b*) Copper plate, length 4·312 metres, width 2·813 metres, thickness 0·0134 metre (density of copper, 8·781).

(*c*) Copper cube, edge 35·35 cm.

(*d*) Steel bar, 8·23 ft. long, 0·0864 ft. square in section (weight of 1 cub. ft. of steel 499·1 lb.).

(*e*) Copper wire, 3061 metres long, radius of section 0·000234 cm., $\pi = 3{\cdot}142$ (density of copper 8·781).

(*f*) The air in a room 19 ft. 4 in. long, 18 ft. 6 in. broad, 11 ft. 9 in. high (density of air $0{\cdot}001291 \times 62{\cdot}32$ in ft. lb. units).

(*g*) 1894 bricks, the weight of a single brick being 8·23 lb.

(*h*) A seam of coal, 3984 yd. long, 1764 yd. wide, average thickness 1·032 yd., assuming 1 cub. ft. of coal to weigh 78·26 lb.

2. If a metre be 39·37 in., calculate by logarithms:

(*a*) The number of metres in a mile.
(*b*) ,, yards in a kilometre.
(*c*) ,, cm. in 23·81 ft.
(*d*) ,, cm. in 1064 inches.
(*e*) ,, inches in 18·62 metres.
(*f*) ,, ,, 5890×10^{-10} metres.
(*g*) ,, sq. inches in 18 sq. metres.
(*h*) ,, cub. inches in 1 cub. decimetre.

3. Using the table below, express (*a*), (*c*), (*h*) and (*k*) as percentages of the total amount of material in solution:

The Material in Solution in 1 cub. mile of average River Water.

	Tons in Cubic Mile.
(*a*) Calcium Carbonate	3267×10^2.
(*b*) Magnesium Carbonate	1129×10^2.
(*c*) Calcium Sulphate	3436×10.
(*d*) Sodium Sulphate	3180×10.
(*e*) Potassium Sulphate	2036×10.
(*f*) Sodium Nitrate	2680×10.
(*g*) Sodium Chloride	1666×10.
(*h*) Silica	7458×10.
(*i*) Ferric Oxide	1301×10.
(*j*) Alumina	1431×10.
(*k*) Organic Matter	7902×10.

4. Using the table below, find the volume of a ton of water at a temperature of (*a*) 32° F., (*b*) 212° F. :

Weight of Water at Different Temperatures.

Temperature.	Weight of 1 cub. ft.	Temperature.	Weight of 1 cub. ft.
32° Fah.	62·417 lb.	70° Fah.	62·302 lb.
40° „	62·423 „	80° „	62·218 „
50° „	62·409 „	90° „	62·119 „
60° „	62·367 „	212° „	59·7 „

5. Employ logarithms to complete the following table :

Quantity of Rainfall corresponding to given depth.

	Tons per sq. mile.	Imperial gallons per acre.	Depth of rainfall.
(*a*)	3626	?	?
(*b*)	10877	?	?
(*c*)	14503	?	?

6. If the matter transported by the rivers mentioned below has an average density 2·23 times that of water, find in each case the number of cubic miles of rock transported per annum.

Dissolved Mineral Matter transported by certain Rivers.

(*a*) Mississippi, 1128×10^5 tons per annum.
(*b*) Danube, 2252×10^4 „
(*c*) Nile, 1695×10^4 „
(*d*) Rhone, 8290×10^3 „
(*e*) Rhine, 5817×10^3 „
(*f*) Thames, 548×10^3 „

7. Find the weight of 172·5 ft. of cast iron piping, outside diameter 14·00 in., thickness of metal 0·625 in., the weight of a cubic foot of cast iron being 474·4 lb. ($\pi = 3{\cdot}1416$).

8. Supply the missing portions of the following table :

	Diameter of circle.	Area of circle.	Circumference of circle.
(*a*)	0·024	?	?
(*b*)	?	0·006940	?
(*c*)	?	0·006082	?

9. Find the lengths in cm. of the sides of the following cubes :

(*a*) Mass, 8967 grams, density 2·312.
(*b*) „ 0·006356 „ „ 8·834.
(*c*) „ 1184×10^3 „ „ 7·726.

10. Evaluate the following :

(*a*) $(33{\cdot}685)^4$, (*b*) $(38{\cdot}125)^{\frac{1}{6}}$, (*c*) $(8{\cdot}763 \times 10^{-3})^{\frac{1}{4}}$, (*d*) $(0{\cdot}0008634)^{\frac{1}{7}}$.

Evaluation of Formulae.

The following solutions are examples on the evaluation of formulae, empirical (that is, based on experience or observation) and otherwise.

EXAMPLE 1.—*In blasting with gunpowder, the number of ounces of powder required to blast any rock is given by* P, *where*

$$P = \frac{XL^3}{8},$$

L *being the length of the line of least resistance,* X *a constant. If* X *be* 4·062, *calculate* P *when* L = 4·336 *ft.*

Since $P = \frac{XL^3}{8}$,

$\log P = \log X + 3 \log L - \log 8$
$= \log 4{\cdot}062 + 3 \log 4{\cdot}336 - \log 8$
$= 0{\cdot}60874 + 3(0{\cdot}63709) + \bar{1}{\cdot}09691$
$= 0{\cdot}60874 + 1{\cdot}91127 + \bar{1}{\cdot}09691$
$= 1{\cdot}61692.$
$P = 41{\cdot}40.$

(Note a proceeding frequently employed, *i.e.* of subtracting log 8 from 0 at once, so that the further process is addition.)

41·40 oz. of powder will be required.

EXAMPLE 2.—*The load, a solid column or pillar of cast iron can support, is frequently calculated by the formula*

$$W = 14{\cdot}9\,\frac{D^{3{\cdot}76}}{L^{1{\cdot}7}},$$

W *being the weight of the load in tons,* D *the diameter in inches,* L *the length of the column in feet. Calculate the load which can be supported by a pillar* 7·3 *ft. high and* 12 *in. in diameter.*

By logarithms.

$$\begin{aligned}\log W &= \log 14{\cdot}9 + 3{\cdot}76 \log D - 1{\cdot}7 \log L\\ &= \log 14{\cdot}9 + 3{\cdot}76 \log 12 - 1{\cdot}7 \log 7{\cdot}3\\ &= 1{\cdot}17319 + 3{\cdot}76(1{\cdot}07918)\\ &\qquad - 1{\cdot}7(0{\cdot}86332)\\ &= 1{\cdot}17319 + 4{\cdot}05772 + \bar{2}{\cdot}53236\\ &= 3{\cdot}76327.\\ W &= 5797.\end{aligned}$$

1·07918	0·86332
3·76	1·7
3·23754	0·86332
75543	0·60432
6475	1·46764
4·05772	

A load of 5797 tons could be supported.

EXERCISES LXXXVI. b.

1. In blowing up walls, soldiers sometimes calculate the necessary charge of powder by the formula

$$P = a \times L^3,$$

where P is the charge in pounds, L the half thickness of the wall in inches, a a constant depending on the nature and so on, of the wall.

Calculate P when $a = 0{\cdot}1672$, $L = 8{\cdot}324$.

2. The deflexion in inches produced by a load uniformly distributed over a beam or girder of rectangular section is given by

$$d = \frac{12WL^3}{8EBT^3},$$

where W = the load in tons, L = length of beam in feet, B the breadth in inches, T the thickness of the beam in inches, while E depends on the nature of the substance of the beam.

In an iron beam in which $E = 1{\cdot}32 \times 10^4$, the length was 18 ft., breadth 12 in., thickness 9·86 in., find the deflexion due to a load of 126 tons.

3. The tractive force of a locomotive is given by

$$T = \frac{D^2PL}{W},$$

where T is the tractive force in pounds weight, W the diameter of the driving wheel in inches, P the mean pressure of the steam in the cylinders in pounds per sq. in., L the length of the stroke in inches, and D the diameter of the piston in inches.

The diameter of a driving wheel of a locomotive is 5 ft., the mean pressure is 65 pounds per sq. in., the length of stroke 27·3 in. and the diameter of the piston 10·6 in. Find the tractive force.

4. The number of cubic feet of water discharged over a sill is given by

$$C = 214\sqrt{H^3},$$

if the water be otherwise at rest, C being the number of cub. ft. discharged over each foot width per minute, H the height of the water surface above the sill in feet.

Calculate the discharge from each foot of width when the sill has a depth of (*a*) 10·3 in., (*b*) 6·4 ft., (*c*) 9·5 ft.

5. The number of gallons of water delivered per hour in pipes is sometimes calculated by

$$G = \frac{\sqrt{(15D)^5H}}{L},$$

where G = no. of gallons delivered per hour,
H = head of water in feet,
D = diameter of pipe in inches,
L = length of pipe in yards.

(*a*) Establish a similar formula where all the lengths are expressed in centimetres, but where G is expressed in litres. (Given 1 lit. = 0·2201 gal. and 1 metre = 39·37 in.)

(*b*) Find the diameter of the pipe if the number of gallons discharged per hour is 2161, the head of water 150 ft. and the length of pipe 1200 yd.

6. A formula sometimes used in calculating the diameter of a single acting pump used in raising water is

$$D = \sqrt{\frac{G}{0{\cdot}34\,LN}},$$

where L is the length of the stroke in feet,
G the number of gallons to be delivered per minute,
N the number of strokes per minute,
D the diameter of the pump in inches.

Calculate (*a*) the number of gallons raised per minute when the length of stroke is 3·24 feet, the diameter of the pump 11·76 in. and the number of strokes per minute 11 ;

(*b*) the diameter of the pump capable of raising 120 gallons per minute when the length of the stroke is 2·94 feet and the pump is working at the rate of 13 strokes per minute.

7. The pressure on a dock gate is given by

$$P = A \times 0{\cdot}278\, D,$$

where P is the force on the gate in cwt.,
A the area exposed to water in square feet,
D the depth of water on the gate in feet.

Find (*a*) the total force when the area exposed is 565 square feet and the depth 24 feet 9 inches ; (*b*) the depth if the force is 31·64 cwt. and the area 616 square feet.

8. The necessary diameter of a balloon, in order that it may be able to raise a given weight, can be calculated by

$$D = \sqrt[3]{\frac{W}{0{\cdot}5236\,(A - G)}},$$

where D = diameter of balloon in feet,
A = weight in lb. of a cubic foot of air,
G = weight in lb. of a cubic foot of the gas used in filling the balloon,
W = weight raised in lb. (including the weight of the balloon itself).

If $A = 0{\cdot}08072$, $G = 0{\cdot}0056$,

calculate (*a*) the diameter in order that the balloon may be capable of raising 12·5 cwt.; (*b*) the weight that could be lifted by a balloon of 35·5 ft. diameter.

9. In warming a building by hot water pipes, the required length of 4 in. pipes is based on the formula

$$L=\frac{(P-t)(T-t)}{P-T}\times 0{\cdot}0045C,$$

where L = the length of the pipes in feet,
P = the temperature (F.) of the pipes,
T = „ required in the building,
t = „ external air,
C = number of cub. ft. to be warmed per minute.

Find the number of feet of piping required when $P=120°$ F., $t=40{\cdot}5°$ F., $T=61{\cdot}5°$ F. and $C=35{\cdot}6\times 10^2$.

10. Calculate the force exerted per sq. ft. by wind blowing directly on a surface according to the formula

$$P=0{\cdot}002288V^2,$$

where P is the pressure in lb. weight per sq. ft.
V „ velocity in feet per second.

	Velocity in ft. per sec.	Description of Wind.
(*a*)	29·3	Brisk gale
(*b*)	73·3	Storm
(*c*)	146·6	Hurricane

11. The elastic force of steam expressed in inches of mercury is given by

$$F=\left(\frac{T+100}{C}\right)^6,$$

where T is the temperature (F.) of the steam,
C is 177
and F is the elastic force.

(*a*) What is the temperature if the elastic force is 60·2 in.?

(*b*) What is the elastic force when the temperature is 279° F.?

12. The number V of cubic feet of steam obtained from 1 cub. ft. of water is given by

$$V=\frac{20559}{0{\cdot}941}P,$$

where P is the pressure in pounds weight per sq. in. (including the pressure of the atmosphere).

Calculate (*a*) V if P be 14·706,
(*b*) P if V be 103·6.

13. The number M of miles per hour travelled by a locomotive

$$= \frac{P.D}{56{\cdot}023S},$$

where P = speed of the piston in ft. per minute,
S = stroke of the engine in ft.,
D = diameter of the driving-wheel in ft.

Complete the following table:

	Diameter of Driving-wheel.	Piston Speed.	Speed of Engine.	Stroke in inches.
(*a*)	3	1867	60	?
(*b*)	?	1232	60	22
(*c*)	4·5	809	?	26

NOTE.—Other exercises on the use of logarithms are given in connexion with Compound Interest and Areas.

The Slide Rule.

EXERCISES LXXXVI. c.

Practical.

If a line AB be drawn to represent the logarithm of any number m, and another BC to represent the logarithm of any other number n, then a line equal in length to (AB + BC) will represent the logarithm of the product $m \times n$ of the two numbers.

1. The common logarithms of the numbers 2 to 10 are as follows:

Number	2	3	4	5	6	7	8	9	10
Logarithm	0·301	0·477	0·602	0·699	0·778	0·845	0·903	0·954	1

On a scale in which 400 mm. represents the logarithm of 10, *i.e.* 1, the preceding logarithms would be represented by the following lengths :

Length in mm., 120, 191, 241, 280, 311, 338, 361, 382, 400.

Calculate to the nearest millimetre the lengths which represent the logarithms of 1·1, 1·2..., 2·1, 2·2..., 9·1..., 9·9 (use a table of logs.).

2. On squared millimetre paper construct a scale, in which are represented, as in Ex. 1, the logarithms of the numbers 1, 1·1, ..., 9·9, 10. Make the divisions of the scale 3 cm. in length. Carefully cut the scale into three others, each of the same length but 1 cm. wide.

The scales can now be used as rough slide rules, and the following exercises may be worked either with these paper scales or a more carefully prepared slide rule.

3. Place two of the scales alongside one another ; slide one scale B along the other A until the division marked 1 in B is opposite to the division marked 2 in A. See if the divisions on A are marked with numbers twice as great as those which mark the opposite divisions in B.

EXAMPLE 1.—*Evaluate* $1{\cdot}4 \times 8{\cdot}5$ *by means of the slide rule.*

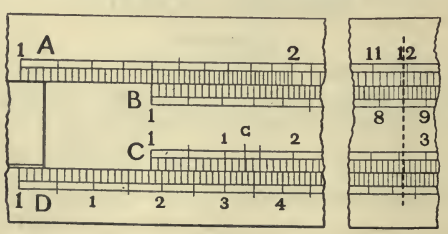


FIG. 72.

Move B along A until the division corresponding to 1·4 in A is opposite to the division marked 1 in B (Fig. 72). Note the

reading on A opposite to 8·5 on B; it is seen to be between 11·8 and 12, and estimated at 11·9 (Fig. 72).

EXAMPLE 2.—*Evaluate* $46 \div 3{\cdot}4$.

It is important to note that in many cases no notice of the decimal point is taken in the work with the slide rule itself; its position is determined by inspection.

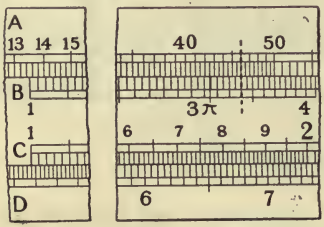


FIG. 73.

Place division 3·4 in B opposite to division 46 in A, read off the division on A opposite division 1 in B; this reading gives the result of the division, and is seen (Fig. 73) to be 13·5.

EXAMPLE 3.—*Evaluate* $58 \times 1{\cdot}3 \div 7{\cdot}9$.

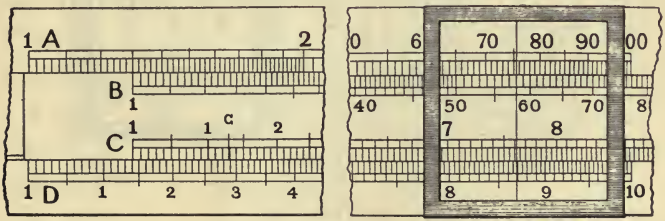


FIG. 74.

(*a*) Set division 1 on B opposite division 1·3 on A, move the *hair slide* until the hair is over the 58 mark on B (Fig. 74).

(*b*) Move the scale B until the 7·9 mark is under the hair (Fig. 75). Note the reading on A opposite the mark 1 on B; this is seen to be about 9·5.

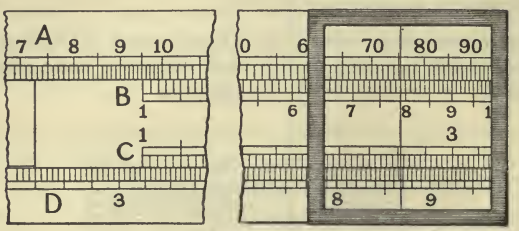


FIG. 75.

EXAMPLE 4.—*Evaluate* $\sqrt{78}$.

Method (*a*). *Using only scales* A *and* B.

Place the hair slide over division 78 on A, slide B until the division on B underneath the hair is the same as the division in A opposite to the mark 1 on B. The reading 8·83 gives the square root (Fig. 76).

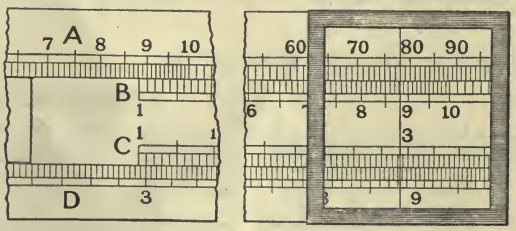


FIG. 76.

Method (*b*). *Using a separate scale.*

Construct a new scale (this will probably be already part of the slide rule) in which all the scale divisions are exactly double the size of the corresponding divisions on A or B; place this scale D against either A or B; if the first marks (1)

are opposite each other, the numbers on D are the square roots of the corresponding numbers on A (or B).

In an actual slide rule, these scales would be as on C and D in Fig. 76.

EXAMPLE 5.—*Evaluate* $\sqrt[3]{76}$.

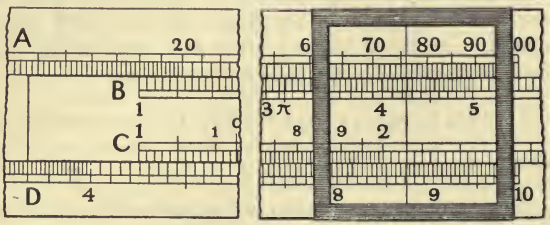


FIG. 77.

Set the hair slide over 76 in A; slide B until the reading on D opposite the 1 on B or C is the same as the reading on B under the hair (Fig. 77); this reading 4·24 gives the cube root.

EXERCISES LXXXVI. d.

Evaluate the following, as accurately as the slide rule permits, and verify by ordinary calculation:

1. $1·3 \times 1·4$.	2. $2·5 \times 1·3$.	3. $4·7 \times 1·22$.	4. $5·4 \times 1·8$.
5. $9·8 \div 2·3$.	6. $113 \div 3·4$.	7. $29 \div 7·4$.	8. $3·5 \div 2·2$.
9. $9 \times 8·3 \div 3·2$.	10. $(8·9 \div 15) \div 3·2$.	11. $76 \div (3·1 \times 4·1)$.	
12. $\sqrt[3]{140}$.	13. $\sqrt{7 \times 4·2}$.	14. $\sqrt{9·3 \div 2·4}$.	
15. $\sqrt{1 \div 3·4}$.	16. $\sqrt{8·9 \times 3·2}$.	17. $\sqrt[3]{7}$.	
18. $\sqrt[3]{7 \div 4}$.	19. $\sqrt[3]{7 \times 4}$.	20. $\sqrt[3]{8·2 \times 2·1}$.	

N.B.—The slide rule should, however, be frequently employed in approximations in general work; accordingly, only a few typical exercises are given here.

EXERCISES LXXXVI. e.

Graphic.

1. The following table gives the volumes and diameters of a number of spheres. Draw a graph, plotting the logarithm of the volume against the logarithm of the diameter; measure the increase in the logarithm of the volume for each unit increase in the logarithm of the diameter; show from your result that the volume is proportional to the cube of the diameter:

Volume	0·524	4·19	14·1	33·5	65·5	113·1	179·6	268	382 cub. in.
Diameter	1	2	3	4	5	6	7	8	9 in.

2. The accompanying table gives the velocity acquired by bodies in falling from various heights. Plot the logarithms of the velocity against the logarithms of the fall. Deduce from your graph that the velocity is proportional to the square root of the fall:

Fall in feet - - -	1	10	20	30	40	50	60	70
Velocity in feet per sec. -	8	25	36	44	51	57	62	67

3. The table below gives the relation between the maximum supply of gas through a certain length of piping and the diameter of the piping, the pressure being the same in each case. Prove that the supply squared is proportional to the diameter raised to the 5th power:

Diameter of pipe in inches -	1	2	3	4	5	6
Supply in cub. ft. per hour -	67	377	1039	2133	3730	5880

4. The following table gives the mean distances from the sun of the chief large planets; also their periods of revolution around the sun in days. Draw a graph connecting the logarithms of the mean distance from the sun with the logarithm of the period. Make what inference you can from your result, and find

(*a*) the period of Neptune—distance 2791×10^6 miles; (*b*) the distance of an asteroid, the period of which is 1200 days:

Planet.	Mean distance in millions of miles.	Period in mean solar days.
Mercury -	36·0	88
Venus -	67·2	225
Earth -	92·9	365
Mars - -	141·5	686
Jupiter -	483·3	4332
Saturn -	886·0	10759

5. The following table gives the ratio of the strength of the magnetic field due to a short magnet at points at varying distances from either pole of the magnet, to the strength of the earth's field. Find how the field varies with the distance (plot the logarithm of the inverse ratio against the logarithm of the distance):

Ratio of field to that of Earth.	Distance from either Pole.
3·63	20 cm.
1·00	30 cm.
0·454	40 cm.
0·240	50 cm.
0·125	60 cm.
0·0816	70 cm.

6. The table below gives the electrical resistance of 1000 yards of platinoid wire of different diameters. Draw a logarithmic graph connecting the resistance and diameter; (*a*) what conclusion do you deduce from it, (*b*) what is the resistance of 1000 yards of platinoid wire of 1 mm. diameter?

Diameter in mm. - -	1·626	1·219	0·914	0·711	0·559	0·457
Resistance in ohms per 1000 yards length	180·3	320·6	870·0	942·2	1026	2280

7. To minimize the risk of fire due to electric lighting in a house, a piece of wire of fusible tin is fixed in a convenient position and the current passes through this ; but if the current becomes too strong, the wire fuses, and the current is stopped. Wires of different thicknesses are used according to the strength of the greatest current which can pass with safety.

The following table gives the thicknesses and corresponding currents :

Thickness in mm.	0·711	0·559	0·457	0·376	0·315	0·274	0·234	0·193
Current in ampères	7·7	5·35	3·96	2·95	2·26	1·84	1·44	1·08

Draw a graph connecting the logarithms of the thickness and current. (*a*) Deduce an approximate connexion between the thickness and current ; (*b*) find the thickness of the fuse if the current is to be not greater than 6 ampères ; (*c*) what current will just fuse a wire of 0·5 mm. diameter ?

61. COMPOUND INTEREST.

In the examples on Simple Interest, dealt with in § **48**, the **principal** is supposed to remain constant, so that in equal intervals of time equal amounts of interest accrue.

In most actual cases, however, the interest is **compound**, and is added to the principal at the end of fixed intervals of time, seldom greater than a year.

The following examples illustrate the more usual methods of procedure in different cases where the interest is compound :

EXAMPLE 1.—*What is the Compound Interest on* £536. 12*s.* *in* $3\frac{1}{2}$ *yrs. at* 4 % *per annum, the interest being payable yearly ?*

£536. 12*s.* = £536·6.

The interest on this sum for the first year is at the rate of £4 on every £100 ; it is therefore $\frac{4}{100}$ of £536·6 = £21·464.

This interest is added to the principal, and the sum serves as the *Principal for the second year.*

Hence, the principal for the second year is

$$£536{\cdot}6 + £21{\cdot}464 = £558{\cdot}064.$$

The interest for the second year

$$= £558{\cdot}064 \times \tfrac{4}{100} = £22{\cdot}32256.$$

The sum of the interest for the second year and the principal for the second year serve as the *Principal for the third year.*

Hence, the principal for the third year

$$= £558{\cdot}064 + £22{\cdot}32256$$
$$= £580{\cdot}38656.$$

The interest for the third year $= £23{\cdot}2154624.$

The principal for the next half year $= £603{\cdot}6020224.$

The interest for the half year is $\frac{1}{2} \times \frac{4}{100}$ of the principal, and

$$\therefore \quad = £12{\cdot}072040448.$$

Hence, the final **amount** is

$$£603{\cdot}6020224 + £12{\cdot}072040448 = £615{\cdot}674062848;$$

and on subtracting the original principal £536·6 from this amount, the total interest is obtained.

$$\therefore \text{ the interest} = £79{\cdot}074062848$$
$$= £79.\ 1s.\ 5\tfrac{3}{4}d.$$

The method of calculation in its more common form is as follows:

Principal for first	year	= £536·6.	
Interest	,,	,, = £21·464	(multiply by 0·04).
Principal for second	,,	= £558·064	(add the 4% interest).
Interest	,,	,, = £22·3226	(work only to nearest figure in 4th decimal place).
Principal for third	,,	= £580·3866.	
Interest	,,	,, = £23·2155.	
Principal for last half	,,	= £603·6021.	
Interest	,,	,, = £12·0720.	
Final amount		= £615·6741.	
Original principal		= £536·6.	
Interest		= £79·0741	(subtract the initial principal)
		= £79. 1s. $5\frac{3}{4}d.$	

In this example, the amount at the end of a full year $=(1{\cdot}04)$ times the principal at the beginning of the year; and the amount at the end of the final half year is $(1{\cdot}02)$ times the principal at the beginning of the year; accordingly, the final amount $= £(536{\cdot}6) \times (1{\cdot}04)^3 \times (1{\cdot}02)$.

It is well to put down the expression for the amount in this form even if the method of evalution be as in the example above.

EXAMPLE 2.—*Find the Compound Interest on £610. 6s. in 4 yrs. at 2½% per annum, the interest being payable quarterly.*

If the rate be 2½% per annum, then in ¼ year the interest on £100 will be ¼ of £2·5 = £0·625, and the interest on £1 will be £0·00625, so that the amount at the end of any quarter = (principal at the beginning of the quarter) multiplied by 1·00625.

The amount (£A) at the end of 4 years, or 16 quarters,

$$= £(610{\cdot}3) \times (1{\cdot}00625)^{16}.$$

Employing logarithms, we obtain

$$\begin{aligned} \log(A) &= \log 610{\cdot}3 + 16 \log 1{\cdot}00625 \\ &= 2{\cdot}7855434 + 16(0{\cdot}0027060) \\ &= 2{\cdot}7855434 + 0{\cdot}0432960 \\ &= 2{\cdot}8288394\,; \end{aligned}$$

$$\therefore\ A = 674{\cdot}279.$$

$$\begin{aligned} \text{The Interest} &= £674{\cdot}279 - £610{\cdot}3 = £63{\cdot}979 \\ &= £63.\ 19s.\ 7d. \end{aligned}$$

N.B.—Four-figure logarithms are of little use in the evaluations of examples on Compound Interest; seven-figure tables should be employed.

EXAMPLE 3.—*Find the Compound Interest on £736. 4s. in 4 yrs. at 3% per annum.*

Frequently it may be convenient to draw up an interest table giving the amounts at the end of various years, the original principal being supposed to be £1. Thus, in the present example,

£1 at the end of 1 year becomes 1·03,
" " 2 years " 1·0609 (add 0·03 times 1·03),
" " 3 " " 1·092727 (add 0·031827 *i.e.* 0·03 times 1·0609),
" " 4 " " 1·125509 (add 0·032782).

true to the 6th decimal place, though this may not always be accurate enough.

The Compound Interest on £736. 4*s* is therefore

$$(£736\cdot2)(0\cdot125509)$$
$$= £92\cdot400$$
$$= £92.\ 8s.$$

	0·125509
	736·2
	87·8563
	3·7653
	0·7531
	251
	92·3998

EXAMPLE 4.—*What sum will amount to* £1389. 3*s. in* 3 *yrs. at* 5 % *per annum, Compound Interest, payable yearly?*

£1 under these conditions will amount to £$(1\cdot05)^3$.
£$(1\cdot05)^3$ has resulted from the investment of £1, and
£1389. 3*s.* from the investment of £$(1389\cdot15)\div(1\cdot05)^3$.
Therefore the sum required = £$(1389\cdot15)\div1\cdot157625$
= £1200.

In finding both the rate per cent. and the time, logarithms are generally necessary.

EXAMPLE 5.—*At what rate per cent. per annum will* £1100 *amount to* £1202. 15*s.* 9*d. in* 3 *yrs., the interest being compound and payable half-yearly?*

If $2r$ be the nominal rate per annum,

£100 becomes £$(100+r)$ at the end of one half year.

Hence,

£1 " £$\frac{100+r}{100}$ " " "

£1 " £$\left(\frac{100+r}{100}\right)^6$ " six half years.

£1100 " £$1100\left(\frac{100+r}{100}\right)^6$ " " "

$$\therefore\ £1202.\ 15s.\ 9d. = £1100\left(\frac{100+r}{100}\right)^6;$$

$$\therefore\ \left(\frac{100+r}{100}\right)^6 = \frac{1202{\cdot}7875}{1100},$$

$$\frac{100+r}{100} = \sqrt[6]{\left(\frac{1202{\cdot}7875}{1100}\right)}.$$

The right-hand expression is then evaluated by logarithms.

$$\log\left(\frac{100+r}{100}\right) = \tfrac{1}{6}(\log 1202{\cdot}7875 - \log 1100)$$
$$= \tfrac{1}{6}(3{\cdot}0801889 - 3{\cdot}0413927)$$
$$= \tfrac{1}{6}(0{\cdot}0387962)$$
$$= 0{\cdot}0064660$$
$$= \log 1{\cdot}015 = \log\left(\frac{100+1{\cdot}5}{100}\right),$$

whence $r = 1{\cdot}5$ and $2r = 3$.

The nominal rate per cent. is 3.

EXAMPLE 6.—*In what time will £1400 amount to £2171. 17s. 2d. at 5 % per annum, Compound Interest, payable yearly?*

If t represent the number of years,

$$£(1400)(1{\cdot}05)^t = £2171.\ 17s.\ 2d. = £2171{\cdot}858.$$

Employ logarithms.

$$t = \frac{\log(2171{\cdot}858) - \log 1400}{\log 1{\cdot}05} = \frac{3{\cdot}3368314 - 3{\cdot}1461280}{0{\cdot}0211893}$$

$$= \frac{0{\cdot}1907034}{0{\cdot}0211893} = 9 \text{ approx.},$$

whence the time is 9 years.

EXERCISES LXXXVII. a.

Mental or Oral.

Give expressions for the following amounts, the interest being compound and payable yearly:

1. £1000 in 3 yrs., at 4 % per annum.
2. £500 in 3 yrs., at 2 %.
3. £500 in 2 yrs., at 3 %.

Give expressions for the following amounts, the interest being compound and payable yearly:

4. £1200 in 4 yrs., at $2\frac{1}{2}$ %.
5. £800 in 5 yrs., at 3 %.
6. £800 in 3 yrs., at 5 %.
7. £900 in 6 yrs., at 4 %.
8. £750 in $6\frac{1}{2}$ yrs., at 4 %.
9. £100 in $4\frac{1}{2}$ yrs., at 6 %.
10. £400 in 10 yrs., at 4 %.
11. £400 in 10 yrs., at $3\frac{1}{4}$ %.
12. £500 in $2\frac{1}{2}$ yrs., at 5 %.
13. £750 in $3\frac{1}{2}$ yrs., at 6 %.
14. £1000 in $4\frac{1}{2}$ yrs., at 4 %.
15. £1000 in $4\frac{1}{2}$ yrs., at $4\frac{1}{2}$ %.

If the nominal rate per cent. per annum be 4, but the interest be payable quarterly, then the rate per cent. per quarter is 1, and £1 becomes $£(1{\cdot}01)^4$ at the end of a year; but $(1{\cdot}01)^4 = 1{\cdot}04060401$, so that 4 % per annum **nominal rate**, the interest being payable quarterly, is the same as 4·060401, **effective rate**, payable yearly.

EXERCISES LXXXVII. b.

Mental or Oral.

Give expressions for the interest in the following cases:

Compound Interest on

1. £500 in 3 yrs., at 4 %, payable half-yearly.
2. £500 in $3\frac{1}{2}$ yrs., at 4 %, payable half-yearly.
3. £500 in $3\frac{1}{4}$ yrs., at 4 %, payable half-yearly.
4. £1000 in 1 yr., at 6 %, payable monthly.
5. £200 in 9 mo., at 3 %, payable quarterly.
6. £1000 in 3 yrs. 4 mo., at 3 %, payable three times a year.
7. £1200 in $2\frac{1}{2}$ yrs., at 6 %, payable monthly.
8. £1 in 9 mo., at 4 %, payable quarterly.

EXERCISES LXXXVII. c.

1. Find the Compound Interest in the following cases :

(*a*) £1500 for 2 yrs., at 5 % per annum, payable yearly.
(*b*) £80. 7*s*. 6*d*. for 4 yrs., at 4 %, „
(*c*) £318. 15*s*. for 2 yrs., at 3 %, „
(*d*) £506. 8*s*. for 3 yrs., at 2½ %, „
(*e*) £700 for 2½ yrs., at 4 %, „
(*f*) £1986. 12*s*. for 2 yrs., at 3⅛ %, „
(*g*) £3218. 1*s*. 6*d*. for 5 yrs., at 2½ %, „
(*h*) £830. 14*s*. for 3 yrs. 3 mo., at 3 %, „
(*i*) £38,626. 6*s*. 6*d*. for 5½ yrs., at 5 %, „

2. Find the *effective* rate, when the *nominal* rate per annum is as follows :

(*a*) 4 %, payable half-yearly. (*b*) 4 %, payable monthly.
(*c*) 6 %, payable monthly. (*d*) 6 %, payable half-yearly.

3. Draw up a set of Interest tables, showing the sums of money to which £1 would amount in 1, 2 ... 10 yrs., at

(*a*) 1 %, (*b*) 1¼ %, (*c*) 2 %, (*d*) 2½ %,
(*e*) 3 %, (*f*) 4 %, (*g*) 5 %,

Compound Interest, payable yearly. State the answers true to the 7th decimal place.

4. Employ the tables drawn up in Ex. **3**, to calculate the Interest on

(*a*) £350 in 2 yrs., at 4 % per annum, payable half-yearly.
(*b*) £440. 7*s*. 6*d*. in 1½ yrs., at 5 %, payable quarterly.
(*c*) £153. 12*s*. in 3 yrs., at 5 %, payable half-yearly.
(*d*) £1080 in 4 yrs., at 2 %, payable half-yearly.
(*e*) £2356. 12*s*. in 2 yrs., at 10 %, payable quarterly.
(*f*) £1875 in 5 yrs., at 4 %, payable half-yearly.
(*g*) £366 in 2 yrs. 3 mo., at 4 %, payable quarterly.

EXERCISES LXXXVII. d.

1. In what time will £306 amount to £372. 7*s*. 9*d*., at 4 % per annum, compound interest, payable yearly?

2. The sum of £5200 was invested by the trustees of an orphan when he was 13 yrs. old. When the orphan became of age (21) the sum had amounted to £6597. 4*s*. 9*d*. What was the rate of interest, payable yearly?

3. At what rate per cent. per annum, compound interest, payable half-yearly, will £333. 6*s*. 8*d*. amount to £375. 7*s*. 9*d*. in 3 yrs.?

4. Find the sum of money which will amount to £1292. 5*s*. 7*d*. in 3 yrs., at $2\frac{1}{2}$ % per annum, compound interest, payable yearly.

5. What sum of money, invested at 2 % per annum, compound interest, at the birth of a boy, would amount to £2000 when the boy attained the age of 25 years?

6. In what time will £1300 amount to £1436. 0*s*. 4*d*., at 4 % per annum, compound interest, payable quarterly?

7. Employ logarithmic tables to find the *effective rate* of interest, if the nominal rate be 1 % per annum, but payable at equal periods.

(*a*) 10 times a year. (*b*) 50 times a year. (*c*) 100 times a year.

N.B.—Note that the rates are all very nearly the same. The answers approximate to what the effective rate would be, if the turn over was immediate.

8. Employ logarithms to calculate what the effective rate would be if the nominal rate was 5 %, payable at the same intervals as in Ex. 7.

The Compound Interest Law. Geometric Progression.

If a sum of money, say £65, be put out at Compound Interest—for example 5 per cent., payable yearly—the amounts at the end of various years would be as follows:

Amount £65 (originally) becomes at the end of the

1st yr. 2nd yr. 3rd yr. 4th yr.

£65(1·05), £65(1·05)^2, £65(1·05)^3, £65(1·05)^4, etc.,

where it is seen that the amount at the end of any year is equal to the amount at the end of the preceding year multiplied by a constant factor, 1·05.

A series of terms of this kind is said to constitute a **Geometric Progression.**

The following examples illustrate the various processes employed in connexion with such series:

EXAMPLE 1.—*Find the nth term of the series in geometric progression, of which the first five terms are* 3, 6, 12, 24, 48.

In this series the 1st term is 3,

2nd ,, 3×2,

3rd ,, 6×2 or 3×2^2,

4th ,, 12×2 or 3×2^3

and 5th ,, 24×2 or 3×2^4,

and so on generally; the 10th for example would be $3 \times 2^{10-1}$.

,, nth ,, ,, $3 \times 2^{n-1}$.

EXAMPLE 2.—*A person possesses an annuity of* £48 *a year, for* 10 *years, payable quarterly, the first payment being made in* 1 *quarter's time. What is the present value of such an annuity, at Compound Interest, reckoned at* 3% *per annum, and also payable quarterly?*

Here, a sum of money is to be found which would be the fair equivalent of the 40 payments.

But, £1 paid now would amount to £(1·0075) in 3 months' time; and

a payment of £1 in 3 months' time would be the equivalent of $£\frac{1}{1{\cdot}0075}$ now; while

a payment of £1 in 6 months' time would be the equivalent of $£\left(\frac{1}{1{\cdot}0075}\right)^2$ now; and

the present value in £ of the 40 payments of £12 would be

$$\frac{12}{1{\cdot}0075}+\frac{12}{(1{\cdot}0075)^2}+\frac{12}{(1{\cdot}0075)^3}+\ldots\frac{12}{(1{\cdot}0075)^{40}}=P \text{ (say)},$$

constituting a Geometric Progression, with $\frac{1}{1{\cdot}0075}$ as the **constant ratio** or **factor.**

Multiply by $\frac{1}{1{\cdot}0075}$, that is, by the constant factor or ratio.

Then

$$\frac{12}{(1{\cdot}0075)^2}+\frac{12}{(1{\cdot}0075)^3}+\ldots\frac{12}{(1{\cdot}0075)^{40}}+\frac{12}{(1{\cdot}0075)^{41}}=P\frac{1}{1{\cdot}0075}.$$

Subtract this from the series for P.

$$\frac{12}{1{\cdot}0075}-\frac{12}{(1{\cdot}0075)^{41}}=P-\frac{P}{1{\cdot}0075};$$

$$\therefore\ P(1{\cdot}0075-1)=12-\frac{12}{1{\cdot}0075^{40}};$$

$$\therefore\ P(0{\cdot}0075)=12-\frac{12}{1{\cdot}0075^{40}}. \quad \ldots\ldots\ldots\ldots(1)$$

But $$\log\frac{12}{(1{\cdot}0075)^{40}}=\log 12-40\log(1{\cdot}0075)$$

$$=1{\cdot}0791813-40(0{\cdot}0032451)=1{\cdot}0791813-0{\cdot}129804$$

$$=0{\cdot}949377$$

$$=\log 8{\cdot}89974;$$

$$\therefore\ \frac{12}{(1{\cdot}0075)^{40}}=8{\cdot}89974;\ \text{and}$$

$$\therefore\ \text{from (1)}\ P=\frac{12-8{\cdot}89974}{0{\cdot}0075}=\frac{3{\cdot}10026}{0{\cdot}0075}=413{\cdot}368.$$

The present value is ∴ £413. 7*s.* 4*d.*

EXAMPLE 3.—*A man puts by the sum of £50 every year at 3% Compound Interest, payable yearly; what will be the accumulated sum just before the 21st payment?*

The first £50 accumulates interest for 20 yrs., and therefore eventually amounts to $£50(1{\cdot}03)^{20}$.

The second £50 accumulates interest for 19 yrs., and therefore amounts to $£50(1{\cdot}03)^{19}$.

The last £50 accumulates interest for 1 yr., and amounts to £50(1·03).

The accumulated sum in £ is therefore

$$50(1{\cdot}03)+50(1{\cdot}03)^2+\ldots\ 50(1{\cdot}03)^{20}=\mathrm{S}\ \text{(say)}.$$

Multiply by (1·03), the constant ratio, and subtract, as in the last example:

$$\therefore\ 50(1{\cdot}03)^{21}-50(1{\cdot}03)=\mathrm{S}(0{\cdot}03).$$

Employ logarithms, and $(1{\cdot}03)^{21}$ is found to be 1·86029.

Whence, $$\mathrm{S}=\frac{50\{0{\cdot}83029\}}{0{\cdot}03}.$$

The accumulated sum = £1383. 16*s.* 0*d.* approx.

EXAMPLE 4.—*The population of the City of London was* 37702 *in* 1891 *and* 26923 *in* 1901. *If the rate of change per* 1000 *per year remained constant, what would you expect the population to be in* 1906?

Suppose the population at the end of a year = population at the beginning of the year multiplied by a factor x. Then

$$\text{the population } 26923=(37702)x^{10};$$

$$\text{the estimated population in } 1906=(26923)x^5$$

$$=(26923)\left\{\frac{26923}{37702}\right\}^{\frac{5}{10}}=(26923)\left\{\frac{26923}{37702}\right\}^{\frac{1}{2}}=\frac{(26923)^{\frac{3}{2}}}{(37702)^{\frac{1}{2}}}$$

$$=\mathrm{P}\ \text{(say)}.$$

Employ logarithms.

$$\begin{aligned}\log \mathrm{P}&=\tfrac{3}{2}\log 26923-\tfrac{1}{2}\log 37702\\&=6{\cdot}6451852-2{\cdot}2881822\\&=4{\cdot}3570030\\&=\log 22751.\end{aligned}$$

Estimated population is 22751.

EXAMPLE 5.—*Find the sum of an infinitely large number of terms of the series* $2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots$.

Let s be the sum.

$s=2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots$ for an infinitely great number of terms.

Multiply by the constant factor $\frac{1}{3}$.

$\frac{1}{3}s = \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$ for an infinitely great number of terms; but the end terms become indefinitely small, whence on subtraction

$$s - \tfrac{1}{3}s = 2\,;$$

$$\therefore\ s = \frac{2}{1 - \frac{1}{3}} = 3.$$

EXAMPLE 6.—*Convert* $0{\cdot}\dot{3}6\dot{5}$ *into a vulgar fraction.*

$$0{\cdot}\dot{3}6\dot{5} = 0{\cdot}365 + 0{\cdot}000365 + 0{\cdot}000000365 + 0{\cdot}000000000365 + \ldots,$$

$$\frac{0{\cdot}\dot{3}6\dot{5}}{1000} = \qquad 0{\cdot}000365 + 0{\cdot}000000365 + 0{\cdot}000000000365 + \ldots,$$

subtract

$$(0{\cdot}\dot{3}6\dot{5})\left(1 - \frac{1}{1000}\right) = 0{\cdot}365\,;$$

$$\therefore\ 0{\cdot}\dot{3}6\dot{5} = \frac{365}{999}.$$

EXERCISES LXXXVII. e.

1. Express the following recurring decimals as series in Geometric Progression, and hence calculate the decimals as equivalent vulgar fractions:

(*a*) $0{\cdot}\dot{4}$; (*b*) $0{\cdot}1\dot{2}\dot{3}$; (*c*) $2{\cdot}2\dot{3}\dot{6}$.

2. Calculate the present value of

(*a*) £109. 5*s*. $8\frac{1}{2}$*d*., due 3 years hence, at 3 % per annum, compound interest, payable yearly.

(*b*) £763. 7*s*. 8*d*., due 20 years hence, at 4 % per annum, compound interest, payable yearly.

3. Verify the entries in the table at the end of the book of present values in the following cases:

(*a*) By direct calculation, for

(i) £1 in 3 yrs., at 3 %, payable yearly.

(ii) £1 in 4 yrs., at 5 %, „ „

(iii) £1 in 6 yrs., at 4 %, „ „

(*b*) By the use of logarithms :

(i) £1 in 15 yrs., at $2\frac{1}{2}$ %, payable yearly.

(ii) £1 in 20 yrs., at $3\frac{1}{2}$ %, „ „

(iii) £1 in 30 yrs., at $4\frac{1}{2}$ %, „ „

(*c*) Assuming the truth of the entries, as verified in (*b*), prove the truth of the entries corresponding to

(i) £1 in 16 yrs., at $2\frac{1}{2}$ %.

(ii) £1 in 21 yrs., at $3\frac{1}{2}$ %.

(iii) £1 in 31 yrs., at $4\frac{1}{2}$ %.

4. Evaluate the present value of the following annuities, the first payment being made at the end of the first year :

(*a*) £100 for 10 yrs., reckoning compound interest at 3 % per annum.

(*b*) £250 „ 14 „ „ „ 4 % „

(*c*) £80 „ 7 „ „ „ $2\frac{1}{2}$ % „

(*d*) £120 „ 16 „ „ „ 3 % „

5. A steel ball, allowed to bounce vertically, rises to heights which gradually decrease in such a way that the height in any one bounce is 80 % of the height in the preceding bounce. What is the height of the 5th bounce, if the ball be first thrown up to a height of 6 ft.? Find also the total distance moved through before finally stopping.

6. A ball bounces as in Ex. **5**, but so that the height of any bounce is a certain multiple (x) of the height of the bounce immediately before. It rises to a height of 2·4 metres when first thrown up, and moves through a total distance of 16 metres before coming to rest. Find x.

7. When a rope is coiled round a post, a person exerting a small pull on one end can withstand a much greater pull at the other end, the pull decreasing, according to the compound interest law, with the number of turns of the rope.

If, when the rope is coiled once round the post, a pull, equal to the weight of 1 lb., can withstand a pull of 16 lb., find the pull

necessary to withstand a pull of 16 tons, if the rope be coiled 4 times round the post.

State the answer true to 3 significant figures.

8. When light passes through glass, a certain fraction of the light falling upon the glass passes through; this fraction varies with the thickness of the glass according to the compound interest law.

A certain piece of glass, 1 mm. thick, allows 95 % of the light to pass through; if the thickness be doubled, only $(0{\cdot}95)^2$ of the light passes through. How much of the light will be transmitted by a plate 2·5 cm. thick?

9. If £1 be put by yearly at 3 %, compound interest, payable yearly, then n contributions will amount to

$$1+1{\cdot}03+(1{\cdot}03)^2+(1{\cdot}03)^3+(1{\cdot}03)^4+\ldots+(1{\cdot}03)^{n-1}$$

directly the last contribution is paid.

Find the amount when n is successively (*a*) 3, (*b*) 4, (*c*) 5.

10. Verify (using logarithms) the entries in the table at the end of the book, giving the amounts in different times of yearly contributions of £1, in the case where the number of contributions is 20 and the rates of interest (*a*) 2·5 %, (*b*) 4 %, (*c*) 5 %.

11. A colliery is estimated to yield an income for 30 yrs., and the purchase money is £54,930. What sum of money should be put annually on one side, so as to redeem the purchase money at the end of 30 yrs., reckoning compound interest at 3 % per annum, payable yearly?

12. A person put aside £20 yearly for his son, beginning at birth. What amount was due to the son on his 21st birthday (including the 22nd instalment)? Assume that the money accumulated at compound interest, at 4 % per annum.

13. What annuity should be paid for 16 yrs. for £100 paid now, the first payment to be made in 1 year's time, and afterwards in yearly payments, reckoning compound interest at 2·5 % per annum?

14. A man, when 30 years old, puts by £40. 4*s.* every year, which sum accumulates at 4 % compound interest. What annuity could he buy with the accumulated proceeds when 55 yrs. old, if, at that age, he can obtain an annuity of £8 on payment of £100?

15. A loan of £1000 can be repaid in 5 years, together with its interest at 4 % per annum convertible half-yearly, by 10 equal half-yearly payments of £111. 6*s.* 6¼*d.* (or more exactly £111·32653). Construct and complete a table as indicated below, showing the amounts of principal and interest contained respectively in each half-yearly repayment :

Half-year. No.	Outstanding Principal in £.	Interest for Half-year in £.	Principal in £ repaid in each Half-yearly Repayment (*i.e.* difference between Interest and Repayment).
1	1000	20	91·327 (*i.e.* 111·327 − 20)
2	1000 − 91·327 (*i.e.* 908·673)	18·174	93·153 (*i.e.* 111·327 − 18·174)
3		?	
4	?	?	
5	?	?	
6	?	?	
7	?	?	
8	?	?	
9	?	?	
10	?	?	109·144
			Total 1000·00

16. A certain insurance company will pay £100 at the end of the given term of years for a single or annual premium as given by the following table:

PREMIUM.

To insure the return of £100 at the expiration of a given term of years.

Terms of Years.	Annual Premium.			Single Premium		
	£	s.	d.	£	s.	d.
10	9	3	8	78	6	6
20	3	18	5	58	5	8
30	2	4	4	43	7	5
40	1	8	0	32	5	5
50	0	18	9	24	0	3
60	0	12	11	17	17	4
70	0	9	2	13	5	11
99	0	3	7	5	12	11

Draw a graph showing the change of premium with the time. Show that the single premiums are not calculated *strictly* accordingly to the Compound Interest Law.

EXERCISES LXXXVII. f.

Practical.

[*Apparatus: Weight. String. Support. Scale. Glass marble. Graduated paper circle.*]

1. Suspend a heavy weight by two strings of the same length (about 6 ft. or more) to two hooks about 2 ft. apart. Fix a pointer to the weight and arrange a metre scale beneath, so that when the weight is allowed to swing, the pointer moves in front of the scale.

Note the extent of, say, 50 successive swings on the same side; determine the law according to which the swings die away.

(This is best done by a graph of the logarithms of the successive swings.)

2. Allow a glass marble to fall upon a stone slab; it rebounds; alter the distance through which it falls until the height of the

rebound is a definite, convenient amount (say 2 ft.). Find the ratio between the height of the rebound and the original fall. Similarly, obtain the ratio of the height of the second rebound to the original fall, and so on.

Arrange these ratios in a series, and determine its character.

3. Attach a weight with horizontal pointer to a support by means of a wire or fine thread; underneath the pointer fix a horizontal graduated circle; as the thread twists and untwists the arrangement will oscillate. Measure the 'sizes' of the successive oscillations, and determine the law according to which they decrease.

EXERCISES LXXXVII. g.

Graphic.

1. In pumping out the air from a vessel by means of an air-pump of the Hawksbee type, the amount of air in the vessel decreases with the number of strokes according to the compound interest law. The initial amount of air is 0·12 gr., and after the first stroke the amount is only 0·096 gr. Draw a graph showing how (*a*) the amount changes with the number of strokes; (*b*) the logarithm of the amount changes with the logarithm of the number of strokes.

Find (*c*) when the amount present will only be 0·001 gr.

2. Use the tables at the end of the book to draw a graph showing the sum to which yearly payments of £1 will amount in different times at (*a*) $2\frac{1}{2}\%$; (*b*) 3%; (*c*) 4%; (*d*) 5%, compound interest payable yearly.

Employ the graphs to find:

(*e*) The time in which the yearly payment of £1 at 3% will amount to £50.

(*f*) The amount after 18 yearly payments of £1 at 3%.

(*g*) The time when the amount at 5% is twice as much as the amount at $2\frac{1}{2}\%$.

3. In an electrical experiment, the discharge from a condenser through a galvanometer gave an initial throw or deflexion of 330 divisions; the deflexion gradually died away according to the compound interest law and the 15th deflexion was only 120 divisions. Draw a graph connecting the extent of the deflexion with the order of the deflexion (use logarithms). What was the amplitude of the 10th deflexion?

4. Draw a graph showing the present value of an annuity of £1 a year at (*a*) 2½ %; (*b*) 3 %; (*c*) 4 %; (*d*) 5 %. Use the tables at the end of the book. Employ your graph to calculate (*e*) the value of an annuity of £1 for 12 years at 2½ %; (*f*) the duration of an annuity of £1, bought for £16, interest 2½ %.

5. Draw a graph showing the present value of an annuity of £1 a year for 11 years as the rate per cent. changes from 2½ % to 5 %.

Rate, - -	2½ %	3 %	3½ %	4 %	4½ %	5 %
Present value,	9·514	9·252	9·002	8·761	8·529	8·306

What is (*a*) the present value of the above annuity at 3½ %; (*b*) the rate per cent. if the present value is 8 guineas?

6. For how many years must a man put by a certain sum yearly in order to obtain afterwards an annuity of double that sum for the same number of years, the rate per cent. being (*a*) 4 %, (*b*) 5 %. (Find the intersection of a graph showing the present value of £1 yearly for 1, 2, 3, ... years, and a graph showing the difference between the present value of £2 for 1 and 2 years, 2 and 4 years, 3 and 6 years and so on.)

7. Draw a graph showing how the present value of £10 varies with the time, the rate of interest being 5 %; on the same piece of paper and to the same scale draw also a graph showing how the present value of yearly payments of £1 varies with the time. Find the point of intersection of the two, and hence find how many payments of £1 yearly are the equivalent of a payment of £10 at the end of the time.

8. From a man A, a bank receives £1000 on deposit at $2\frac{1}{2}\%$ payable yearly. The bank, however, lends this £1000 to B at 5 % payable yearly. The interest received by the bank can also yield 5 % payable yearly. Find when the bank can repay the £1000 to A, together with the accumulated interest, and still have the £1000 to its credit on account of the loan to B.

(Draw a graph showing the amount of £1000 at various times at $2\frac{1}{2}\%$, also a graph showing the amount on account of yearly payments of £50 at 5 %. Find the point of intersection.)

9. The table below gives the number of years' purchase of a lease as the duration of the lease changes, if the rate of interest be assumed at 10 %.

Find (*a*) the years' purchase if the duration be 11 years,
(*b*) the duration if the years' purchase be 8.

Years' duration	24	20	16	12	8	4	1
Years' purchase	9	8·5	7·8	6·8	5·34	3·2	0·91

62. THE AREA OF A TRIANGLE.

It has already been seen that **the area of a triangle is equal to half the product of the length of its base and its altitude.** The area can also be calculated when the lengths of the three sides are given.

EXAMPLE 1.—*Find the area of a triangle* ABC *of which the sides* AB, BC, CA *are* 26, 42 *and* 40 *units of length respectively.*

In Fig. 78 let AD be at right angles to BC, O the middle point of BC.

$$AB^2 = BD^2 + AD^2,$$

$$AC^2 = CD^2 + AD^2;$$

$$\therefore\ AC^2 - AB^2 = CD^2 - BD^2.$$

But $$(CD - BD)(CD + BD) = 2(OD)(CB).$$

Hence, $$OD = \frac{CD^2 - BD^2}{2BC} = \frac{AC^2 - AB^2}{2BC}$$

and $BD = BO - DO$.

Since $AD^2 = AB^2 - BD^2$, AD is now known, and the area of the triangle $= \frac{1}{2}\{AD \times BC\}$.

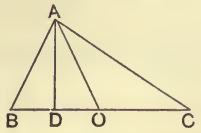


FIG. 78.

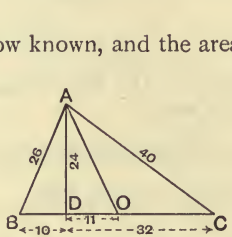


FIG. 79.

In this example $CD^2 - DB^2 = 40^2 - 26^2 = 66 \times 14$;

and $2BC \cdot OD = 2(42)(OD)$;

$\therefore\ OD = \frac{66 \times 14}{84} = 11$;

$\therefore\ BD = \frac{42}{2} - 11 = 10$,

$AD^2 = 26^2 - 10^2 = 36 \times 16$; $\therefore\ AD = 24$,

and the area $= \frac{1}{2} \times 24 \times 42 = 504$ units.

In a great many cases application is made of the fact, that **the areas of similar triangles** (*i.e.* triangles of the same shape) **are proportional to the squares of their corresponding sides.** (Exercises LXXIX. b.)

EXAMPLE 2.—ABC *is a triangle, and the portions* ADE, CFG, BHI *are cut off;* DE *being parallel to* BC.
FG ,, ,, AB.
IH ,, ,, AC.

If AB *is* 7 in., BC 6 in., CA 8 in., *and if* AD *is* 1 in., BI 2 in., CF 1 in., *what fraction is the area of the remaining portion* DEFGHI *of the whole area* ABC?

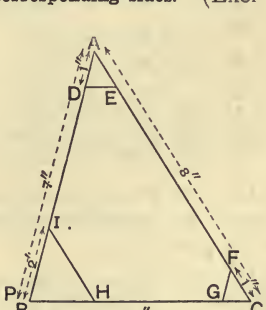


FIG. 80.

Since AD = 1 in. and AB = 7 in. and the triangles ADE, ABC are of the same shape, the area of ADE $= \frac{1^2}{7^2}$ of the area of ABC.

Similarly, the area of BIH $= \frac{2^2}{7^2}$ of the area of ABC.

„ CFG $= \frac{1}{8^2}$ „ „

The area of the three triangles together $= \left(\frac{1}{7^2} + \frac{2^2}{7^2} + \frac{1}{8^2}\right)$ of the area of ABC.

Hence DEFGHI has an area of $1 - \frac{369}{3136}$ of the whole triangle $= \frac{2767}{3136}$ of the triangle ABC.

EXERCISES LXXXVIII.

ABC is a triangle, D the foot of the perpendicular from A to BC, O the middle point of BC.

Calculate (*a*) OD, (*b*) AD, (*c*) the area of the triangle ABC in the following cases (the lengths are given in inches):

1. AB= 65, BC= 57, CA=68.
2. „ =156, „ =176, „ =68.
3. „ = 34, „ = 42, „ =20.
4. „ = 51, „ = 55, „ =26.
5. „ = 51, „ = 77, „ =40.
6. „ = 20, „ = 21, „ =13.
7. „ =100, „ = 91, „ =61.
8. „ = 65, „ = 36, „ =61.
9. AB=b, BC=a, CA=b.
10. AB=a, BC=a, CA=a.

11. The area of a triangle ABC is 5000 sq. in., the base BC is 100 in., the side AB is 125 in.; calculate the length of the side AC to the nearest inch.

12. If in the preceding exercise the area=2730 sq. in., BC=91 in., AB=61 in., find the value of AC.

13. Let the sides of a triangle ABC be represented by BC=a, CA=b, AB=c, and let AD be the perpendicular drawn from A to BC, and O the middle point of BC.

Show that (i) $\mathrm{OD} = \frac{b^2 \sim c^2}{2a}$, [*The symbol $\sim$ means that the smaller of the two quantities is to be taken from the greater.*]

(ii) $\mathrm{AD}^2 = \frac{a^2c^2 - (a^2 - b^2 + c^2)^2}{4a^2}$,

(iii) $\mathrm{ABC} = \left\{\frac{4a^2c^2 - (a^2 - b^2 + c^2)^2}{16}\right\}^{\frac{1}{2}}$.

Also show that the last expression

$$= \left\{\frac{(a+b+c)(a+b-c)(b+c-a)(a-b+c)}{16}\right\}^{\frac{1}{2}}.$$

This last formula is suitable for logarithmic computation.

14. Apply the expression developed in the preceding exercise to the calculation (true to four significant figures) of the area of the triangles, the sides of which are respectively:

(*a*) 203, 218, 324. (*b*) 96·02, 89·3, 52. (*c*) 825·4, 1018, 918·2.
(*d*) 93·7, 135, 141. (*e*) 671, 528, 609. (*f*) 42·3, 59·3, 37·6.
(*g*) 972, 971, 63·2. (*h*) 361, 258, 372. (*i*) 1096, 1342, 1723.
(*j*) 1562, 437, 1485. (*k*) 2374, 1836, 1972. (*l*) 692, 347, 560.

15. ABC is a triangle, D and E two points on AB and AC respectively, such that $\mathrm{AD} \div \mathrm{AB} = \mathrm{AE} \div \mathrm{AC}$. Find the ratio of the portions into which the triangle is divided by DE when

$\frac{\mathrm{AD}}{\mathrm{AB}}$ is (*a*) $\frac{1}{2}$, (*b*) $\frac{1}{3}$, (*c*) $\frac{2}{3}$, (*d*) $\frac{3}{4}$, (*e*) $\frac{4}{5}$, (*f*) $\frac{x}{y}$, (*g*) $\frac{1}{x}$.

16. ABC is a triangle, such that BC = 10 in., CA = 11 in., AB = 12 in. DEF are points on BC, CA, AB respectively, such that BD = 3 in., CE = 4 in., AF = 5 in.

Calculate the areas of the triangles

(*a*) BDF, (*b*) CDE, (*c*) AFE, (*d*) DEF

in terms of that of the triangle ABC.

17. In a triangle ABC, BC = 13 in., CA = 15 in., AB = 16 in. DE is a line parallel to BC, dividing the triangle ABC into two equal parts and meeting AB, AC, at D and E respectively: find the length of AD true to 3 significant figures.

63. BILLS OF EXCHANGE AND BANKER'S DISCOUNT.

Debts are frequently cancelled by means of **Bills of Exchange.** A bill of exchange may be drawn up in the following fashion:

London, 13th March, 1905.

£375. 10*s.*

Twenty-five days after date pay to the order of Mr. A. B. the sum of Three Hundred and Seventy-five Pounds Ten Shillings, value received. *C. D.*

Mr. E. F., Sheffield.

Such a bill would be **drawn** by C. D. and **accepted** by E. F., who would write his acceptance across the bill, while A. B. is the **payee.**

Once the bill has been accepted, it becomes negotiable, and may pass through several hands before being presented finally at a Bank. If presented before the date of maturity, together with three days of Grace (*i.e.* 13th March + 25 days + 3 days), *i.e.* April 10th, the banker deducts an amount of **Discount, calculated in the same manner as ordinary interest on the number of days, through which the bill has still to run.**

Thus, in the above example, if the rate of discounting be 3 %, and if the bill be discounted on March 31st, that is 10 days before April 10, the banker deducts

$$\frac{10}{365} \times \frac{3}{100} \times £375.\ 5s. = £0{\cdot}308 = 6s.\ 2d.$$

The **Banker's Discount** is therefore 6*s.* 2*d.*

EXERCISES LXXXIX.

Find the Banker's discount on the following bills (3 days' Grace to be included):

	Bill for	Maturing on	Date of Discounting.	Rate °/₀
1.	£890. 12*s.* 6*d.*	May 4, 1905	April 3, 1905	3
2.	£1896. 10*s.*	3 mo. from April 17, 1905	June 18, 1905	3
3.	£250. 17*s.* 6*d.*	June 10, 1906	June 1, 1906	$2\frac{1}{2}$
4.	£3864	6 mo. from April 3, 1906	June 10, 1906	4
5.	£7876. 5*s.*	6 mo. from Feb. 2, 1906	May 8, 1906	$3\frac{3}{4}$
6.	£1750	March 10, 1904	Feb. 3, 1904	4
7.	£387. 17*s.* 6*d.*	March 12, 1904	Feb. 16, 1904	$4\frac{1}{2}$
8.	£2750	6 wks. from Ap. 3, 1904	Mar. 28, 1904	$3\frac{1}{2}$
9.	£1568	Sept. 15, 1905	Sept. 1, 1905	$2\frac{3}{4}$
10.	£18,763	Oct. 14, 1905	Sept. 29, 1905	4
11.	£1266. 13*s.* 4*d.*	Dec. 11, 1905	Dec. 1, 1905	$3\frac{1}{2}$
12.	£836. 17*s.* 6*d.*	Dec. 23, 1905	Dec. 10, 1905	$2\frac{3}{4}$
13.	£9451. 12*s.* 6*d.*	Nov. 26, 1904	Nov. 23, 1904	3
14.	£8615	4 wks. from Ap. 15, 1905	April 1, 1905	$2\frac{1}{2}$
15.	£1020. 11*s.* 9*d.*	July 28, 1906	July 3, 1906	3
16.	£1237. 8*s.* 4*d.*	Aug. 3, 1906	July 1, 1906	$3\frac{1}{2}$
17.	£1193. 1*s.* 6*d.*	70 days from July 2, 1906	Sept. 1, 1906	4
18.	£8765	25 days from May 2, 1906	May 15, 1905	$2\frac{3}{4}$
19.	£9876	4 mo. from Dec. 4, 1904	Jan. 24, 1905	$4\frac{1}{2}$
20.	£219. 12*s.* 3*d.*	5 mo. from Aug. 1, 1904	Nov. 1, 1904	3

Find the terms missing from the following tables :

	Bill.	Date of Maturing.	Date of Discounting.	Banker's Discount.	Rate °/₀
21.	£376	April 3, 1906	?	£3. 15*s.* 2½*d.*	5
22.	?	July 28, 1905	July 10, 1905	£1. 5*s.* 2½*d.*	2½
23.	?	May 30, 1904	Mar. 21, 1904	£8. 15*s.*	3½
24.	£1200	?	June 1, 1905	£2. 19*s.* 9½*d.*	2¾
25.	£3750	Sept. 17, 1905	?	£2. 11*s.* 9*d.*	2½
26.	£1872. 10*s.*	Aug. 28, 1906	Aug. 1, 1906	£4. 13*s.* 6*d.*	?
27.	£4015	Oct. 10, 1905	Aug. 31, 1905	£14. 3*s.* 9½*d.*	?
28.	£456. 5*s.*	July 28, 1905	?	£0. 18*s.* 9*d.*	2½
29.	£608. 6*s.* 8*d.*	?	Feb. 26, 1905	£1. 8*s.*	3½
30.	£406. 13*s.* 4*d.*	Jan. 26, 1905	?	£0. 15*s.*	4½
31.	£501. 17*s.* 6*d.*	May 22, 1905	May 1, 1905	£0. 16*s.* 6*d.*	?

64. CALCULATION OF AREAS (*continued*).

Conical Surfaces.

Suppose AOB represents a sector of a circle (*i.e.* an area bounded by the circular arc AB and the two radii OA and OB).

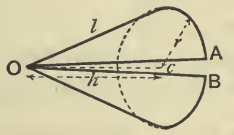
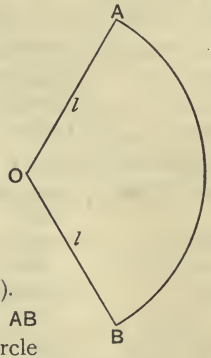


FIG. 81.



Its area = ½OA (length of arc AB).

Wrap up the surface so that AB becomes the circumference of a circle and AO coincides with BO. The surface now becomes **Conical**, and it is clear that

The curved surface of the cone = area of the sector,
and ∴ ,, ,, $= \frac{1}{2}$**(slant edge)(circumference of base).**

∴ If h be the distance between the apex O and the centre c of the base (Fig. 81),

r = the radius of the base
and l = the slant edge.

The area of a cone

$$= \tfrac{1}{2} l \times 2\pi r = \pi l r$$
$$= \sqrt{h^2 + r^2} \,.\, \pi r.$$

In dealing with frusta of a cone the principle of similar figures can often be used.

(The **frustum** of a cone is the part between the base and a plane parallel to the base.)

If a frustum of a cone be unwrapped, it will be seen that the curved surface is the difference between the areas of two similar circular sectors (Fig. 82).

If n be the number of degrees formed by the two bounding radii of the sector and L_1 and L_2 the radii,

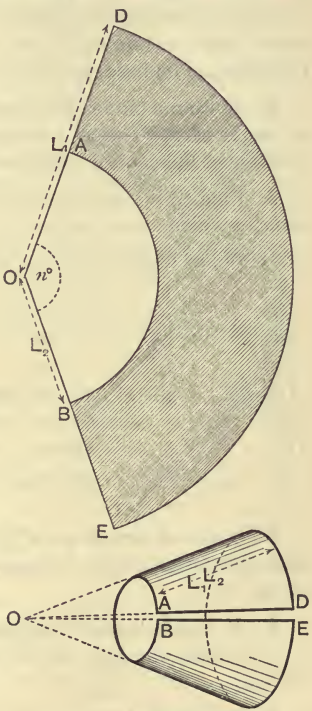


FIG. 82.

the area of the sector $= \frac{n}{360}\pi L_1^2 - \frac{n}{360}\pi L_2^2$ (see p. 333)

$$= \pi(L_1 - L_2)(L_1 + L_2)\frac{n}{360}$$

= (difference in radii)(mean of the two arcs),

whence **the curved surface of the frustum of a cone**
$= \pi$**(slant edge) × (mean of the diameters of the ends).**

EXERCISES XC.

1. A circle of 4 in. radius is divided into four equal sectors; if one of these sectors be folded into the form of a cone, what is (*a*) the radius of its base and (*b*) its altitude?

2. If, from a circle of 6 in. radius, a sector be formed containing an angle of 288°, what will be (*a*) the altitude, (*b*) the radius of the base of the conical surface which can be formed from it?

3. Find (i) the altitudes and (ii) radii of the bases of the conical surfaces which can be formed from the following sectors:

(*a*) Radius of circle 15 cm., angle subtended at the centre 30°
(*b*) ,, 18 cm., ,, ,, 45°
(*c*) ,, 9 cm., ,, ,, 50°
(*d*) ,, 36 cm., ,, ,, 160°
(*e*) ,, 14 cm., ,, ,, 209°
(*f*) ,, 13 cm., length of arc 31·41 cm.
(*g*) ,, 61 cm., ,, 69·11 ,,
(*h*) ,, 17 cm., ,, 50·26 ,,
(*i*) ,, 25 cm., ,, 94·24 ,,
(*j*) ,, *l*, ,, *s*.

4. A conical tent is to be 8 ft. high, the radius of its base 6 ft.; what amount of canvas is needed for its construction (neglecting seams, etc.)?

5. An old-fashioned candle snuffer was in the form of a cone; if its altitude was 3 in., radius of base 1 in., what was its outside surface?

6. The bearing of a shaft is in the form of a cone, 5 in. long, radius 3 in.; what is its total surface?

7. Find the curved surface of the following cones:

(*a*) Slant edge 10·125 in., radius of base 8 in.
(*b*) ,, 15 ft. ,, 5 ft.
(*c*) ,, 8 ft. ,, 7 ft.
(*d*) ,, 9 ft. ,, 6·7 ft.
(*e*) ,, 8 ft. ,, 4·6 ft.

8. A tin-plate basin is in the form of a frustum of a cone, with the diameters of the upper and lower rims 18 in. and 12 in. respectively, the slant edge is 5 in.; how many square inches of tin-plate does it contain?

9. A tin funnel is in the form of two frusta. The upper one has its upper rim 6 in. in diameter and lower rim 1·2 in. diameter; the lower frustum has its upper rim 1·2 in. diameter and its lower rim 0·6 in. diameter; the slant edges of the upper and lower frusta are respectively 5 in. and 4 in. What amount of tin-plate does the funnel contain?

10. A tin can consists of (1) an upper cylindrical portion 4 in. long, 6 in. diameter, (2) a lower cylindrical portion 18 in. long, 12 in. diameter, (3) a connecting conical frustum, axial length 4 in., and (4) a circular base. What is the total surface of tin-plate used in its construction?

11. Find the outside surface of a flower pot, of which the outer dimensions are: height 12 in., upper rim diameter 9 in., lower rim diameter 6 in., drainage hole in base 1 in. diameter.

12. Find the curved surface of the following conical frusta:

	Diameter of upper rim.	Diameter of lower rim.	Altitude.
(*a*)	1 cm.	18 cm.	4 cm.
(*b*)	13·5 in.	18·5 in.	6 in.
(*c*)	20 in.	42 in.	60 in.
(*d*)	3·4 in.	6·6 in.	3 in.
(*e*)	100 in.	340 in.	22 in.

Spherical Surfaces.

Suppose the sphere in Fig. 83 to be divided by parallel planes into a number of thin zones, such as PQ. The surface of all these zones make up the whole surface of the sphere. But the zone PQ may be also regarded as a portion of a

conical surface which touches the sphere, and with its apex, say, at T.

$\therefore$ the area of the zone $=$ PQ $\times$ mean perimeter (see p. 439) $=$ PQ $\times 2\pi$AP (PQ being very small).

But since PQT is perpendicular to OP, O being the centre of the sphere, and since AP is perpendicular to OA the $\triangle$s OAP, PTA have the same shape, and

$$\frac{AA'}{PQ} = \frac{AT}{PT} = \frac{AP}{OP},$$

where A, A′ are the centres of the circles through P and Q.

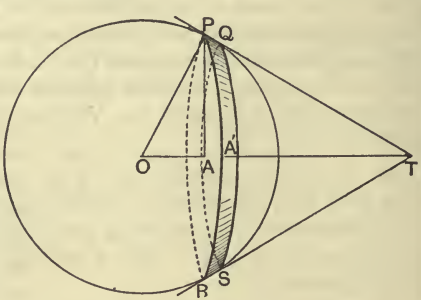


FIG. 83.

$\therefore$ AA′ $\times$ OP $=$ AP $\times$ PQ, and the surface of the zone

$$= 2\pi \text{OP} \times \text{AA}'.$$

Hence **the surface of a zone of a sphere of radius $r = 2\pi rd$, d being the distance between the two planes which bound the zone; and the surface of the complete sphere $= 2\pi r \cdot 2r = 4\pi r^2$.**

EXERCISES XCI. a.

Find the surfaces of the spheres of which the diameters are:

1. 3·11 in.	**2.** 12·3 in.	**3.** 2·02 cm.
4. 248 cm.	**5.** 30·2 cm.	**6.** 24·0 in.
7. 1·1 ft.	**8.** 50·2 cm.	**9.** 42 cm.

Find the diameters of the spheres of which the surfaces are:

10. 5728·84 sq. cm.	**11.** 6734·61 sq. ft.
12. 81·7130 sq. in.	**13.** 333·293 sq. cm.
14. 2865·26 sq. ft.	**15.** 1269·23 sq. cm.
16. 725·835 sq. in.	**17.** 4584·35 sq. yd.

EXERCISES XCI. b.

(π may be taken as 3·1416.)

1. If the earth be regarded as a sphere, having a radius 4000 miles in length, what is its surface?

2. What is the surface of a cricket ball, diameter 3·75 in.?

3. The curved surface of a hemisphere is 10331·48 sq. cm.; what is its radius?

4. A hollow brass sphere is 4 in. in diameter; what is the area of its outside surface? What also is the mass of the sphere, if the thickness be 0·01 in. (density of brass 8·816)?

5. If the surface of the earth (regarded as a sphere, radius 4000 miles) be supposed ruled by the 360 meridians or lines of longitude, and so divided into 360 equal parts, find in square miles the surface of the earth between longitudes

(*a*) 30° E. and 30° W. (*b*) 25° 6′ E. and 16′ E.
(*c*) 140° E. and 90° E. (*d*) 76° W. and 15° E.
(*e*) 25° 10′ W. and 45° 20′ E. (*f*) 126° 10′ W. and 10° W.
(*g*) 79° W. and 78° W.

6. The table below gives the distances from the centre of the earth (radius 4000 miles approx.) to the centres of the following parallels of latitude, while

$23\frac{1}{2}°$ N. is the Tropic of Cancer
$23\frac{1}{2}°$ S. „ Capricorn } These bound the Torrid Zone.
$66\frac{1}{2}°$ N. is the Arctic circle.
$66\frac{1}{2}°$ S. „ Antarctic circle.

Latitude - -	10°	20°	$23\frac{1}{2}°$	30°	40°	North
Distance = radius multiplied by	0·1736	0·3420	0·3987	0·5000	0·6428	
Latitude - -	50°	60°	$66\frac{1}{2}°$	70°	80°	North
Distance = radius multiplied by	0·7660	0·8660	0·9171	0·9397	0·9848	

Calculate in square miles to four significant figures the surface of

(a) each of the Frigid Zones.

(b) „ Temperate Zones.

(c) the Torrid Zone.

(d) the Earth between latitudes 20° N. and 40° N.

(e) „ „ 20° N. „ 40° S.

(f) the portion between parallels 20° N. and 40° S. and longitudes 10° E. and 11° E.

(g) a portion of a Frigid Zone between 80° E. and 85° 30′ E.

(h) a portion of the Torrid Zone between 75° E. and 10° E.

7. The inside of a glass goblet is made in the form of a portion of a hemisphere of 5 in. radius, with the radius of the lower and smaller end 3 in.: what is its total inside curved surface?

8. A boiler is in the form of a cylinder with hemispherical ends: its total length is 12 ft., its diameter 4 ft.; what is its surface?

9. What is the total surface of a hemisphere, of diameter 8·26 cm.?

10. A hollow spherical shell is 1 in. thick and its outer diameter is 6 in.; what is its total outside and inside surface?

11. In Exercise **10**, what would be the total surface if the shell were cut in halves and separated?

12. If each square centimetre of each surface of a film of water be the seat of energy sufficient to raise 1 gram weight through a vertical distance of 0·7 mm., what vertical distance could 1 gr. be lifted through by utilizing the energy of a soap bubble 24 cm. in diameter?

13. A cauldron is in the form of a hemisphere, its diameter is 6·2 ft. and thickness 1 in.; find its weight approximately, the weight of 1 cub. ft. of the substance being 450 lb.

EXERCISES XCI. c.

Practical.

[*Apparatus: Sheets of metal or cardboard. Shears. Balance.*]

1. Cut out a number of triangles from a piece of sheet metal or cardboard. Cut out also a square of, say, 5 cm. side. Calculate the areas of the triangles (*a*) by measuring the length of the sides, and then computing; (*b*) by comparing the weights of the triangles with that of the square.

2. Draw a quadrant of a circle, radius 2 in. Beginning from one end, mark off a number of small arcs (AB may be taken as a typical one). Draw perpendiculars (such as AN) from the ends

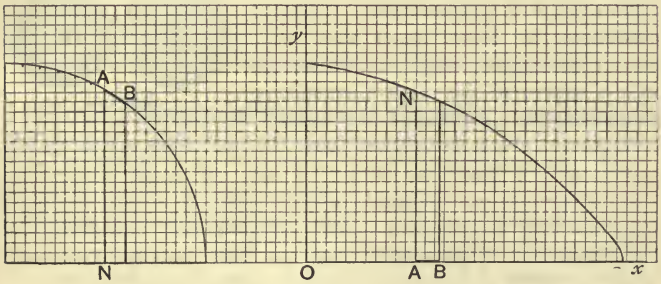


FIG. 84.

to one of the bounding radii. Draw a graph by marking along one line Ox the length of the arcs and setting off perpendiculars equal to the perpendiculars found above.

Find the area contained between Ox, Oy and the graph.

Calculate the ratio of the area to that of the quadrant above.

3. Draw a circle upon squared paper, and determine its area. Find also the area of a square drawn upon the radius of the circle. Divide the former by the latter result, so as to determine the ratio between the area of a circle and the area of the square on the radius.

4. Draw a circle upon squared paper, taking as radius the distance between 10 lines. Count the number of squares in the circle, estimating, as before (p. 127), the fractions of squares. Now calculate the area from the formula, area of circle $= \pi \times \text{rad.}^2$, the radius being 10. Thus verify practically the accuracy of the formula.

It is difficult to determine experimentally the area of the surface of a sphere, but the following device provides a very approximate illustration of the rule for calculating the area from the radius :

5. Make or procure the apparatus shown in Fig. 85. To make the instrument, a narrow metal ring with a neck is obtained. Thin sheet india-rubber is fastened upon the ring, so as to form a hollow box with flexible ends. A square centimetre is drawn upon the india-rubber. A flat disc, which can be inflated into a sphere of the same diameter, is thus obtained.

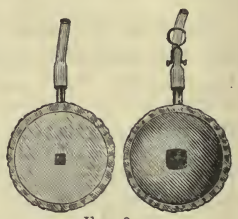


Fig. 85.

By means of a tube fastened upon the neck blow out the india-rubber until it takes the form of a ball. While it is stretched measure the area of the square marked upon it. It will be found approximately double what it was originally, thus indicating that **the surface of a complete sphere has four times the area of the hemispherical section** or flat part obtained by cutting it into halves.

6. That the surface area of a sphere is four times the area of a circle of the same diameter as the sphere can also be demonstrated by weighing hemispherical shells of sheet brass and circular discs of the same material,* diameter and thickness as the sphere. If the hemispheres are fairly large, say three inches in diameter, two of them together weigh, within a fair degree of accuracy, four times as much as a disc. Here is an actual result :

Mass of disc, - - - -	22·1 grams.
Mass of two hemispheres, - -	88·8 grams.

*[Brass hemispheres and discs can be obtained from Messrs. Griffin and Sons, Ltd., Kingsway, London, W.C.]

7. The area of the surface of a sphere may also be found in the following way: Procure an old tennis ball. Measure its diameter and then cut off the flannel covering; flatten each half section down on squared paper and find the area. Add the two to get the area of the whole surface. Hence prove that the area of the sphere is four times that of a circle of the same diameter.

A fives ball may be similarly employed, the four coatings being cut off and their areas measured.

8. Weigh a thin hollow spherical shell of metal, measure its diameter (d), also the thickness of the metal; calculate and compare the weight of the shell with πd^2 multiplied by the weight of unit area of the sheet metal.

65. CALCULATION OF VOLUMES (*continued*).

Volumes of Cones and Pyramids.

ABCDEFGH (Fig. 86) is a cube, O is the central point. The cube could be divided into 6 exactly equal parts by the planes OAB, OBC, and so on.

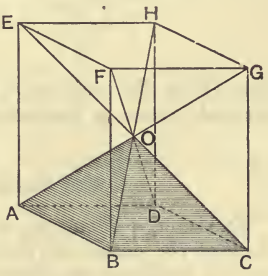


FIG. 86.

Hence, in this particular case, the volume of a single pyramid, such as OABCD, is $\frac{1}{3}$(area of base)(altitude), being $\frac{1}{6}$th that of the cube.

The pyramid may be regarded as the sum of an exceedingly great number of square strips, such as *abcd* (Fig. 87), and each strip may be supposed slipped along until the form of the pyramid is changed from Fig. 87 to Fig. 88, so that in this case also the volume$=\frac{1}{3}$(area of base)(altitude). Reasoning in this fashion, it can be shown that the volume of any square pyramid $=\frac{1}{3}$(area of base)(altitude).

Again, the series of strips, such as *abcd* (Fig. 88), might have been replaced by a series of other shaped strips of the same

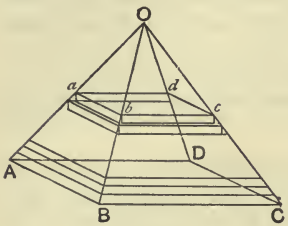


FIG. 87.

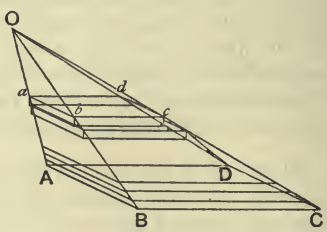


FIG. 88.

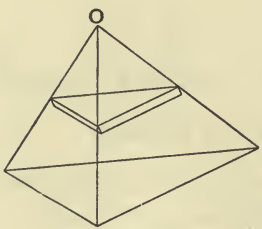


FIG. 89.

FIG. 90.

area (Figs. 89 and 90). So that **the volume of any cone or pyramid = ⅓ (area of base)(altitude).**

Volumes of Frusta.

EXAMPLE. *Find the volume of the frustum of a cone, the radii of the plane ends being* 7 *inches and* 10 *inches respectively, and the height of the frustum* 9 *inches* (Fig. 91).

In this and in most cases of a similar character, the problem is best solved by the use of similar figures.

If a cone be imagined cut across by a number of parallel planes all perpendicular to the axis, but at different distances from the apex of the cone; the radius of any section is proportional to the distance of the section from the apex, and

the increase in the radius is proportional to the increase in the distance of the section from the apex.

Thus, in this example, if ABCD (Fig. 92) be half a section of the frustum through the axis, O the position of the apex of the cone of which the frustum forms a part, and if CE be parallel to the axis OAB,

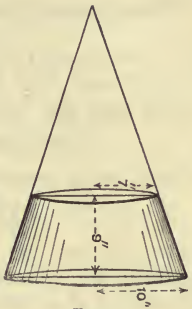


FIG. 91.

$$\frac{OA}{AC}=\frac{CE}{ED};\ \therefore\ OA=\frac{(AC)(CE)}{(BD-AC)}$$

$$=\frac{7\times 9}{10-7}=21,$$

whence also $OB=21+9=30$.

The frustum is therefore the difference between two cones, one of height 30 inches and end radius 10 inches, the other of height 21 inches and end radius 7 inches.

The volume of a cone of altitude 30 inches and end radius 10 inches is $\frac{1}{3}\pi\times 10^2\times 30$ cubic inches. .....(i)

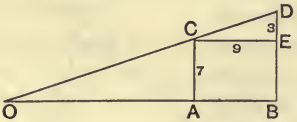


FIG. 92.

The volume of a similar cone of altitude 21 and end radius 7 $=(\frac{21}{30})^3\times\frac{1}{3}\pi\times 10^2\times 30$ cub. in.,

or $=(\frac{7}{10})^3\times\frac{1}{3}\times\pi\times 10^2\times 30$ cub. in.; .....(ii)

therefore subtracting (ii) from (i) the volume of the frustum

$$=\left\{1-\left(\frac{7}{10}\right)^3\right\}\frac{\pi\times 10^2\times 30}{3}=657\pi.$$

Similarly, in the general case where the radii of the ends are R . r and the altitude h,

The complete cone, of which the frustum may be regarded as part, has an altitude $=\frac{R}{R-r}\cdot h$.

The volume of this complete cone $= \frac{1}{3}\pi R^2 . \frac{R}{R-r} . h.$

The volume of the frustrum

$$= \left(1 - \frac{r^3}{R^3}\right)\frac{1}{3} \times \pi R^2 \times \frac{Rh}{R-r}.$$

This may also be written

$$= \tfrac{1}{3}[\pi R^2 + \pi Rr + \pi r^2]h,$$

and still more generally

$$= \tfrac{1}{3}h(S + \sqrt{Ss} + s),$$

where $S . s$ are the areas of the ends; and in this form the expression is suitable for calculating the volume of a frustum (Fig. 93) of a pyramid or of any conical body.

FIG. 93.

Volumes of Spheres.

Suppose a very small triangle ABC drawn on the surface of a sphere and all the points on its perimeter joined to O, the centre of the sphere. A small pyramid will be formed, of which the volume = $\frac{1}{3}$(area of ABC)(radius of sphere).

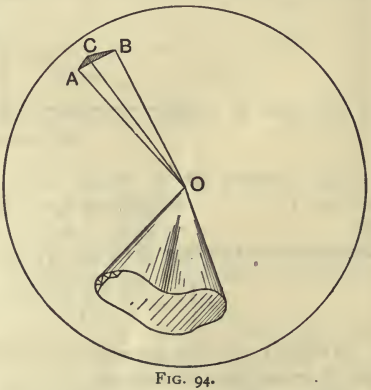


FIG. 94.

So, if any other surface be marked out on the sphere, it may be regarded as made up of a great number of very small triangles; and if the points on its boundary be joined to the centre of the sphere, the volume of the solid thus formed $= \frac{1}{3}$ area of the portion

of the spherical surface × radius of the sphere. In particular,

the volume of the whole sphere

$$= \tfrac{1}{3}\text{(surface of the sphere)(radius)} = \tfrac{1}{3} \cdot 4\pi r^3.$$

EXAMPLE 1.—*What is the volume of the substance of a spherical shell, external diameter* 12 *in., internal diameter* 8 *in.?*

The volume = the difference between the volumes of two spheres of diameters of 12 in. and 8 in. respectively.

∴ the volume in cubic inches

$$= \frac{1}{3}\pi \times 12^2 \times \frac{12}{2} - \frac{1}{3}\pi \times 8^2 \times \frac{8}{2} = \frac{\pi}{6}\{12^3 - 8^3\}$$

$$= \frac{3{\cdot}1416}{6}\{1728 - 512\}$$

$$= 0{\cdot}5236\{1216\}$$

$$= 637 \text{ cub. in. approx.}$$

EXAMPLE 2.—*What is the volume of the spherical sector obtained by joining the centre of a sphere* 15 *cm. in radius, to the points on the circumference of a small circle of* 12 *cm. radius drawn on its surface?*

Clearly, the distance between the centres of the sphere and the small circle $= \sqrt{15^2 - 12^2}$ cm. = 9 cm.

Whence the spherical surface of the sector

$= 2\pi \times 15(15 - 9) = 180\pi$ sq. cm. (p. 442),

and the total volume of the sector

$= \frac{1}{3} \times 15 \times 180\pi$ c.c.

$= 900\pi$ c.c.

$= 2827{\cdot}4$ c.c.

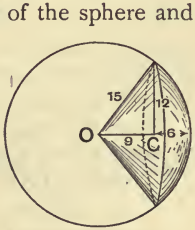


FIG. 95.

EXAMPLE 3.—*A sphere, radius* 9 *cm., is divided into two portions by a plane, the distance of which from the centre of the sphere is* 6 *cm. What are the volumes of the two portions or segments?*

In Fig. 96 let ABC represent the section, O the centre of the sphere.

The volume of the spherical sector OABCD $= \frac{1}{3} \times 9 \times 2\pi 9 \times 3$, *i.e.* ($\frac{1}{3}$ radius × area of ABCD).

The volume of the cone OABC

$= (\frac{1}{3}$ altitude × area of base)

$= \frac{1}{3} \times 6 \times \pi(9^2 - 6^2)$ c.c.

The volume of the spherical segment ABCD

$= \frac{1}{3} \times 9 \times 2\pi 9 \times 3 - \frac{1}{3} \times 6 \times \pi(9^2 - 6^2)$ c.c.

$= \pi(162 - 90)$ c.c.

$= 72\pi$ c.c.

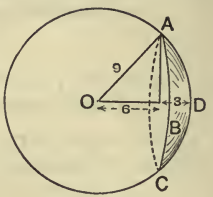


FIG. 96.

EXAMPLE 4.—*Find the volume of the portion of a sphere, radius* 12 *in., comprised between two planes, distant* 8 *in. and* 6 *in. from the centre (and on the same side of the centre).*

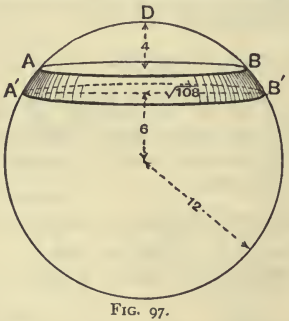


FIG. 97.

Here the required volume is the difference between the volumes of two spherical segments, *i.e.* between DA′B′ and DAB.

Volume of DA′B′

$\therefore = \frac{1}{3} \times 2\pi \times 12 \times 6 \times 12 - \frac{1}{3}\pi(\sqrt{12^2 - 6^2})^2 6$ cub. in.

Volume of DAB

$= \frac{1}{3} \times 2\pi \times 12 \times 4 \times 12 - \frac{1}{3}\pi(\sqrt{12^2 - 8^2})^2 8$ cub. in.

$\therefore$ the volume of the required spherical frustum in cub. in.

$$= 576\pi - 216\pi - 384\pi + \frac{640}{3}\pi$$

$$= \frac{568\pi}{3}.$$

EXERCISES XCII.

1. Find the volumes, altitudes, or bases of the following cones :

	Volume.	Diameter of base.	Altitude.
(*a*)	?	5·3 ft.	6 ft.
(*b*)	?	10·2 ft.	5 ft.
(*c*)	265·905 cub. in.	?	12 in.
(*d*)	220·898 cub. in.	?	15 in.

What is the content, in gallons, of each of the following vessels?

2. A basin in the form of a frustum of a cone. Upper rim 24 in. diameter, lower rim 18 in. diameter, depth 6 in.

3. A tin can, lower portion cylindrical, 9 in. diameter, 14 in. high; higher „ „ 5 in. „ 4 in. „ connecting portion a frustum of a cone 5 in. in height.

4. A vat 2 ft. deep, the top of which forms a rectangle 8 ft. × 6 ft., its base a rectangle 5 ft. 4 in. by 4 ft.

5. Show that if the earth is regarded as a sphere of 4000 miles radius, its volume is 268×10^9 cubic miles approx.

6. Twenty small shot are dropped into a burette containing water, and they produce a rise of level in the water corresponding to an increase of volume of 0·281 c.c.: find the average diameter of the shot.

7. Find the diameters, volumes, or surfaces of the following spheres :

	Diameter.	Volume.	Surface.
(*a*)	54 cm.	?	?
(*b*)	2·3 ft.	?	?
(*c*)	?	137·258 cub. in.	?
(*d*)	?	448·921 cub. in.	?
(*e*)	?	?	167·415 sq. cm.

8. A sphere, of 5 in. radius, is cut into two parts by means of a plane, distant 4 in. from the centre. Find the volume of the smaller of the two portions. (Take π as $\frac{22}{7}$.)

9. A spherical sector is obtained from a sphere of 6 in. radius, and the spherical cap bounding it has a height of 2 in.; what is the volume of the sector?

10. The interior of a glass goblet has the form of a portion of a sphere; the upper rim of the glass is 6 in. diameter, the base is 3·6 in. diameter; what volume of water could the glass hold, the height of the glass being 2·4 in.?

11. A spherical shell, external and internal radii 8 in. and 6 in. respectively, is cut in two by a plane bisecting at right angles a radius of the external sphere. What is the volume of the smaller of the two portions?

12. A cylindrical boiler has hemispherical ends: its total length is 16 ft., diameter 6 ft. (inside measurements). What is its cubical content?

13. A double convex lens has two spherical surfaces, both of 50 cm. radius, and its central thickness is 1 cm. What is its weight, the density being 2·3?

14. Find the cubical content of a mug in the form of a frustum of a cone, the upper radius being 2 in., the lower 2·5 in. and the height 5 in.

15. What is the mass of a conical bearing of iron, 6 in. long, diameter of the end 4·8 in.? (1 cub. ft. of the iron weighs 450 lb.)

16. A hay rick forms a frustum of a cone surmounted by another cone. The height of the frustum is 10 ft., the lower radius 10 ft., the upper 12·5 ft., while the height of the cone is 6 ft. What is the volume of the rick?

Graphic Calculation of Volumes.

Suppose a solid to be divided up (Fig. 98) into several parts by a number of equidistant planes; thus forming a number of thin slices, a typical example of which is seen in the middle

of Fig. 98. The slice being very thin its volume is obtained by multiplying its mean end area by the thickness, *i.e.*

$$\text{volume of slice} = \left(\frac{A + A'}{2}\right) \text{(thickness), approx.}$$

where A, A′ are the areas of the two plane ends.

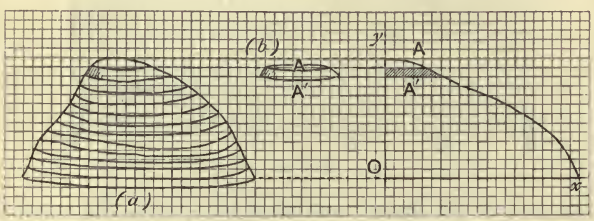


Fig. 98.

Draw an auxiliary graph (Fig. 98), such that the height of any plane section of (*a*) is marked along the line O*y*, and the area of that section represented by a line perpendicular to O*y*; it is clear that, corresponding to the thin slice shown in (*b*), we have a small trapezium (shown shaded in Fig. 98), the parallel sides of which are A, A′, and the width is equal to the thickness of the slice.

$$\text{The area of this strip} = (\text{thickness})\left(\frac{A + A'}{2}\right).$$

Therefore the area of the strip represents the volume of the slice, and the area of the whole figure represents the volume of the whole solid (*a*).

Example 1.—*Find graphically the volume of a cone, base* 1*-inch radius, altitude* 2 *inches.*

Suppose ABC (Fig. 99) represents the section of the cone and AO the axis. At a point N on the axis, the section perpendicular to the axis is a circle, NP being the radius.

The area of the section is πNP^2; but since π is a constant multiplier it is convenient to disregard it for the present.

At the corresponding point n of the axis Oy of the graph, set off a horizontal line to represent NP^2 on any convenient scale, and so for the other points.

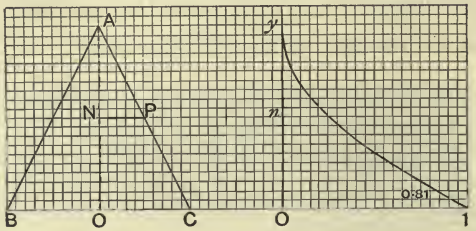


FIG. 99.

The area of the figure, multiplied by π, gives the volume of the cone. Hence the volume $= \pi(0{\cdot}66)$ cub. in. (By counting the squares.)

EXAMPLE 2.—*Find the volume of a hemisphere of 2-inch radius.*

Suppose the hemisphere placed with its plane base horizontal; the plan is a circle of 2-in. radius; the contours are also circles. (Fig. 100, which is drawn to $\frac{1}{4}$ scale.)

Set off the auxiliary figure, and mark the square of the radius of any contour against the height of the contour; the constant factor π need not be taken into account until the end. Count the squares. The volume $= 5{\cdot}3\pi$.

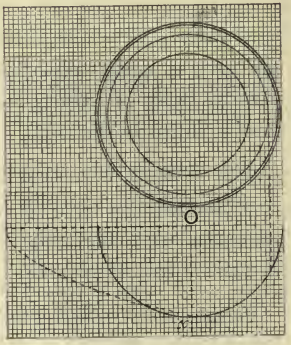


FIG. 100.

EXAMPLE 3.—*Find the volume of a hillock from its given contoured plan.* (1 *square represents* 10^2 *sq. yds.*)

The contours (Fig. 101) give the areas of sections at heights of 0, 10 ft., 20 ft., 30 ft. above the plane of reference.

The areas of these sections are seen on counting the squares to be 399×10^2, 178×10^2, 61×10^2, 7×10^2 sq. yd.

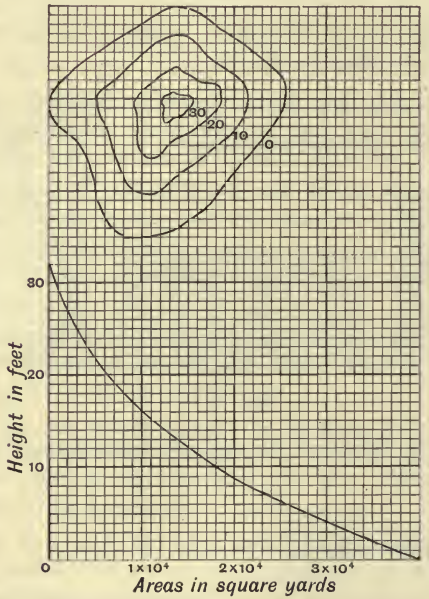


FIG. 101.

Plot a graph of these areas against the heights; in Fig. 101 each square will correspond to the volume of a block of earth 1 ft. in height and 10^3 sq. yds. in section, and, therefore, to obtain the volume of the hill, count the number of squares in the figure and multiply by $10^3 \times 9$ to express in cubic feet.

On counting, the squares are seen to be 431 in number.

The volume of the hill is therefore $10^3 \times 9 \times 431$ cub. ft.

$$= 3{\cdot}9 \times 10^6 \text{ cub. ft. about.}$$

EXERCISES XCIII.

Graphic.

1. Find the cubic contents of the vessels represented in section by (*a*), (*b*), (*c*) in Figs. 102, 103, 104.

(Scale—1 square represents 1 sq. cm.)

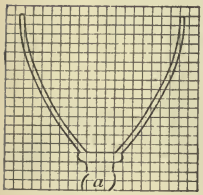


FIG. 102.

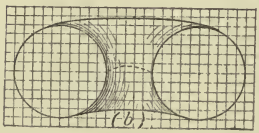


FIG. 103.

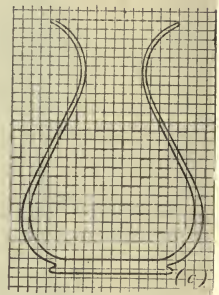
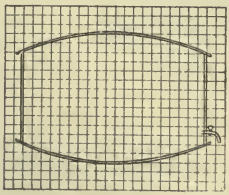
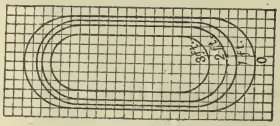


FIG. 104.

2. Find the volume of the cask the dimensions of which are shown in Fig. 105.



(Scale—1 square represents 1 sq. in.)
FIG. 105.



(Scale—1 square represents 1 sq. yd.)
FIG. 106.

3. Find the volume of the mound shown in Fig. 106, by its contoured plan.

Exercises **10, 11, 13, 16** on p. 454 lend themselves to this graphic method of calculation.

REVISION EXERCISES XCIV.

Miscellaneous: mainly from Examination Papers.

A 1. A boy multiplied a number by 495 and obtained the product 1925617. He was told that the figures 9 and 7 were wrong, and the other figures right. What was the correct product?

2. If a rectangle 43·86 yards long and 49·02 yards wide is divided into 129200 equal squares, how long (in inches and decimal of an inch) will the side of each square be?

3. A had a number of florins and B had the same number of half-crowns, and neither had any other money. B paid 3*s.* 6*d.* to A by giving 3 half-crowns to A and receiving 2 florins from A. If after the payment was made A had £4. 5*s.* 6*d.*, how much money had B then?

4. A person buys a triangular plot of ground, of which the sides are, respectively, 140, 143, 157 yards. Find its area in acres and the cost at 7*s.* 6*d.* per square yard.

5. If 500,000 rabbits bought in Belgium, at the rate of 75 francs per 100 are sold in England at an average price of 10*d.* each, how much do the importers gain after paying 2 per cent. on their outlay for freight, etc.? [25 francs = £1.]

6. One man with his team could plough a field in 3 days. Another with his could do the work in $2\frac{1}{2}$ days. In what time would the two men and the two teams together accomplish the work?

7. What difference in amount of simple interest shall I obtain by lending £5250 for 2 years at $3\frac{1}{2}$ or $4\frac{1}{2}$ per cent. per annum?

8. A cistern 31 metres long and 4 metres wide holds water weighing 185752 kilograms. Remembering that 1 cubic decimetre of water weighs 1 kilogram, find the depth of the cistern in metres, decimetres, etc.

9. The yearly birth-rate of a certain town exceeds the yearly death-rate by three per thousand; what time will elapse before the population increases by 50 per cent. (neglecting changes due to immigration and emigration)?

10. The field CDFE from which the following measurements (given in links) were taken is bounded by straight lines. Draw it on the scale of 1 chain to the inch and calculate its area.

	⊙ B	
	220	114 ⊙ F
⊙ D 78	160	
⊙ C 93	75	
	0	56 ⊙ E
	⊙ A	

B 1. If the price of coal be raised 10 per cent., find by how much per cent. a man must reduce his consumption of that article so as not to increase his expenditure.

2. A capitalist, having a sum of money to invest, places half in a bank which offers depositors $4\frac{1}{2}$ per cent. compound interest added yearly to the deposits; and the other half in a bank, which also allows compound interest, where the rate is $\frac{1}{2}$ per cent. less, but the additions of interest are made every six months. At the end of two years he finds there is a difference of £249. 16*s*. $3\frac{1}{4}$*d*. in the two accounts. Find the sum originally deposited.

3. A tradesman sells cloth at 10*s*. a yard, making a profit of 8 per cent. on the cost price; if he sold the same cloth at 10*s*. 6*d*. per yard, what per cent. profit would he then make?

4. A hollow closed cubical box is made of iron and is 2 inches thick, the external edge being 4 feet long; it being given that iron weighs 7·112 times as much as water, and that a cubic foot of water weighs 1000 ounces, find in tons the weight of the box.

5. A merchant sold 90 cwt. 3 qr. 14 lb. of madder, which cost him 38*s*. 6*d*. per cwt., to another merchant at the rate of 42*s*. per cwt., the purchase price to be one quarter in cash and the remaining three quarters in hemp: if the hemp cost the other merchant 24*s*. 10*d*. per cwt., find how much money he would pay and the price at which he would reckon the hemp, on the supposition that he is to make the same net profit as the first, so that the exchange is equal.

6. Find, in cubic feet, the volume of the frustum of a cone, the height of which is 30 feet, the greater diameter 6 feet, and the less 3 feet ($\pi = \frac{22}{7}$).

7. A train, going at the rate of 72 miles an hour, overtakes another train 192 yards long, going in the same direction on a parallel line at the rate of 54 miles an hour, and completely passes it in three-fourths of a minute: find the time in which the trains would have completely passed one another, if they had been going in opposite directions; find also the length of the faster train.

8. Find, with as little work as possible, the value, to the nearest shilling, of $£2{\cdot}684 \times (1{\cdot}035)^4$.

9. From a cylinder of wood, with a diameter of 11 inches, a rectangular beam is cut. If the breadth of the beam is 8 inches, find the depth from the formula $d^2 = D^2 - b^2$, where b is the breadth and d the depth of the beam, and D the diameter of the cylinder.

If the strength of a beam varies as bd^2, compare the strengths of beams 3, 4, 5, 6, 7, 8 in. broad cut from a cylinder 11 inches in diameter, and so find, roughly, the breadth of the strongest beam which can be cut from the cylinder.

10. A certain sum of money (say £100) is put out at 10 per cent. compound interest, payable yearly. Draw a graph showing how the interest depends upon the time. Use your graph to find in what time the interest would amount to £30.

C 1. How many pieces of wire, each 4·13 in. long, can be cut from a rod whose length is 8·24 ft., and what will be the length of the piece left over?

2. Having given that a metre is 39·3708 in., show that the difference between 35 yards and 32 metres is less than a centimetre.

3. One strip of carpet is 1 ft. $10\frac{1}{2}$ in. wide, and another strip is 2 ft. 3 in. wide. A square yard of the former costs 8*d.* more than a square yard of the latter, and a yard of the latter costs 5*s.* 6*d.* How much does a yard of the former cost?

4. If £100. 7*s*. 6*d*. was allowed by a banker as simple interest on £2555, deposited with him for $1\frac{1}{4}$ years, what average rate per cent. per annum of interest was obtained by the investor?

5. A can do a piece of work in $2\frac{1}{2}$ days which B can do in $3\frac{1}{3}$ days. If A's wages are £1. 16*s*. 8*d*. a week and B's wages are £1. 8*s*. 9*d*., what would A have charged for doing a piece of work for which B received £11. 10*s*.?

6. A workman is able to save $12\frac{1}{2}$ per cent. of his wages, but if his wages were raised 2*s*. a week and his expenses were increased by 10 per cent., his annual savings would be diminished by 17*s*. 4*d*.; what are the man's weekly wages, a year being taken as 52 weeks exactly?

7. A person subscribes the sum of £20 yearly to an insurance society, beginning on his 30th birthday; to what sum would he be entitled just before he attains his 55th birthday, reckoning compound interest at 4 per cent. per annum?

$$\log 1{\cdot}04 = 0{\cdot}0170333,$$
$$\log 26658 = 4{\cdot}4258276,$$
$$\log 26659 = 4{\cdot}4258439.$$

8. Water has to be diverted from a river through a six-inch diameter pipe, running full bore at a velocity of one foot per second, to irrigate a field of 20 acres. How long will it take to deliver an inch of water over the whole area? (Area of circle $= \pi r^2$.)

9. ABCDEF is a field bounded by the straight lines AB, BC, CD, DE, EF, FA. The points B and C are on one side of AD, and the points E and F are on the other side of AD. A person wishing to find the area of the field, notes the position of points *f*, *b*, *e*, *c* on AD, such that the corresponding directions *f*F, *b*B, *e*E, *c*C are all of them perpendicular to AD; he also measures the lengths of these lines *f*F, *b*B, etc., so obtaining the following data:

A*f* = 8 chains,	*f*F = 9 chains,
A*b* = 10 chains,	*b*B = 7 chains,
A*e* = 23 chains,	*e*E = 3 chains,
A*c* = 25 chains,	*c*C = 8 chains.
AD = 29 chains.	

Draw a plan of the field and find the area in acres, etc.

10. The Great Wall of China is said to be 2400 km. long, 61 dcm. high, 457·5 cm. thick at the top, and 7625 mm. thick at the bottom. How many cubic metres of building material does it contain?

D 1. Divide 73438·6445 by 4·12345, and 0·004321 by 0·00043212, correct to three places of decimals.

Add together 0·4 of a mile, 0·424 of a rod, 0·4246 of a yard; giving the result (i) as a decimal of a mile, and (ii) in feet and the decimal of a foot.

2. In calculating the amount of wages lost by workmen in a strike, it was assumed that 200,000 men lost 42 days' work valued at 4*s.* 6*d.* a day. If the error in the estimate of the number of men may be 10 per cent. of the true value, and that in the estimate of the value 20 per cent. of the true value, find the greatest possible error in the whole amount as a percentage of the true amount.

3. If £12. 17*s.* 3½*d.* is divided among 4 men, 6 women and 8 children, so that each man may have twice as much as a woman, and each woman three times as much as a child, what sum will each man receive?

4. If a piece of land in France cost 7350 francs per hectare, find its price per acre in English money; 5 francs being equal to 4 shillings and 100 hectares to 245 acres.

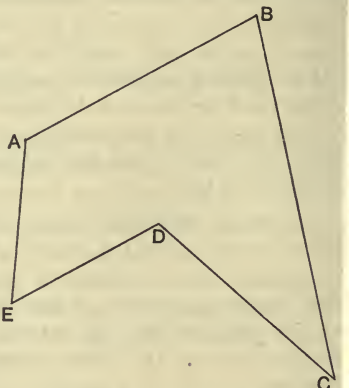
5. A can just give B a start of 20 yards and C a start of 27 yards in a walking contest of a quarter of a mile. How much can B give C in 120 yards?

6. A shopman bought eggs at the rate of 7 for a shilling, and sold them at a profit of 40 per cent. How many eggs would a customer get for a shilling?

7. A closed cubical copper box, the sides of which are everywhere three inches thick, encloses a cubical space whose edge is eight inches long. What would be the total cost of the material of the box at fourpence per cubic inch?

8. If numbers are formed by arranging *all* the digits 1, 2, 3, 5, 6, 7, 8 so that the odd and even digits occur alternately, show that each number is divisible by 11.

9. A sum of money is accumulating at compound interest at a certain rate per cent. If simple interest instead of compound were reckoned, the interest for the first two years would be diminished by 13*s.* 4*d.*, and that for the first three years by £2. 0*s.* 4*d.*; what is the sum?



(The figure is shown here at half size, *i.e.* 2·5 ft. to the mile.)
FIG. 107.

10. Find, in acres and decimals of an acre, the area of a field which is represented, on a scale of 5 feet to the mile, by ABCDEA (Fig. 107); *or*, if you prefer, take the scale as 1 to 1000, and give the area in hectares. If you want to draw perpendiculars, judge their positions by the eye.

E 1. Find the square of $1\frac{129}{187}$, correct to six places of decimals. Express the result as a decimal of 3·141593.

2. Find the value of 71 tons 7 cwt. 2 qr. at 30 guineas a ton.

3. At 6 a.m. on November 11th, a watch which keeps fairly accurate time indicated 5 hr. 59 min., and at 6 p.m. on November 25th, it indicated 6 hr. 0 min. 56 sec. What was its daily gain?

4. Two men undertake to do a piece of work for £6. One of them could do it by himself in 16 days, while the other would require 20 days. They employ an assistant; and, all three working together, they complete the work in 8 days. How much of the money has each of them earned?

5. A train 99 yards long, and travelling at uniform speed, overtakes in succession two trolleys on the other line of rails, one at rest and the other travelling in the same direction as the train. If the train pass the first trolley in $7\frac{1}{2}$ seconds, and the second one 10 minutes later in $6\frac{3}{4}$ seconds, how far ahead will the train be when one trolley overtakes the other? (The length of the trolley is to be neglected.)

6. Give careful instructions how to set out the boundary lines of a lawn tennis court 78 feet by 27 feet.

7. A cask is $30\frac{1}{2}$ inches long, its bung diameter is $26\frac{1}{2}$ inches and its head diameter is 23 inches; find how many gallons of wine it contains. (Assume the form to be a double frustum of a cone.)

8. A man buys 500 eggs for £1. 15*s.* and sells 200 of them at 8 a shilling. At what rate must he sell the remainder if he is to make 25 per cent. profit on his outlay?

9. Find, to the nearest penny, the compound interest on £1257. 16*s.* in 4 years at $2\frac{1}{2}$ per cent. per annum.

10. The safe weight, in tons, which may be attached to a hempen rope of diameter d inches is given by $0{\cdot}987d^2$. Find the safe weights which may be attached to ropes of 0·5, 1, 1·5, 2, 2·5, 3, 3·5 4 inches in diameter, and plot a curve on squared paper to show the nature of your result.

Determine, by inspecting the curve, whether the following arrangements are safe: diameter 1·7 in., weight 3·5 tons; diameter 2·8 inches, weight 7 tons.

F 1. If 12 tons 2 cwt. 2 qr. 10 lb. of coal is to be divided among 25 men and 20 women, giving each man three times as much as a woman, how much will each man receive?

2. Find three numbers lying between 500,000 and 510,000, each of which is exactly divisible both by 37 and by 73.

3. On a map drawn to the scale of an inch to a mile, a certain estate covers an area of 2·44 square inches. Find the real area of the estate in acres.

4. A landlord spends $\frac{1}{5}$ of the gross rent of a house in repairs and pays income-tax at 11*d.* in the £ on the balance. If he then have £85. 17*s.* 6*d.* left, what was the gross rent?

5. The interval between the ticks of a pendulum l cm. long is either $\pi\sqrt{\frac{l}{g}}$ seconds or $2\pi\sqrt{\frac{l}{g}}$ seconds, where $g=981$. Determine which; given that, roughly, a pendulum a metre long beats seconds.

6. The New York Department of Agriculture has estimated the total farm wealth for the current year at $6,415,000,000. This total is stated to be 4% higher than in 1904, 8% higher than in 1902 and 36% higher than in 1899, each of these percentages being reckoned on the earlier year concerned. Calculate the total farm wealth for each of those years.

Assuming that the percentage increase on 1904, as stated above, is correct to the nearest integer only, what are the limits of error in your result for that year?

7. The distance from X to Y by a footpath is to the distance from X to Y by road as 5 to 6, and driving from X to Y by the road at the rate of 10 miles per hour takes 1 hr. 9 min. less time than walking from X to Y by the footpath at the rate of $4\frac{1}{2}$ miles per hour. How far is it from X to Y by the footpath?

8. The value of a piece of machinery depreciated yearly in such a way that the value at the end of any year was only 90% of the value at the beginning of that year. The cost price was £120, and it was sold eventually as waste metal for £7. 10*s.* Obtain the number of years during which the machine was in use.

9. A cubical box, of external dimensions 17 inches each way, would contain crushed ore of the value of £421. 17*s.* 6*d.* if it were made of material 1 inch thick; but by mistake it has been made of thicker material, and the difference in the value of the ore which it will hold is consequently £78. 17*s.* 6*d.*; what is the real thickness of the material?

10. The diagonal AC of a quadrilateral garden ABCD is 975 links; the perpendiculars on AC, from B and D, are respectively 385 and 567 links: find the area in acres, roods, perches.

G 1. If a whole number be added to its square, show that their sum can only end in 0, 2, or 6; and if a whole number be added to its cube, the sum can only end in 0, 2, or 8.

2. The time of swing of a simple pendulum varies as the square root of the length of the pendulum. If a pendulum 1 metre in length swings once in a second, find to the nearest millimetre the length of a pendulum which swings 75 times in one minute.

3. What is the interest for two years to the nearest penny at $3\frac{3}{4}$ per cent. compound interest on £2113. 7*s.* 7*d.*?

4. What is the value of 7 tons 15 cwt. 1 qr. 27 lb. of copper at £58. 6*s.* 8*d.* a ton?

5. State and explain the effect of moving the decimal point to the right or to the left of its original position. What is the value of 2·109375 of £2. 4*s.*?

6. A walks from X to Y at the rate of 4 miles per hour; B, who starts $4\frac{1}{2}$ hours later from X and rides on a bicycle at the rate of 12 miles an hour, reaches Y at the same time as A. What is the distance between X and Y?

7. A cistern, whose length is 8 ft. 9 in. and breadth 7 ft. 4 in., is filled with water to a depth of 4 ft. 6 in. If a cub. ft. of water weigh 1000 oz., express as a decimal of a cwt. the weight of water in the cistern.

8. A pair or scales is improperly balanced, so that 1 lb. in one pan balances 15·75 oz. in the other. If a merchant in selling goods places them in the latter pan, how much per cent. does he gain by such a proceeding?

9. A man buys 76 gallons of wine at 5*s.* $7\frac{1}{2}$*d.* per gallon; he sells 13 gallons at a loss of 20 per cent., whilst two gallons are lost through leakage. At what price per gallon must he sell the remainder in order that he may gain 10 per cent. on the whole transaction?

10. A person borrows £1500, promising to repay the sum borrowed and the proper interest by ten equal yearly instalments, the first to fall due in one year's time.

Reckoning compound interest at 5 per cent. per annum, find the value of the annual instalment. [$(1{\cdot}05)^{10}$ may be taken as 1·629.]

H 1. The metre was originally determined so that forty million metres should be equal to the earth's circumference: a metre is 39·37 inches. Compute from these data the mean radius of the earth in miles, being given that the circumference of a circle is to its diameter as 355 to 113.

2. A landlord's net rental, after paying 5 per cent. for collection, and 8*d.* in the pound income-tax *on the remainder*, is £580: find his gross rental.

3. Selling articles at 6*s.* 6*d.* each, a dealer makes a certain percentage of profit; on increasing the price to 7*s.* 1*d.* his percentage of profit is increased by 10. What profit is he making at the latter price?

4. A triangular field ABC is to be levelled. The side BC is level and A is 2·3 metres higher than BC. Find how much the field is lowered at A, and how much it is raised along BC.

5. A litre of water weighs a kilogram, a litre of another liquid weighs 1·340 kilograms. A mixture of the two weighs 1·270 kilograms per litre. Determine the volume of each liquid in a litre of the mixture.

6. Employ logarithms to evaluate $(0{\cdot}41568)^{3{\cdot}25}$. Find the side of a cubical block of iron weighing 7892 lb., assuming that a cubic foot of water weighs 62·5 lb., while iron is 7·726 times as heavy as water.

7. A sum of £3826. 10*s.*, put out at compound interest payable yearly, amounted to £5441. 13*s.* 4*d.* in 8 years. What was the rate per cent. per annum?

8. A length of 60 centimetres is divided into equal parts. What is the number of these parts if, when this number is increased by unity, the length of each part is decreased by 1 millimetre?

9. The Snowdon Tramway has a continuous rise of 3140 ft. in $4\frac{5}{8}$ miles. The average gradient is said to be 1 in 7·83. Compare these statements, and determine the true average gradient from the first statement to three places of decimals.

10. Draw, as accurately as you can, a fair sized plan of a field ABCDEF from the given Field-Book entries. Then find the area in acres, roods, poles.

	Links.	
	3350	D
	2760	1000 to C
to E 500	2400	
	2080	400 to B
to F 1700	1800	

From A go North.

I 1. (*a*) Reduce $\frac{1\frac{0}{3}+\frac{4}{3}-1-\frac{24}{7}(\frac{4}{5}+\frac{1}{9})}{\frac{3}{2}(\frac{7}{4}-1)-\frac{4}{5}(\frac{1}{7}-\frac{1}{10})}$ to its simplest form.

(*b*) Multiply 5·61177 by 0·71279 correctly to five places of decimals, employing by preference a contracted method of multiplication.

2. How many square feet of timber 1 inch thick are required to make a closed box with a base 4 feet 6 inches × 3 feet, internal measurement, which shall contain 400 gallons of water? (Take 277·3 cubic inches to the gallon.)

3. What interest does a purchaser pay, if he adopts credit terms in buying the following article: "Military case will be sent on receipt of 6*d.* deposit and the payment of 17 weekly instalments of 6*d.* each, making 8*s.* 6*d.* in all, or 7*s.* 6*d.* cash."

4. How many feet in height is an isosceles triangle of which the base is 506 inches, and its area equal to that of a scalene triangle the sides of which are 368, 330 and 314 inches?

5. Divide by 17 lineal feet (1) 2023 lineal feet, (2) 2023 square feet, (3) 2023 cubic feet. State carefully what the quotient is in each case, and give reasons for your answers.

6. Find the cubical content of a block 2 ft. 9 in. long, 1 ft. 8 in. wide and 1 ft. 4 in. deep. If the weight of this block is 11 cwt., find what would be the length of a bar of the same material, having a sectional area of $\frac{5}{8}$ of a square foot and weighing 18 cwt.

7. From the equation $t=2\pi\sqrt{(l\div g)}$ find l in terms of the other quantities, and calculate its value to three significant figures when

$$t=1,\ g=32{\cdot}18,\ \pi=3{\cdot}1416.$$

8. If 15 cwt. 2 qr. 7 lb. cost £32. 13*s.* 7½*d.*, what will be the cost of 2 tons 3 qr. 21 lb. 8 oz. at $\frac{2}{3}$ of the former rate?

9. A circular basin of large dimensions, filled with water, is surrounded by a gravel path of uniform width: find the area of the walk and the diameter of its outer circumference, from dimensions of the paths only.

10. What sum to the nearest shilling paid now, together with £300, £250 and £200 paid respectively at the ends of 1, 2 and 3 years from the present time, will be equivalent to three sums of £350 due at the ends of 1, 2 and 3 years from the present time, if money is worth 3 per cent. per annum and compound interest payable yearly is reckoned?

J 1. (i) Express $\frac{22}{7}$ as a decimal fraction of 3·14159.

(ii) Find what decimal fraction of 7 cwt. 2 qr. 13¼ lb. is equal to 5 cwt. 1 qr. 11 lb. 3 oz.

2. A tank contains 9100 kilo. of water, and another tank of the same horizontal section, but 103 centimetres deeper, has a capacity of 10,750 litres. Find the depth of the first tank in metres, and the area of its section in square metres. (1 c.c. of water weighs 1 gram: a litre is 1000 c.c.)

3. A rectangular area, the length being five times the breadth, contains 70,000 acres. How many miles of fencing would be necessary to enclose it?

4. A cylindrical pontoon, 3 feet in diameter, having conical ends, is so far sunk under water that there is a mean pressure of 2·45 lb. upon each square inch of its surface: find the total

pressure (in tons), the extreme length of the pontoon from the extremities of the cones being 14 feet and the height of each cone being 2 feet. ($\pi = \frac{22}{7}$.)

5. The number of first-class passengers is two-fifths of the number of third-class passengers, but the fares are respectively 8*d.* and 2½*d.* How many are there of each class if the total receipts be £575. 14*s.* 0*d.*?

6. Find the compound interest on £800 for three years, the rate of interest being 5 per cent. per annum.

7. A manufacturer sells to a merchant, who in his turn sells to a retailer. If the retailer sell the goods to the consumer for £865. 3*s.* 0*d.*, and each trader makes ten per cent. profit, what is the prime cost of the goods?

8. What is meant by an *average*?

The following are the figures of twelve divisions of the House of Commons on a certain evening: Government, 201, 219, 255, 170, 195, 207, 223, 244, 250, 250, 248, 236; Opposition, 182, 205, 247, 101, 124, 136, 202, 243, 242, 235, 236, 183. What is the *average majority* of the Government for that evening?

9. A man puts by ¼*d.* on the 1st January, and doubles the deposit every succeeding year. How much will he have saved at the end of 25 years?

10. Draw a *plan* of the field, and find its area in acres, roods, perches, from the following Field-Book entries in links:

	to D	
0	1050	0
	840	250 to E
to C 180	750	
to B 320	600	
	520	450 to F
0	000	0

From A go North.

K 1. In 1901 the National Debt was £673,608,200. How much would be required to pay interest on this for a year at $2\frac{3}{4}$ per cent.?

2. Manitoba has an area of 47,188,500 acres; of this 1,965,200 acres were under wheat in 1901, producing 18,353,013 bushels of wheat. What percentage was under wheat, and how many bushels an acre did it produce? Give each result to the nearest whole number.

3. (*a*) A and B run a mile race, and A wins by 60 yards; afterwards B and C run a mile race, and B wins by 60 yards. If A and C run a mile race, by how much would A win, it being assumed that each runs at the same rate as before?

(*b*) A tub is filled with water by two taps running together in one minute forty seconds, and it would be filled by one of the taps alone in three minutes; how long would the other tap, running alone, take to fill the tub?

4. A steamer A is in distress and stationary, and fires a gun which is heard on another steamer B coming direct towards her. A fires another gun ten minutes after the first, and this is heard on B 9 mins. 48 secs. after the first. At this moment B is 4 miles from A. How soon will the two steamers be alongside of each other? Give the answer to the nearest second, supposing sound to travel at the rate of 1130 feet per second.

5. Yellow brass consists of 66 parts of copper to every 34 parts of zinc; spelter of 1 part copper to 1 of zinc. What amount of spelter would you melt with 324 tons of yellow brass, so as to produce an alloy suitable for sheathing, the composition of which is 6 parts of copper to every 4 of zinc?

6. A frustum of a rectangular pyramid has a base of 27 inches by 24, while the top is 18 inches by 16, and the height of the frustum is 32 inches. Find its volume.

If a plane passing through the smaller edges of the top and base divide the frustum into two wedges, find their respective volumes.

7. The yearly output of a gold mine decreases every year by 13 per cent. of its amount during the previous year. Given that the first year's output is £260,000, and that $(0{\cdot}87)^{10}=0{\cdot}24842$ approximately, find

(a) the total output for the first ten years ;

(b) the total output for all time.

8. To make an article takes 20 hours of a workman's time paid at $7\frac{1}{2}d.$ per hour. The material, 10 % of which is wasted in the working, costs 10$d.$ a pound. The final weight of the article is 20 lb. At what price must it be sold to gain 8 % profit on the total cost?

9. A and B are partners in a trading venture, A contributing £2000 and B £3000. A, however, acts as manager, the understanding being that of the profits A shall receive 25 per cent. for his services as manager, the remainder to be divided in the ratio of their contributions. B's profits are £120 ; what do A's amount to?

10. The following table gives approximations to the interest which would accrue to £200 in the specified times, the turnover being immediate, and the nominal rate 5 per cent. per annum.

Time in years	0	1	2	3	4	5	6
Interest in £	0	10·3	21·0	32·4	44·3	56·8	69·9

Exhibit the relation between interest and time, by means of a graph.

Employ your graph to find

(i) in what time the interest would amount to £50·1 ;

(ii) the interest which would accumulate in 3 years 3 months.

L 1. The product of two numbers, whose difference is 3, is 142504. Find the square root of 142504, correct to whole numbers, and hence, by trial, find the two numbers.

2. A rectangular vessel, 3·4 metres long and 2·6 metres wide, contained wine 7 decimetres deep. If as many casks as possible, each capable of holding 3 hectolitres, were filled from the vessel,

how many such casks were there; and what, to the nearest millimetre, was the depth of the wine left in the vessel? (A litre = a cubic decimetre, and a hectolitre is 100 litres.)

3. A builder has two houses which cost him equal amounts; he sells the first to A at a profit of 30 per cent., and the second to B at a loss of 20 per cent. A then buys B's house from him, giving the same price as he did for the first house. What profit per cent. did B make?

4. The capacity of an oil can may be calculated approximately by regarding the lower part as a cylinder, and the upper part as a cone. If the total height is $14\frac{1}{2}$ inches, the height of the cylindrical portion 10 inches and circumference 27 inches, how many gallons will it hold?

If the circumference of the can were a inches, what would be the circumference of another of the same shape but of double the capacity? ($\pi = \frac{22}{7}$, 1 gal. = 277 cub. in.)

5. An express train 332 feet long and going at 62 miles per hour overtakes a goods train 526 feet long running in the same direction on a parallel line at 17 miles per hour. Find how many seconds the trains take to pass each other.

6. There is a row of 21 trees planted at intervals of one year. The youngest tree is 10 feet high and the middle tree is 25·9 feet high. Assuming that during each year of growth a tree adds to its height the same fraction of its height at the beginning of that year, find the heights of the oldest tree and of the third tree.

(N.B.—$1{\cdot}1^{10} = 2{\cdot}59$ approximately.)

7. What would be the cost of the material of a closed cubical box, 3 inches in thickness on all sides, whose interior is also cubical and contains 27 cubic feet, reckoned at 2*s.* 6*d.* per cubic foot of material?

8. What sum at compound interest will amount to £650 at the end of the first year and £676 at the end of the second year?

9. A dealer buys oranges of two qualities, one at a shilling a dozen and the other at 8 pence a dozen. He then mixes them and sells the lot at 15 oranges for a shilling. He makes a profit of 5 per cent. on his outlay: find the ratio of the number of oranges of the two kinds.

10. A rifle, sighted to 1000 yd., rests on a support 5 ft. from the ground and is fired. The height of the bullet above the support is given by the following table:

Distance in yd. from firing point:									
100	200	300	400	500	600	700	800	900	1000
Vertical height above support in ft.:									
7·3	11·2	15·0	18·5	21·0	23·3	25	22·5	16·5	0

On squared paper give a representation of the path of the bullet, showing also the ground level and the height of the support, and find in what positions a butt of height 20 ft. would stop the bullet.

M 1. Find the number of gallons in a cylindrical vessel the diameter of which is 6 feet and height 7 feet. Given that the area of a circle equals $\frac{22}{7}$ times the square of the radius, and that a gallon is equal to 277·2 cubic inches.

2. A hare starts to run, at 12 miles per hour, when a dog is 44 yards off. After half a minute the dog sees her, and pursues at 16 miles per hour. How soon will he catch her?

3. Find the present value of an annuity of £300 payable every year for 3 years at 5 per cent. per annum simple interest.

4. If the wages of 45 women amount to £207 in 48 days, how many men must work 16 days to receive £76. 13*s.* 4*d.*, the daily wages of a man being double those of a woman?

5. If the annual increase in the population of a state is 25 per thousand, and the present number of inhabitants is 2,624,000, what will the population be in 3 years' time? And what was it a year ago?

6. A man having 3 sons, left £9656 to be divided among them in proportion to their ages at the time of his death; when he died, their ages were 25, 22 and 21 years respectively; what was the share of each? And what difference would it have made to each of the sons if the father had lived a year longer?

7. A merchant commenced business with a certain sum of money, and at the end of each year increased his capital by 10 per cent. At the beginning of the fifth year 5 per cent. of his capital was worth £732. 1*s*. What sum did he commence with?

8. In 1891 the population of the East Riding of Yorkshire was 400,085, the Poor Rate £166,260, and the number of paupers 11,133; in the North Riding the figures were, respectively, 354,382, £167,574 and 10,483; and in the West Riding 2,464,415, £1,031,537 and 45,806. Find, to the nearest penny, the average cost of a pauper for the whole of Yorks; and, to two decimal places, the average number of paupers per cent. of population.

9. ABCD is a quadrilateral plot of ground; AB is 204 yards, BC 204, CD 253, DA 325, and the angle ABC is 60°. Join AC: show that the triangle ABC is equilateral; then find the area of each triangle in square yards. Give the area of the whole in acres.

10. A, B, C, D are four stations on a railway; the distance AB is 10 miles; BC, 10; CD, 8. The following is an extract from a time table:

Up.	Down.
A dep. 7.57 a.m.	D dep. 8.29 a.m.
B arr. 8.15 a.m. / dep. 8.18 a.m.	C —
C arr. 8.37 a.m. / dep. 8.40 a.m.	B —
D arr. 8.55 a.m.	A arr. 9.10 a.m.

Plot, in one diagram on squared paper, graphs to show the positions of both trains at any time between those given, assuming that they run uniformly between the stations. When, and where, do they pass one another?

N 1. Two straight lines AB, CD, cut one another at right angles. If ACBD represent a four-sided field, find how many roods, poles, etc., there are in its area, the lengths AB, CD being 45 and 80 yards respectively.

2. Find the Least Common Multiple of 539, 1183 and 1573.

3. How many doses, each weighing 0·00176 grain, can be obtained from a powder weighing 0·096 grain, and what is the weight of the remaining portion of the powder?

4. Find, to the nearest integer, avoiding unnecessary figures in the calculations, the value of $\frac{1{\cdot}7696 \times 3{\cdot}04517}{0{\cdot}7 \times 0{\cdot}00641}$.

5. Two rectangular cisterns, the areas of whose bases are 36 sq. ft. and 45 sq. ft. respectively, stand at different levels, and are placed in communication by a pipe through which water flows from the first cistern to the second at the average rate of $3\frac{3}{4}$ gallons per minute. If a cubic foot of water weigh $62\frac{1}{2}$ lb. and 10 lb. of water measure one gallon, in what time will the difference of the levels of the water be changed by 3 inches?

6. With wheat at 28*s.* per quarter, 6 men can be fed for 15 days at a certain cost. How long can 7 men be fed for the same cost, when wheat is 30*s.* per quarter?

7. A racecourse is 11 miles in circumference, and 3 men start together to travel the same way round it. A travels 4 miles an hour, B $5\frac{1}{2}$ miles an hour and C 8 miles. When will they all come together again?

8. 33 per cent. of the candidates in an examination pass in mathematics and classics; 42 per cent. fail in mathematics, and 52 per cent. fail in classics. What per cent. of the whole of the candidates fail in both mathematics and classics?

9. Find to within 1*d.* how much a trader should pay for 100 yards of silk so that, when selling it at 2*s.* $11\frac{3}{4}$*d.* a yard, he may make a profit of 35 per cent. on his outlay.

10. The three sides of a triangular field are, respectively, 85, 132 and 157 yards. Find its area in acres. It can be shown that this triangle is right-angled. Do this, and find the area again by an easier method.

66. EQUATION OF PAYMENTS.

When various lots of goods are bought on the same date, but with varying terms of credit, it is frequently convenient to know the **date** on which **one** payment of the **total nominal** amount may be made, *i.e.* with no charge for extra credit and no deduction for discount.

The question is equivalent to that of finding the **average** number of days of credit appertaining to each £, and this number is known as the **Equated term.**

EXAMPLE 1.—*Find the equated term in the following:*

Goods to the value of £800, term of credit 62 days.
" " £600, " 31 "
" " £600, " 31 "
" " £500, " 61 "

The average number of days' credit each £ receives

$$= \frac{800 \times 62 + 600 \times 31 + 600 \times 31 + 500 \times 61}{800 + 600 + 600 + 500}$$

$$= \frac{49600 + 18600 + 18600 + 30500}{2500}$$

$$= \frac{117300}{2500}$$

$$= 46{\cdot}92 \text{ days.}$$

The equated term is therefore taken as 47 days, and the **equated time** as 47 *days* after the date of the transaction.

More frequently the goods would be bought on different dates, as in the following example:

EXAMPLE 2.—*Find the equated time in the following case:*

	Goods.	*Term of Credit.*
April 1	£500	2 *months.*
April 10	£600	3 "
July 15	£500	2 "

It is desired to know the date on which a single payment of £1600 would clear the debt.

The sums are due as follows:

June 1, July 10, Sept. 15,

and the example can be evaluated as in Ex. 1, working from some **zero** date (here conveniently June 1).

There are therefore bills of

£500 due in 0 days,
£600 „ 39 „ from June 1 to July 10,
£500 „ 106 „ „ „ Sept. 15.

The equated term

$$= \frac{500 \times 0 + 600 \times 39 + 500 \times 106}{500 + 600 + 500} \text{ days}$$

$$= \frac{23400 + 53000}{1600} = \frac{76400}{1600}$$

$= 48$ days.

The equated time is 48 days after June 1, *i.e.* July 19 (use Table of Days).

Instead of full payment of the sum total of the bills, the balance only may be required, certain cash payments having been made before the term of expiration of the various bills.

EXAMPLE 3.—*Find the equated time in the following example:*

Date.	*Goods.*	*Term of Credit.*	
March 1	£500	2 *months*	*April 1, by cash* £350
April 16	£600	3 „	
June 7	£500	2 „	*July 1,* „ £750
July 3	£400	2 „	

Assume the same rate of interest throughout, remembering that cash payments tend to neutralize debts. Take March 1st as zero date.

Equated term

$$= \frac{500 \times 61 + 600 \times 137 + 500 \times 159 + 400 \times 186 - 350 \times 31 - 750 \times 122}{500 + 600 + 500 + 400 - 350 - 750}$$

$$= \frac{30500 + 82200 + 79500 + 74400 - 10850 - 91500}{900} = \frac{164250}{900}$$

$= 182{\cdot}5$ days,

i.e. equated term $= 183$ days.

Equated time is August 31. Balance £900.

EXERCISES XCV.

1. A person buys goods on April 10 to the amounts, and under terms of credit, as stated :

Value of goods, £550; credit, 2 mo.: goods, £650; credit, 3 mo.· goods, £739; credit, 2 mo.: goods, £640; credit, 2 mo.

Find the average number of days' credit each £ receives, if the goods are paid for by a single payment of

(£550+£650+£739+£640).

2. A person buys goods as follows :

Goods, £800.	Credit, 2 mo.	Goods, £812.	Credit, 80 days.
„ £736.	„ 2 mo.	„ £400.	„ 3 mo.
„ £720.	„ 70 days.		

Date of purchase June 12.

Find the average number of days' credit each £ receives, if the goods are paid for by a single payment of the nominal sum of the values of the goods.

3. A person buys goods as follows :

June 1.	Goods, £760.	Credit, 2 mo.
„ 20.	„ £650.	„ 60 days.
„ 28.	„ £720. 10*s*.	„ 80 days.
July 1.	„ £364.	„ 2 mo.

Find the average number of days' credit from June 1 that each £ receives, as in Ex. **1** and **2**.

This average number is known as the equated term.

The date obtained by adding the equated term to June 1 (or whatever the zero date be chosen) is known as the equated time.

4. In Ex. **3** find the equated term and time if the zero date be August 1.

5. Find the equated time on account of the following purchases:

(*a*)	July 2.	Goods,	£862. 5*s.*	Credit,	80 days.
	„ 4.	„	£760.	„	2 mo.
	„ 15.	„	£390.	„	2 mo.
	Aug. 10.	„	£800.	„	60 days.
(*b*)	May 1.	Goods,	£61. 7*s.* 6*d.*	Credit,	2 mo.
	„ 16.	„	£76. 3*s.* 6*d.*	„	60 days.
	June 3.	„	£59. 10*s.*	„	80 days.
	July 4.	„	£80. 10*s.*	„	2 mo.
	„ 11.	„	£112. 8*s.* 6*d.*	„	2 mo.

6. Find the equated time of the following purchases and payments, and also the balance on account:

(*a*)

Purchase.			Payment.	
Date.	Amount.	Credit.	Date.	Amount.
May 3	£700	2 mo.	June 1	£1000
„ 10	£800	60 days		
June 14	£750	60 days	July 1	£1000
„ 26	£300	2 mo.		
July 3	£600	2 mo.	Sept. 1	£500

(*b*)

Purchase.			Payment.	
Date.	Amount.	Credit.	Date.	Amount.
Feb. 1	£600	2 mo.		
June 3	£1800	3 mo.	Feb. 28	£500
July 4	£785	60 days	June 30	£1000
„ 17	£900	3 mo.	July 31	£1000

(*c*)

Purchase.			Payment.	
Date.	Amount.	Credit.	Date.	Amount.
Jan. 10	£500	2 mo.	Mar. 1	£600
Feb. 16	£750	2 mo.	May 1	£800
Mar. 10	£1000	4 mo.	June 15	£800
May 18	£1350	3 mo.	July 1	£1000

(*d*)

Purchase.			Payment.	
Date.	Amount.	Credit.	Date.	Amount.
Aug. 7	£600	3 mo.		
Sept. 10	£800	3 mo.	Sept. 1	£500
Oct. 12	£900	2 mo.	Oct. 1	£500
Nov. 10	£700	3 mo.	Nov. 1	£500

67. FOREIGN EXCHANGES.

Money Exchange between different Countries.

Different countries use different money systems; and accordingly it is important to be able to express any sum of money in the systems of different countries. For example, suppose a London merchant owes a Paris merchant 10,000 francs; he requires to know how many francs are equivalent to £1. In calculating, it must be borne in mind that

1st. The *amount of gold* in the standard gold coin of one country must always bear a definite ratio to the amount of gold in the standard gold coin of another country. But

2nd. The exchange value of the two coins fluctuates between certain limits.

The tables below give particulars respecting the coinage of a few countries.

Coinage of the United Kingdom.

The authorized coinage of the United Kingdom consists of the following pieces, exclusive of those issued only on special occasions:

Denomination.	Standard Weight.	Least Current Weight.	Remedy of Weight.
Gold:	Grains.	Grains.	Grains.
Sovereign - -	123·27447	122·500	0·20000
Half-Sovereign -	61·63723	61·125	0·15000
Silver:			
Crown - -	436·36363	—	2·000
Double Florin -	349·09090	—	1·678
Half-Crown -	218·18181	—	1·264
Florin - -	174·54545	—	0·997
Shilling - -	87·27272	—	0·578
Sixpence - -	43·63636	—	0·346
Threepence -	21·81818	—	0·212
Bronze:			
Penny - -	145·83333	—	2·91666
Halfpenny- -	87·50000	—	1·75000
Farthing - -	43·75000	—	0·87500

Standard Gold contains twenty-two twenty-fourths of fine metal and two twenty-fourths of alloy.

The fractional amount of pure or fine metal in a 'gold' or 'silver' coin determines its **fineness.** Since out of 1000 parts of standard gold, 916·67 are pure gold; or out of 24 parts of standard 22 parts are pure gold; the fineness of standard gold is said to be 916·67 millièmes or 22 carats.

Standard Silver consists of thirty-seven-fortieths of fine metal and three-fortieths of alloy; fineness, 925.

Bronze is an alloy of copper 95 parts, tin 4 parts and zinc 1 part.

The Remedy is the amount of variation permitted in fineness and in weight of coins when first issued from the Mint.

FOREIGN COINAGE.

France.	Germany.	United States.
GOLD:		
100 franc piece		double eagle
50 "		
20 "	20 mark piece (doppelkrone)	eagle
10 "	10 mark piece (krone)	half eagle
SILVER:		
5 franc piece	5 mark piece	dollar
2 "	2 "	half dollar
1 "	1 "	quarter "
50 centimes	50 pfennigs	dime
NICKEL:		
	20 pfennig piece	5 cents
	10 "	
	5 "	
BRONZE:		
10 centime piece		1 cent
5 "		(100 cents = 1 dollar)
2 "	2 pfennig piece	
1 "	1 "	
(100 centimes = 1 fr.)	(100 pfennig = 1 mark)	

The following are particulars respecting the standard coins in the above countries:

	British.	French.	American.	Germany.
	Sovereign	**20 Francs**	**Eagle**	**20 Mark**
Weight in grains Troy	123·27	99·56	258	122·918
Fineness, *i.e.* proportion of pure gold in total	0·91667	0·900	0·8993	0·900
Weight of pure gold in grains Troy	113·0016	89·6	232	110·626
	Florin	**5 Franc**	**Dollar**	**Mark**
Weight of pure silver in grains Troy	161·45	347·23	371·25	77·16
Fineness - - -	0·920	0·900	0·900	0·900
Total weight in grains Troy	175·5	385·81	412·5	85·73

EXERCISES XCVI.

1. Find the weight in grams of £1 sterling of the (*a*) standard weight, (*b*) least current weight.

2. Calculate the weight in grams of (*a*) 1*s.* and of (*b*) 1*d.*

3. What is the value per oz. Troy of (*a*) the gold used in coinage, (*b*) pure gold?

4. What is the value of 1 oz. Avoirdupois of pure gold?

5. How many sovereigns could be coined from 20 lb. (Troy) of pure gold? (State to the nearest sovereign.)

6. How many shillings could be coined from 2 lb. (Troy) of pure silver?

The following (Exx. **7, 8** and **9**) are quotations from mining returns. Find the fineness of the gold from the mines mentioned:

7. ASHANTI GOLDFIELDS AUXILIARY.—The clean up for April from Dredges Nos. 4 and 5 is as follows: Gold recovered, 194 oz.

(approximate value £776); total recovered for four months ending April 30, 1906, 885 oz. (Approximate value £3540.)

8. N. BONSOR MINE.—The tributors crushed 18,576 tons of ore at the Bonsor battery, yielding 10,525 oz., and 12,422 tons of tailings were treated, yielding 4413 oz. Troy, or a total of 14,938 oz., of a value of £42,687.

9. BONSOR AMALGAMATION.—The tributors ran the mill for 179 days, crushing 15,784 tons, yielding 3505 oz., and 8906 tons of tailings were treated, yielding 1451 oz., or a total of 4956 oz., of a value of £14,060. 14*s.* 6*d.*

10. Find the value in English money of the gold in (*a*) a 10 mark piece, (*b*) a 10 franc piece, (*c*) an American eagle.

11. Calculate the fineness of the standard coins of the countries named in the following tables:

(Great Britain—sovereign, weight 123·27 grains, $\frac{22}{24}$ fine.)

	Country.	Gold Coins.	Legal Weight in Grains.	Sterling Value.
				£. s. d.
(*a*)	Argentine Republic	Argentino or 5 peso piece	124·44	0 19 10
(*b*)	Austria-Hungary	Ducat	53·85	0 9 4
(*c*)	Brazil - - -	10 milreis	138·35	1 2 5½
(*d*)	Denmark - -	10 krone piece	69·14	0 11 0¼
(*e*)	Egypt - -	100 piastre piece (Egyptian £)	131·18	1 0 3½
(*f*)	Japan - - -	20 yen piece	257·21	2 0 11¾
(*g*)	Mexico - -	10 peso piece	261·12	2 0 5½
(*h*)	Ottoman Empire	Turkish pound of 100 piastres	111·36	0 18 0¾
(*i*)	Persia - -	Toman of 200 shahis	57·90	0 9 5
(*j*)	Peru - - -	Libra of 10 sols	123·27	1 0 0
(*k*)	Russia - -	Imperial of 15 roubles	199·10	1 11 9

12. What would be the value in francs of the gold in an English sovereign (weight 123·27 grains, 22 carat fine), the 20 franc piece being 99·56 grains in weight and 900 fine? In other words, find the *Mint gold par* of exchange between England and France.

13. Using the tables in Ex. **11**, find the Mint gold par of exchange between Great Britain and (*a*) Russia, (*b*) Japan, (*c*) Denmark.

14. If only 25·11 francs be regarded as the Paris equivalent of £1 sterling in London, it just pays to ship gold from London to Paris, *i.e.* to send across English sovereigns and have them recoined into French money. If 25·33 francs in Paris are equivalent to £1 in London, it just pays to ship gold from Paris to London and have it recoined there. These values for the exchange are known as the **Specie Points**. The actual course of exchange between London and Paris generally varies within these limits.

Find (*a*) the cost per sovereign of transmitting gold from Paris to London and coining, or of transmitting from London to Paris, also (*b*) what would be the cost of so transmitting £35,700.

15. If the cost of transmitting gold to Berlin, and having it recoined there, be £11; when the amount transmitted is £2042, what are the specie points between London and Berlin?

68. FOREIGN BILLS OF EXCHANGE.

Even when the buying and selling of goods take place in one country, money payments are seldom made, except when in one town, and when the amounts are comparatively small; more usually cheques, bills, etc., are employed instead.

In dealing with foreign countries, similar or modified methods are employed, except that cheques are most frequently not now available. Also, it is clear that since trading between two countries consists generally in an exchange of *goods*, it is not actually necessary that much money or bullion should pass at all between two countries trading with one another.

Thus the table below gives for the United Kingdom the actual imports and exports of bullion and specie, also the value of imports:

Imported Bullion.	Exported Bullion.	Imports.	Year
£ 45,261,468	£ 35,491,184	£ 413,434,242	1899
39,513,173	31,972,039	523,075,163	1900
32,217,306	26,015,102	521,090,198	1901
31,393,345	26,125,206	528,391,274	1902
28,967,723	39,233,238	542,600,289	1903

The following examples illustrate two of the most common methods which may be employed by a person in London or England in liquidating a debt abroad, say at Paris:

EXAMPLE 1.—*Calculate the cost of settling a debt of* 12,000 *francs by buying from a bill broker,* 3 *months' bills on Paris at* 25·50, *discount* $3\frac{1}{2}\%$.

Debt = 12000 fr., if paid now.

3 months' interest at $3\frac{1}{2}\%$ = 105

12105 = the equivalent of debt in 3 months, at the end of which time the bills mature.

In addition there must be paid

Brokerage at 1 per mille* = 12·11 = fees charged by the

Stamp duties $\frac{1}{2}$ per mille = 6·06 bill broker.

12123·17 fr. total.

Amount to be paid to settle the debt

$$= £\frac{12123 \cdot 17}{25 \cdot 50} = £475 \cdot 416$$

= £475. 8*s.* 4*d.* to the nearest penny.

* 1 per mille is 1 per 1000 or 0·1 %.

NOTE.—£1 paid now buys bills to value 25·50 fr., or more than the par rate of exchange, on account of the fact that 25·50 fr. is the value of a bill maturing in 3 months' time, which can be bought for £1 now. The exchange, however, depends upon the relative demand of London on Paris, and *vice versa*, and not on the interest alone.

EXAMPLE 2.—*Find the rate of cost of settling a debt by employing an agent in Paris to draw a 3 months' bill on the debtor.*

The agent draws up a 3 months' bill or draft for the debtor to accept; this bill the agent then negotiates in Paris.

The rate of exchange will now be less than the par rate, say 25·17 fr.

i.e. 25·17 fr. is now the present value in Paris of each £1 on the draft, payable in London in 3 months.

By expending £1 in 3 months' time, the debtor therefore can liquidate a present debt of 25·17 fr.,

less, however, by 0·02517 on account of brokerage

and „ „ 0·012585 stamp duties.

∴ the net amount is 25·132245 francs.

A *Present* Payment of £1 at the same rate of discount as in Ex. 1 would liquidate 25·132245 fr., increased by $\frac{7}{8}\%$ (the percentage increase at 3 months at $3\frac{1}{2}\%$ per annum), *i.e.* £1 now would liquidate a debt of

$$(25{\cdot}132245 + 0{\cdot}219907) \text{ fr.} = 25{\cdot}352152.$$

EXERCISES XCVII.

1. Find the cost to a London merchant of settling debts of (i) 385,000 fr., (ii) 43700 fr., (iii) 58200 fr., assuming the following rates of exchange for 3 months' bills:

(*a*) London on Paris 25·47.

(*b*) London on Paris 25·5.

(*c*) Paris on London 25·12.

(*d*) Paris on London 25·14.

Rate of discount 3%. Brokerage 1 per mille. Stamp duty 0·5 per mille.

2. Find the cost to a London merchant of settling a debt of 450,000 marks, assuming that the rates of exchange for 3 months' bills are as follows:

London on Berlin (*a*) 20·63, (*b*) 20·52. Berlin on London (*c*) 20·36, (*d*) 20·33. Rate of discount, brokerage and stamp duties as in Ex. 1.

3. Find the rate of discount when a 3 months' bill, £1236. 11*s.* 9*d.*, endorsed 25·18, is worth £1227. 0*s.* 4*d.* (Brokerage and stamp duty as above.)

4. Find the exchange in London on Paris for 3 months' bills, if a debt of 12617·54 fr. can be liquidated for £500. (Rate of discount 2%, brokerage and stamp duty as above.)

69. STOCKS AND SHARES.

In medieval and earlier times, financial companies of the type met with now could not exist, and for many reasons:

(*a*) laws were in force against usury;

(*b*) no opportunity existed for storing up wealth, other than in the form of corn, cattle, and other perishable commodities;

(*c*) very little business was carried on between town and town;

(*d*) most of the labour was expended upon material furnished by the consumers, and therefore a large amount of capital was unnecessary.

These conditions gradually changed however; the restrictions with regard to lending money broke down; stores of the precious metals were discovered, towns became less isolated, while capital became more and more necessary for carrying on wars and for other purposes. The goldsmiths started banking business, and banking firms gave interest to small depositors.

The earliest borrowers were kings, the money being expended generally upon wars, and to a smaller extent on Public Works. In this way arose the **National Debt.** At the time of the

Revolution in 1688, the National Debt was only £666,400; a terminable annuity, charged upon certain branches of revenue; increasing after the Ten Years' War, it amounted to £14·5 millions in 1697; at the end of the reign of Queen Anne it stood at £36·1 millions, while at the present time it is about £800 millions. The early rate of interest was high, but gradually it has become less, while attempts have been, and are being made to reduce the amount of the debt by sinking-funds and other means. Examples on page 504 furnish illustrations of this.

In addition to the money invested in all the securities comprised in the National Debt, money was invested in various companies.

The earliest companies were known as Regulated Companies, each member trading on his own capital, but observing certain rules (somewhat similar to the Stock Exchange at present). The members could compete against one another; but the membership offered much prestige, and many advantages with respect to tolls, customs, and recovery of debts.

Later, many of the Regulated Companies became converted into **Joint Stock Companies**. A Joint Stock Company is a single corporation, with one capital, and trading as a single individual. The most famous of the early Joint Stock Companies was the East India Company, formed as a Regulation Company in 1600, and afterwards converted into a Joint Stock Company, as the distance to the East Indies made the risk of trading too great for a single individual. Other well known companies were started, but not with quite the same success.

The "South Sea Bubble" was founded to trade with Spanish America, in the hope that large profits might be made from the slave trade and the whale fishery. In April 1720 the £100 shares of the Company stood at £120, in July at £1020, but

eventually became practically worthless. The company being Joint Stock, each member was liable for the debt of the company; many thousands of people were ruined by the bursting of the "Bubble," and shortly afterwards it was made illegal to form companies with transferable shares, unless the company was incorporated by Royal Charter. In these new companies members were only liable for their contributions to the Share Capital. Later, companies of this kind were abolished, and Joint Stock Companies became again all prevalent; but in 1855 public opinion once more deemed Limited Liability reasonable, and now there exists a very large number of Limited Liability Companies, in which the individual member is not responsible for the debts of the company, his responsibility ceasing entirely when once his shares are fully paid up.

With reference to various companies and stocks, the following terms may be noted in addition to "Joint Stock Company" and "Limited Liability Company":

Ordinary Stock. When the capital of a company, or amount of loan, is divisible for the purpose of sale into amounts denoted by their *nominal* money value, it is spoken of as **Stock**. Thus Consols are Stock. Sometimes any amount of Stock can be dealt in; *e.g.*, you may buy or sell £68. 13*s.* 7*d.* nominal of Consols; in other cases, dealing can occur only with multiples of £1, £5, and so on.

Shares. When the capital is divided into a certain number of parts, these parts being indivisible, then these parts are known as **Shares.**

Preference Stock is stock that has priority, or preference, over ordinary stock, not only with regard to dividend, but also on liquidation.

Bonds are documents, in which the issuer promises to pay a sum of money and interest at some time, either fixed in

advance, or by drawing by ballot the number of the bond (according to the terms on issue).

Debentures are bonds, the repayment of which, together with interest, is secured by a charge upon the assets of the company which issues the Debentures.

Scrip (short for Scrip Certificates) are provisional documents which are issued by companies and others, as evidence that the holder is entitled to receive stock, bonds, etc., when issued.

In 1694, the Stock Exchange came into existence, by means of which people might exchange their money for stock, or stock for money. In fact, the Stock Exchange was then, as now, a **Market** in which various shares or stock are bought or sold; these shares or stock are virtually sleeping partnerships.

To the Stock Exchange market ordinary people may not go: their transactions are effected by intermediaries or agents, known as **Stock Brokers**, and through them with buyers or sellers known as **Jobbers.** These dealers or jobbers are generally willing to sell or to buy, but, as in all other cases with people who buy or sell, the two prices are different; *i.e.* the jobbers are willing to sell stock at one price, but only to buy at a slightly lower price.

Examples, furnished from extracts from a daily paper:

(*a*) "Consols" ($2\frac{1}{2}\%$) $90\frac{1}{2}$ $90\frac{5}{8}$.
(*b*) "Brighton" 134 136.
(*c*) "Aerated Bread" $6\frac{3}{8}$ $6\frac{5}{8}$.

(*a*) Means that the jobbers offered to sell to the brokers £100 Consols (note carefully £100 Consols does not mean £100 money) for £$90\frac{5}{8}$ sterling, or offered to buy each £100 Consols for £$90\frac{1}{2}$ sterling, and that the holder of each £100 Consols receives dividend at the rate of £$2\frac{1}{2}$ per annum.

(*b*) Means that jobbers offered to sell Brighton Railway Stock at £136 sterling for every £100 stock, or to buy Brighton Railway Stock and pay £134 for every £100 Stock.

(*c*) Means that for one share in the Aerated Bread Company a jobber asked £6⅝, but that for one share he only offered to pay £6⅜.

EXERCISES XCVIII. a.

STOCKS AND SHARES.

British Funds.

Consols 2½ per cent.	90½	90⅝
Ditto, May Account	90 9/16	90 11/16
New Transvaal Loan	99⅝	99⅞
Local Loans 3 per cent.	99⅝	99⅞
India 3 per cent.	96⅝	96⅞
Ditto, 2½ per cent.	81¼	81¾
Ditto, 3½ per cent.	105⅛	105⅜
Rupee Paper 3½ per cent.	66⅛	66⅜
Irish Land Stock	92⅛	92⅜
National War Loan 2¾ per cent.	98⅝	98⅞
Water Board 3 per cent. Stock	95⅝	95⅞

Foreign Government Stock.

Argentine, 1886	102½	103
Buenos Aires Waterworks 5 per cent.	102¼	102¾
Brazilian 4 per cent., 1889	87	87½
Ditto, West of Minas	99¼	99¾
Chilian, 1896, 5 per cent.	100½	101½
Chinese 5 per cent., 1896	103	103½
Ditto, 5 per cent. Railway Loan	102½	103
Egyptian Unified	106	106½
French 5 per cent. Rentes	97	99
German 3 per cent.	86	86½
Greek 4 per cent. Monopoly	53½	54
Italian 5 per cent. taxed	104¼	104¾
Japanese 6 per cent., 1904	101	101½
Ditto, 6 per cent. (Second Series)	100¾	101¼

Home Railways.

Brighton	134	136
Ditto, Deferred	120½	121½
Caledonian	110½	111½
Ditto, Deferred	36¾	37
Central London	90½	91½
Chatham and Dover	15⅛	15⅜
Ditto, 2nd Preference	60½	61½
Furness Ordinary	63	66
Great Central Preference	39¼	39¾

Canadian and Foreign Railways.

Antofagasta and Bolivia	221	223
B.A. Great Southern,	137	138

Miscellaneous.

Aerated Bread	6⅜	6⅝
Allsopp New Ord.	20	21
Ditto, Pref.	34	38
Anglo-American Tele. Ord.	66	68
„ „ „ Pref.	114	115
„ „ „ Def.	23¾	24
Apollinaris	6⅞	7⅛
Armstrong, Whitworth	58/6	59/6

MINING MARKET.

Africans.

Anglo-French	1⅞	2	Kleinfontein	1⅝	1¾
Boksburg	1 1/16	1 3/16	Knights	3⅛	3¼
Bonanza	⅝	¾	Langlaagte	2⅝	2¾
City and Sub.	4¼	4½	May Consolidated	2 1/16	2 3/16
			Modderfontein	7⅛	7¼

Use the foregoing list, and find:

1. What would be the price asked for by a jobber in British Funds for

(*a*) £1500 Indian 3 per cents.

(*b*) £2300 Irish Land Stock.

(*c*) £1200 Water Board 3 % Stock.

2. What a jobber would offer to pay for the following Foreign Government Stock:

(*a*) £2350 Argentine, 1886.

(*b*) £4100 Chilian, 1896, 5 %.

(*c*) £5000 French 5 % Rentes.

3. (*a*) If a jobber buys and sells 1760 shares of the Aerated Bread Co. as above, what profit he makes by the transaction.

(*b*) What is the difference between the buying and selling price of

(i) 2760 shares in Apollinaris?

(ii) 3725 „ Armstrong, Whitworth?

(iii) 160 mining shares in the Bonanza?

Brokerage and other Charges.

Since the dealings with a jobber are always conducted through an intermediary stock-broker, certain charges must be paid, whether for selling or for buying stock. These charges vary with the value of the stock or share. They are regulated partly by competition, partly by custom; generally they are: $\frac{1}{8}$ % in the case of Consols, *i.e.* £$\frac{1}{8}$ for every £100 stock bought or sold; $\frac{1}{2}$ % in the case of railway stock; 3*d.* per £1 share (nominal) in mining shares. In addition to these commission fees, further charges are incurred, as will be evident from the procedure described below.

The stock is bought from the jobber by the broker, and on a certain day a **Transfer Deed** is prepared by the broker, showing that the actual seller (the holder of the stock at present) is to transfer the stock to the buyer; and within 10 days after, the buyer has to pay, and the transfer deed is delivered. This, after being signed by the buyer, is sent back to the broker, who sends it on to the company whose stock or shares have been purchased; they note the transfer and send the certificate back. For the transfer deed, a small amount is paid, and certain stamp duties, varying with the amount of the transfer and its value. In the following examples no account is taken of stamp or transfer duties:

EXAMPLE 1.—*Find the cost of buying £5700 stock in Chinese 5 %, 1896; 103–103½. Brokerage ¼ %.*

Here £100 stock is bought from the jobber for £103½ money; the broker's fee brings its cost up to $(£103\frac{1}{2} + £\frac{1}{4})$

$$= £103\frac{3}{4}.$$

$$\therefore \text{ the cost of £5700 stock} = £\frac{5700}{100} \times 103\frac{3}{4} \text{ money}$$

$$= £5913.\ 15s.\ 0d.$$

EXAMPLE 2.—*What amount of French 5 % Rentes, 97–99, can be purchased for £3980? What yearly dividend would the holder be entitled to, and what would be the rate of interest received on his money? Brokerage ½ %.*

Here $£(99 + \frac{1}{2})$ money would purchase £100 Rentes, yielding £5 yearly.

$$\therefore £3980 \text{ would purchase } £100 \times \frac{3980}{99{\cdot}5} \text{ Rentes, yielding } £5 \times \frac{3980}{99{\cdot}5} \text{ yearly.}$$

∴ the amount is £4000 Rentes and the income £200 yearly.

The rate of interest is $\frac{200}{39{\cdot}8}$ %, *i.e.* 5·025 % approx.

EXAMPLE 3.—*What change of income would be occasioned to an investor on selling £10,000 Chilian 5 % stock, 100½–101½, and investing the proceeds in Japanese 6 %, 101–101½? Brokerage ¼ % on each transaction.*

On the sale of £10,000 Chilian 5 %,

$$£(100\tfrac{1}{2}-\tfrac{1}{4}) \times \frac{10000}{100} \text{ money},$$

i.e. £10,025, would be received.

The income on this if invested in Japanese 6 % would be

$$£\frac{10025}{101\frac{1}{2}+\frac{1}{4}} \times 6 \text{ per annum, } \textit{i.e. } £\frac{60150}{101{\cdot}75}, \textit{ i.e. } £591{\cdot}155.$$

The income originally was $£\frac{10000}{100} \times 5$, or £500.

The change = £91. 3*s.* 1*d.*

EXERCISES XCVIII. b.

Find the cost or proceeds of the following transactions (brokerage ⅛ % in all cases):

1. Buying £1230 Consols at 91⅜–⅞.
2. Selling £580 L. & N.W.R. at 143½–4.
3. „ £1000 G.E.R. at 104–105.
4. Buying £20,000 G.W.R. at 114¼–⅜.
5. „ £1180 Consols at 91¼–⅜.
6. Selling £380 L. & N.W.R. at 143–145.
7. Buying £4200 G.E.R. at 87½–88.
8. Selling £6000 Argentine at 74⅛–⅝.
9. Buying £980 Stock 46¼–⅜.
10. Selling £10,000 Stock at 98½–99.
11. „ £1120 „ at 112⅛–⅝.
12. Buying £6000 „ at 96¼–⅜.
13. Find, correct to 3 places of decimals, the net rate per cent per annum on the money invested in the following, assuming the full dividend paid (brokerage neglected in all cases):

COLONIAL GOVERNMENT SECURITIES.

	Stocks.	Closing Prices.	
(*a*)	Canada 3 % Reg. - -	97½	98½
(*b*)	Cape of G.H. 3½ % Ins. -	97½	98½
(*c*)	Natal 3 % Ins. - - -	86½	87½
(*d*)	N.S.W. 3½ %, 1924 - -	99½	100½
(*e*)	„ 3 % Ins. - -	89	90
(*f*)	N. Zealand 4 % Ins. - -	107	109
(*g*)	„ 3½ % Ins. -	101	102
(*h*)	„ 3 % Ins. - -	89½	90½
(*i*)	Queensland 3 % Ins. -	88½	89½
(*j*)	Rand Water Board 4 % -	95	97
(*k*)	S. Australia 4 %, 1916-36 -	101½	102½
(*l*)	Tasmania 3½ % Ins. - -	100	101
(*m*)	Victoria 3½ % Ins. - -	100½	101½
(*n*)	„ 3 % Ins. - -	90	91
(*o*)	W. Australia 3 %, 1915-35 -	87½	88½

14. Find the change in the income due to selling out from the stocks in the table in Ex. **13**, and reinvesting as follows, assuming brokerage ¼ % in each case :

(*a*) £12,000 Canadian 3 % into N.S.W. 3½ %.
(*b*) £28,000 N. Zealand 4 % „ W. Australian 3 %.
(*c*) £56,000 Tasmanian 3½ % „ Victorian 3 %.
(*d*) £19,500 N.S.W. 3 % „ Rand Water Board 4 %.
(*e*) £2500 Natal 3 % „ Victoria 3 %.
(*f*) £12,250 Cape of G.H. 3½ % „ „ 3 %.
(*g*) £2350 Canadian 3 %. „ Cape of G.H. 3½ %.
(*h*) £36,000 Queensland 3 % „ New Zealand 3 %.
(*i*) £12,500 New Zealand 3 % „ Queensland 3 %.
(*j*) £80,000 Rand Water Board „ New Zealand 4 %.

State your answer to the nearest shilling.

15. The yield per cent. on the *money* invested in the following Indian Railway Companies is stated below; find the price to the nearest shilling of the stock:

	Railway.	Dividend per cent.	Yield per cent. on Money.	Price of Stock.
(*a*)	Assam Bengal - -	£3	£3. 3*s*. 10*d*.	?
(*b*)	East Indian - -	£4	£2. 19*s*. 3*d*.	?
(*c*)	Madras - - -	£5	£4. 0*s*. 6*d*.	?
(*d*)	Southern Mahrattas -	£5	£4. 18*s*. 0*d*.	?
(*e*)	Gt. Indian Peninsula	£3	£2. 15*s*. 5*d*.	?
(*f*)	Bombay, Baroda -	£5	£3. 6*s*. 0*d*.	?

16. What was the percentage declared on the following Railway Stock, if the rate of interest on the money invested was as in the accompanying table?

	Stock.	Approx. Price.	Dividend for 1905. Per cent.	Yield per cent.
(*a*)	Great Western - -	$135\frac{1}{4}$	?	£3. 19*s*. 0*d*.
(*b*)	Brighton Def. - -	120	?	£4. 7*s*. 6*d*.
(*c*)	Lanc. and York. -	105	?	£3. 10*s*. 0*d*.
(*d*)	Midland Def. - -	67	?	£3. 17*s*. 0*d*.
(*e*)	North-Eastern - -	140	?	£3. 17*s*. 6*d*.
(*f*)	North-Western - -	156	?	£3. 19*s*. 0*d*.
(*g*)	South-Western Def. -	$52\frac{1}{2}$	?	£3. 15*s*. 6*d*.

17. State equivalent percentage returns on the stocks (*a*) to (*f*) (all supposed of the same security): and find the equivalent % on stock at the prices given on the right.

(*a*) $2\frac{3}{4}$ %; price of stock 110,.....|.....(i) 160, (ii) 220.
(*b*) 5 %; „ „ 155,.....|.....(i) 100, (ii) $77\frac{1}{2}$, (iii) 248.
(*c*) 3 %; „ „ 44,.....|.....(i) 100, (ii) $80\frac{2}{3}$, (iii) 110.
(*d*) 9 %; „ „ 198,.....|.....(i) 66, (ii) 132.
(*e*) $7\frac{1}{2}$ %; „ „ 100,.....|.....(i) 40, (ii) $73\frac{1}{3}$, (iii) $93\frac{1}{3}$.
(*f*) £3. 12*s*. 9*d*. %; „ 100,.....|.....(i) $75\frac{5}{8}$, (ii) 110, (iii) 165.

18. A Forest, Land &c. Co. advertises for subscriptions for 450,000 six % Preference Shares of £1 each, 400,000 five % First Mortgage Debentures of £1, and a certain number of Ordinary Shares.

If the gross profits be estimated at £238,000, and if from this there be paid out for general expenses, reserve fund, etc., the sum of £158,000 (not including interest on Debentures), and if the interest on the Ordinary Shares be estimated at 11 % on the nominal capital, how many of such shares are offered for subscription?

19. A Motor Car Company offers for subscription

100,000	seven % Preference Shares of £1 each		£100,000
100,000	Ordinary	„ „	£100,000
			£200,000

The Company, although it anticipates a profit of 3*d.* per mile run per day, is content to estimate a profit of 2*d.* per mile per day, with runs of 100 miles per day, for each of 100 motors, and an average of 300 days to the year. It allows £3000 for administrative expenses. Calculate the net estimated profits ; also what approximate dividend could be declared on the Ordinary Shares if £5000 be put by as a reserve fund.

20. On a Company being floated, a man receives an 'option' for the 'call' of 3000 shares at $1\frac{3}{8}$ within 3 mo. When the shares stand at $1\frac{15}{16}$–2, the man exercises the right of his option, and sells 3000 shares at the market price. The shares necessary for the sale are supplied by the Company in accordance with the terms of the option. What does the man make? (Brokerage 3*d.* per share for the sale.)

N.B.—The 'call' entitles the receiver of it to the right of calling upon the giver to sell to him, within or at an agreed time in the future, at an agreed price, a certain number of shares.

21. What does a man make by exercising his option for a call at $2\frac{1}{8}$ for 2500 shares, when the shares stand at $2\frac{3}{16}$–$2\frac{15}{16}$, if he has paid £200 for the option, and if brokerage be estimated at 3*d.* per share on the sale?

22. A man pays £120 for the option of a 'Put,' *i.e.* for the right to call upon the giver of the option to buy from the receiver, at an agreed time in the future, a definite number of shares, or a definite amount of stock, at a fixed price. The option is for 3600 shares at $1\frac{3}{8}$; the option is exercised when the price of the shares in the open market is $1\frac{1}{16}$–$1\frac{3}{16}$. What is the gain or loss due to the exercising of the option (brokerage 3*d.* per share)?

23. "——" Tramways and Carriage Company. For the year 1905 the gross receipts were £256,741 and the expenses £171,661. A dividend at the rate of $9\frac{1}{2}$ per cent. per annum is proposed for the last half-year, making, with the interim of $8\frac{1}{2}$ per cent. per annum, 9 per cent. for the year. (An *interim dividend* is one declared in the middle of a working year.) The sum of £12,053 is added to the reserve fund.

If the balance was used up in paying dividend, what was the share capital, and what was the sum paid as dividend the first half-year?

24. Great Central and Midland Three and a Half Per Cent. Guaranteed Stock. Issue of £2,000,000. Messrs. Glyn, Mills, Currie & Co., as bankers of the Joint Committee, are authorized to receive subscriptions for the Stock at the price of £103 per centum.

Payable on application - - - - - -	5 per cent.
On allotment (the premium of £3 and an instalment of £20 of capital) - - - - - -	23 „
„ 15th May, 1906 - - - - - - -	25 „
„ 15th August, 1906 - - - - - -	25 „
„ 15th November, 1906 - - - - -	25 „
	£103

A man buys stock calculated to yield him £250 per year; what are his several instalments?

25. "R. and J. H——." The accounts show a profit of £19,348 on the year's trading, and an available balance of £15,683. The

directors recommend that £1250 be added to the reserve account, that a dividend be paid at the rate of 4 per cent. per annum, and that the balance, £1356, be carried forward.

What was (*a*) the share capital, (*b*) the market price of a £1 share; if the interest earned on the actual money invested was $4\frac{1}{4}$ %? (The payment to the reserve account, together with the amount carried forward, is taken from the available balance.)

26. London Electric Supply. The quantity of electrical energy sold in 1905 amounted to 13,042,932 units, producing a net revenue of £78,721. From this a sum of £30,000 was carried to reserve accounts, and £3141 carried forward. The directors proposed a dividend of 4 per cent.

What was the (*a*) share capital, (*b*) the net profit per unit?

27. Harrod's Stores. The directors recommend a dividend of 15 per cent. and a bonus of 2 per cent. for the six months ended January 31, which, with the interim dividend, will make 22 per cent. for the year, leaving £22,261 to be carried forward.

Find the net interest received for the year on the money invested by a buyer of the £1 share at the current price. Harrod's Stores $4\frac{1}{4}$–$4\frac{3}{8}$. (Brokerage, 6*d.* per share.)

28. A mine is estimated to give a net profit of £2500 yearly for 20 years. The owners endeavour to float a company with a certain number of £1 shares. How many shares should be issued, on the understanding that the yearly profit will be enough to repay the money invested at the end of 20 years, with interest at the rate of 5 % per annum?

(*N.B.*—A yearly payment of £8. os. $5\frac{3}{4}$*d.* for 20 years would repay £100 with the interest thereon at 5 % per annum.)

29. Home and Colonial Stores. The net profits for the year amounted to £133,419, which with the balance brought forward makes £141,679. The sum of £13,341, being 10 per cent. of the net profits, has been added to the reserve fund. After due payment of the dividend on the 15 per cent. cumulative ordinary

shares, the directors propose a dividend of 15 per cent. per annum on the A shares, leaving £1838 to be carried forward.

Find the total share capital (cumulative ordinary together with the A shares).

Many attempts have been made at reducing the National Debt and interest thereon, as shown by the following exercises :

30. In 1822, £152,422,143 Five per cents. was converted into Four %. £2,794,276 was paid off to those people unwilling to accept the reduction.

£100 of the 5 % was, however, converted into £105 of the 4 %. Find the yearly saving to the country.

31. Find the yearly saving on the conversion of £1,153,846 Bank of Ireland 5 per cents. into 4 per cents.

32. In 1824, 5 per cents. issued in 1797, were converted into 3 per cents. £133. 6*s.* 8*d.* 3 per cents. was given for each £100 in 5 per cents.

The amount of 5 % stock was £1,013,668.

The amount held by dissentients £41,011.

Find the amount of interest saved.

33. In 1830, 4 per cent. annuities were exchanged for £100 Three and a half per cents., or for £70 Five per cents.

The amount of the 4 % annuities was £153,671,091.

The amount held by dissentients was £2,880,915 (paid off).

The amount converted into $3\frac{1}{2}$ % was £150,119,609.

Find the yearly saving.

34. What was the effective rate per cent. of a loan of 36 millions of pounds, issued in 1815, when, for £100 sterling, £174 stock at 3 % might be obtained together with £10 stock at 4 % ?

35. Draw graphs showing the fluctuations in the highest, and in the lowest prices of Consols :

(*a*) From 1838-1847 inclusive ; (*b*) from 1848-1857 inclusive ;
(*c*) „ 1858-1867 „ (*d*) „ 1868-1877 „
using the data in the accompanying table :

	Consols.			Consols.	
	Highest.	Lowest.		Highest.	Lowest.
1838	$95\frac{1}{4}$	$90\frac{5}{8}$	1858	$98\frac{3}{4}$	$93\frac{7}{8}$
1839	$93\frac{7}{8}$	$89\frac{1}{4}$	1859	$97\frac{3}{8}$	$88\frac{1}{4}$
1840	$93\frac{1}{8}$	$85\frac{3}{4}$	1860	$95\frac{7}{8}$	$92\frac{1}{2}$
1841	$90\frac{1}{2}$	$87\frac{1}{4}$	1861	$94\frac{1}{4}$	89
1842	$94\frac{7}{8}$	$88\frac{1}{2}$	1862	$94\frac{3}{4}$	$90\frac{3}{8}$
1843	$97\frac{1}{8}$	$92\frac{1}{8}$	1863	94	90
1844	$101\frac{3}{8}$	$96\frac{1}{2}$	1864	92	$87\frac{1}{8}$
1845	$100\frac{5}{8}$	$91\frac{7}{8}$	1865	$91\frac{1}{2}$	$86\frac{3}{4}$
1846	$97\frac{3}{4}$	94	1866	$90\frac{3}{8}$	$84\frac{5}{8}$
1847	$93\frac{7}{8}$	$78\frac{3}{4}$	1867	$96\frac{3}{8}$	$89\frac{3}{4}$
1848	90	80	1868	$96\frac{1}{8}$	$91\frac{7}{8}$
1849	$98\frac{7}{8}$	$88\frac{5}{8}$	1869	$94\frac{1}{4}$	$91\frac{3}{4}$
1850	$98\frac{3}{8}$	$94\frac{3}{8}$	1870	$94\frac{1}{2}$	$88\frac{1}{4}$
1851	$99\frac{1}{8}$	$95\frac{5}{8}$	1871	94	$91\frac{3}{8}$
1852	102	$95\frac{7}{8}$	1872	$93\frac{3}{8}$	$91\frac{1}{4}$
1853	101	$90\frac{3}{4}$	1873	94	$91\frac{5}{8}$
1854	$95\frac{7}{8}$	$85\frac{1}{8}$	1874	$93\frac{3}{4}$	$91\frac{1}{4}$
1855	$93\frac{3}{4}$	$86\frac{1}{4}$	1875	$95\frac{3}{4}$	$91\frac{5}{8}$
1856	$95\frac{7}{8}$	$85\frac{3}{4}$	1876	$97\frac{1}{2}$	$93\frac{3}{8}$
1857	$94\frac{1}{4}$	$86\frac{1}{2}$	1877	$97\frac{3}{8}$	93

70. STATISTICS.

Averages.

The term **arithmetic average** or **mean** has already been employed many times, and is the name given to the result which is obtained when the sum of a number of quantities is divided by the number of such quantities. Two other means are occasionally employed, thus: Suppose there are 8 quantities, the numerical values of which are 3, 5, 4, 6, 3, 8, 2, 6.

The arithmetic mean

$$= \text{sum} \div \text{number} = \tfrac{1}{8}\{3+5+4+6+3+8+2+6\} = \tfrac{37}{8} = 4{\cdot}625.$$

The **geometric mean**

$$= \sqrt[8]{3\times5\times4\times6\times3\times8\times2\times6} = \sqrt[8]{103680}.$$

The use of logarithms shows the value to be 4·23606, and generally, if n be the number of quantities, their geometric mean is the nth root of their product.

Again, suppose the reciprocal numbers be employed, a mean can be obtained as follows:

The reciprocal of the arithmetic mean of the reciprocals of the quantities $= \frac{1}{\frac{1}{8}\{\frac{1}{3}+\frac{1}{5}+\frac{1}{4}+\frac{1}{6}+\frac{1}{3}+\frac{1}{8}+\frac{1}{2}+\frac{1}{6}\}}$

The result, or **harmonic mean** $= \frac{8}{2\cdot075} = 3\cdot8 \ldots .$

As theoretical quantities, the meanings of the above results are clear enough, but in common parlance the word average is used in a very wide sense.

The following examples afford material for the calculation of different *means* and also of various other *averages*.

EXAMPLE 1.—*The week-day trains between Waterloo and Feltham run as below. What is the average duration of the journey?*

W'loo.	Felt.	W'loo.	Felt.	Felt.	W'loo.	Felt.	W'loo.
12.10 a.m.	12.57	3.45 p.m.	4.35	4.45 a.m.	5.46	4.11 p.m.	4.48
5. 5	6. 1	4. 7	4.49	5.51	6.48	5.11	5.50
6.30	7.19	4.25	5.15	6.46	7.44	5.30	6.20
7.15	8. 9	4.55	5.31	7.53	8.31	6. 8	6.49
7.30	8.17	5. 5	5.49	8.18	8.52	6.21	7. 0
8.10	8.51	5.40	6. 5	8.31	9.19	6.50	7.40
9.15	10. 4	5.47	6.26	9. 3	9.34	7.29	8.10
9.30	10.12	6. 8	6.55	9.27	10.17	7.35	8.25
10. 5	10.55	6.40	7.14	9.42	10.30	8. 8	8.45
10.50	11.26	7.10	7.58	10.23	11. 6	8.20	9.11
11.20	12.12	7.45	8.25	10.30	11.23	9. 2	9.44
11.50	12.30	7.50	8.40	11.10	12. 1	9.16	10. 7
12.15 p.m.	1. 5	8.10	8.50	11.52	12.35	9.58	10.34
12.40	1.20	8.35	9.24	12.53 p.m.	1.31	10. 0	10.52
1.15	1.52	9.10	9.48	1.18	2. 7	11. 6	11.47
1.30	2.24	9.58	10.50	1.39	2.12	11.15	12. 9
1.40	2.20	10.35	11.10	1.54	2.46	—	—
2.10	2.50	11.50	12.18	3. 4	3.44	—	—
3.10	3.49	12.10	12.57	3.11	4. 4	—	—
3.15	3.57	—	—	—	—	—	—

The actual times in minutes are:

47, 56, 49, 54, 47, 41, 49, 42, 50, 36, 52, 40, 50, 40, 37, 54, 40, 40, 39, 42, 50, 42, 50, 36, 44, 25, 39, 47, 34, 48, 40, 50, 40, 49, 38, 52, 35, 28, 47, 61, 57, 58, 38, 34, 48, 31, 50, 48, 43, 53, 51, 43, 38, 49, 33, 52, 40, 53, 37, 39, 50, 41, 39, 50, 41, 50, 37, 51, 42, 51, 36, 52, 41, 54.

Analyse the results as below:

Duration of journey, -	25	28	31	33	34	35	36	37	38	minutes.
No. of trains, - -	1	1	1	1	2	1	3	3	3	
Totals, - - -	1	2	3	4	6	7	10	13	16	

Duration of journey, -	39	40	41	42	43	44	47	48	49	minutes.
No. of trains, - -	4	7	4	4	2	1	4	3	4	
Totals, - - -	20	27	31	35	37	38	42	45	49	

Duration of journey, -	50	51	52	53	54	56	57	58	61	minutes.
No. of trains, - -	9	3	4	2	3	1	1	1	1	
Totals, - - -	58	61	65	67	70	71	72	73	74	

The arithmetic average

$$= \{25 + 28 + 31 + 33 + (2 \times 34) + 35 + (3 \times 36) + (3 \times 37) + (3 \times 38) + (4 \times 39) + (7 \times 40) + (4 \times 41) + (4 \times 42) + (2 \times 43) + 44 + (4 \times 47) + (3 \times 48) + (4 \times 49) + (9 \times 50) + (3 \times 51) + (4 \times 52) + (2 \times 53) + (3 \times 54) + 56 + 57 + 58 + 61\} \div 74$$

(74 being the number of trains)

$$= (25 + 28 + 51 + 33 + 68 + 35 + 108 + 111 + 114 + 156 + 280 + 164 + 168 + 86 + 44 + 188 + 144 + 196 + 450 + 153 + 208 + 106 + 162 + 56 + 57 + 58 + 61) \div 74$$

$$= 3310 \div 74$$

$$= 44{\cdot}7 \text{ nearly.}$$

Thus the arithmetic average time is 44·7 min. No single train is timed exactly to this—times, however, are not given to fractions of minutes.

The calculation of the geometric mean is too long to be employed often in cases of this kind. For certain purposes it is more useful to take the **mode** than the arithmetic average, *i.e.* the time which *most* trains take, or the most common time. 9 trains take 50 minutes on the journey, 7 take 40 min.; these are the times occurring most frequently, and the mode = 50 min.

Perhaps more generally useful than either the arithmetic average, or the mode, is the **median**, a number calculated as follows: There are 74 train journeys; arrange them as above in order of magnitude. It is seen from the line marked 'totals' that there are 36 trains taking more than 44 minutes, and 37 trains taking less; also, that there are 35 trains taking less than 43 minutes, and 37 trains taking more, and the median lies between 43 and 44 minutes.

If a graph be drawn it shows that practically there are two groups of trains, "fast" and "slow."

EXAMPLE 2.—*The following table gives the times of the trains from London to Runcorn (Cheshire). Find the arithmetic average, the mode, and the median.*

Runcorn (Cheshire) from Euston, 180½ miles.

EUSTON.	RUNC.	RUNC.	EUSTON.
5.15 a.m.	10. 0	12.20 a.m.	5.50
7.10	11.46	7.25	12. 0
8.30	1.10	8.45	1.40
10.15	2.45	10.30	3.15
12.10 p.m.	4. 7	12.24 p.m.	5.15
12.15	5.13	1.47	6.10
2.40	7.31	2.46	7.10
5.35	10. 5	4.34	8.50
12. 0	8.17	6.39	10.45
—	—	7.35	3.50

The times in minutes of the 19 journeys are: 285, 276, 280, 270, 237, 298, 291, 270, 497, 330, 275, 295, 285, 291, 263, 264, 256, 246, 495.

Arrange the times in ascending order, thus: 237, 246, 256, 263, 264, 270, 270, 275, 276, 280, 285, 285, 291, 291, 295, 298, 330, 495, 497.

The arithmetic average $= \frac{5704}{19} = 300{\cdot}2$ minutes.

Note the great effect of the two very slow trains; without them the arithmetic mean would be 277·2 nearly.

The Mode = 282 minutes. There are two journeys of 270 min., two of 285 and two of 291.

The Median is the time such that the number of longer times is the same as the number of shorter times = 10th in above arrangement = 280 min., and is not much affected by one or two abnormal times.

EXAMPLE 3.—*The volumes and dimensions of ten cubes are measured, and found to be as follows: volumes in c.c.* 1·03, 1·16, 1·52, 1·03, 1·19, 1·73, 1·29, 1·73, 1·44, 1·23; *edges in cm.* 1·01, 1·05, 1·15, 1·01, 1·06, 1·20, 1·09, 1·20, 1·13, 1·07.

Two people work out the average size of the cubes, one by dealing with the edges, the other with the volumes. Find the difference between the two averages.

The arithmetic average of the edges $= \frac{10{\cdot}97}{10} = 1{\cdot}097$ cm.

„ „ volumes $= 13{\cdot}35 \div 10 = 1{\cdot}335$ c.c.

The edge of a cube of 1·335 c.c. $= \sqrt[3]{1{\cdot}335} = 1{\cdot}101$ cm. (employ logarithms).

The average length of the edge = 1·101 cm. deduced from the volume

and = 1·097 cm. deduced from the edges themselves.

Arrange the values of the edges and volumes in ascending order.

Edges—

1·01, 1·01, 1·05, 1·06, 1·07, 1·09, 1·13, 1·15, 1·20, 1·20 cm.

Volumes—

1·03, 1·03, 1·16, 1·19, 1·23, 1·29, 1·44, 1·52, 1·73, 1·73 c.c.

The median lies between the 5th and 6th values, *i.e.* between 1·07 and 1·09, and has the same value whichever method has been employed.

The geometric mean also would be the same for both methods (if the measurements were exact), and

calculated from the edges $= \sqrt[10]{\text{the product of all the edges}}$ and

$$\begin{aligned}\text{the logarithm of the mean} &= \tfrac{1}{10}\{0{\cdot}008632 + 0{\cdot}021190 \\ &\quad + 0{\cdot}025306 + 0{\cdot}029384 + 0{\cdot}037427 \\ &\quad + 0{\cdot}053078 + 0{\cdot}060698 + 0{\cdot}158362\} \\ &= \tfrac{1}{10}\{0{\cdot}394077\} = 0{\cdot}0394077,\end{aligned}$$

whence the mean or average edge $= 1{\cdot}095$ cm. approx.

Several quantities are used above in expressing an average. The ideal average should

(1) Show a type if any type exists.

(2) Give proper influence to extreme cases.

(3) Be easily calculated, but not greatly affected by small errors in the data.

Errors in Practical Work.

Method of Mean Squares.

In measuring a certain quantity, the following results are obtained: 4·02, 3·98, 4·00, 4·03, 4·01, 4·01, 3·99, 4·02, 3·97, 3·97.

Here there are ten results and the arithmetic mean is 4·00.

The differences between the quantities and the arithmetic mean are 0·02, −0·02, 0, 0·03, 0·01, 0·01, −0·01, 0·02, −0·03, −0·03.

The sum of the squares of the differences

$$\begin{aligned}&= 10^{-4}\{4 + 4 + 0 + 9 + 1 + 1 + 1 + 4 + 9 + 9\} \\ &= 10^{-4}\{42\}.\end{aligned}$$

The product of the number of readings, and the number reduced by unity $= 10 \times 9 = 90$. Divide this product into the sum of the squares of the differences; extract the square root of the quotient; it can be proved that the final answer gives the likely, or probable, error in taking the arithmetic mean as the true result.

That is, the error $= \sqrt{\frac{42 \times 10^{-4}}{10 \times 9}} = \frac{\sqrt{4 \cdot 2}[10^{-2}]}{3} = \frac{1 \cdot 764 \times 10^{-2}}{3}$,

$= 0 \cdot 006$ approx.

$\therefore$ the likely error in taking 4·00 as the true value is $\pm 0 \cdot 006$. The true value probably lies between 4·006 and 3·994, and it is clear that limits of this kind approach more closely as the number of the readings increases.

EXERCISES XCIX.

Find, from the data below, the average times of the given train journeys, stating (*a*) the arithmetic average, also (*b*) the most common time, or 'mode,' and (*c*) the 'median' time.

1. London to Harrogate (Yorks) from King's Cross, 203 miles.

King's Cross.	Harrogate.	Harrogate.	King's Cross.
5.20 a.m.	10.42	7. 0 a.m.	11.30
7.15	12.59	8. 5	1. 5
9.45	2·14	9. 5	1.55
10. 0	2.30	10.10	2.20
10.35	4.19	10.27	3.55
1.40 p.m.	5.42	10.57	4.10
2.20	6.58	12.27 p.m.	5.30
3.45	8. 0	12.50	6.15
5.45	10.58	2.30	7. 0
6.15	12. 5	6. 0	10.45
10.45	5.53	8. 5	2.40
11.30	8.20	8.35	3.20
—	—	10 25	5.50

2. London to Harrogate. Another route from St. Pancras.

St. Pancras.	Harrogate.	Harrogate.	St. Pancras.
2.45 a.m.	10.10	7.30 a.m.	1. 5
5.15	10.57	9.45	2.15
9.30	2.14	10.23	3.35
11.30	3.56	12. 0	4.55
12. 0	5.53	12.27 p.m.	6.20
1.30 p.m.	6.30	2.40	8. 5
2.15	8. 8	5.40	10.20
3. 0	8.38	6. 5	11.15
5. 0	10. 0	8.40	4.20
5.45	12. 5	10.25	5.30
12. 0	5.53	—	—

3. Leeds (Yorks) from King's Cross, 186¼ miles.

King's Cr.	Leeds.	Leeds.	King's Cr.
5.15 a.m.	9.17	2.48 a.m.	8. 5
5.20	9.38	6.30	10.40
7.15	11.15	7.50	11.30
9.45	1.20	9. 0	1. 5
10.35	3. 0	10.15	1.55
1.30 p.m.	5.12	11.20	3.55
1.40	6. 3	11.33	4.10
2.20	7.11	2. 0 p.m.	5.30
3.45	7.45	2.55	7. 0
5.45	9.35	3. 5	8.30
6.15	10.34	5.30	9.25
10.45	3.33	6.30	10.45
11.30	4.35	10.15	3.20

4. Leeds from St. Pancras, 196 miles.

St. Pan.	Leeds.	Leeds.	St. Pan.
2.45 a.m.	9.12	12. 5 a.m.	5.30
5.15	9.52	2.37	7. 0
8.30	1.12	2.47	7.15
9.30	1.22	3. 0	7.35
10.30	3. 0	3.18	7.30
11.30	3.22	4. 2	8. 0
12. 0	4.55	6.10	10.40
1.30 p.m.	5.30	6.35	10.40
2.15	6.37	8.30	1. 5
3. 0	7.37	10.20	2.15
4. 0	8.57	11.10	3.35
5. 0	9.10	12.45 p.m.	4.55
5.45	10.15	2.24	6.20
7.15	11. 6	2.45	7.40
8.30	12.22	4. 0	8. 5
9.30	1.40	6.22	10.20
12. 0	4. 0	7.17	11.15
—	—	10. 0	4.20

5. Draw graphs in each of the cases **1–4**, showing the number of trains which perform the journey in each time which occurs.

6. Tabulate the results above, grouping the times within 15 minute limits (thus, in (1) the first time is greater than 5 hrs. 15 min. and less than 5 hrs. 30 min.). Find the mode as calculated from the groups.

7. Find the (*a*) arithmetic, (*b*) geometric, and (*c*) harmonic means of the following numbers (the reciprocals are also stated).

Numbers, -	22·1	23·5	24·2	23·9	24·3
Reciprocals,	0·045045	0·042533	0·041322	0·041841	0·041152

Numbers, -	25·1	23·2	22·8	24·0	23·6
Reciprocals,	0·039841	0·043103	0·043860	0·041667	0·042373

8. The table below gives the annual premiums charged per £100 insurance by various Assurance companies :

ANNUAL PREMIUMS FOR ASSURANCE, ETC.

NAME OF OFFICE.	(*a*) Age 21.			(*b*) Age 30.			(*c*) Age 40.			(*d*) Age 50.			(*e*) Age 60.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
Friends' Provident, -	1	17	11	2	8	0	3	4	0	4	9	7	6	13	11
General, - - -	2	0	0	2	9	10	3	5	4	4	12	8	6	18	0
Gresham, - -	1	19	8	2	9	0	3	5	8	4	14	3	7	6	5
Guardian, - -	1	18	2	2	8	10	3	4	6	4	9	3	6	14	6
Law Life, - -	2	0	1	2	9	4	3	4	10	4	11	0	6	17	6
Law Union & Crown,	1	18	6	2	8	4	3	4	0	4	9	10	6	16	0
Legal and General, -	2	1	2	2	10	9	3	5	11	4	10	9	6	19	5
Life Assoc. of Scot.,	1	19	8	2	10	0	3	5	4	4	13	4	7	2	4
L'pool & Lond. & Gl.,	1	18	7	2	9	3	3	5	6	4	11	3	7	5	11
London & Lancashire,	1	16	9	2	6	10	3	2	4	4	6	10	6	18	11
London Assurance, -	2	0	8	2	9	6	3	4	11	4	11	5	7	2	11
Lond., Edin. & Glas.,	1	19	11	2	8	11	3	4	7	4	12	0	7	2	0
Marine & General, -	1	19	0	2	8	10	3	5	11	4	11	11	7	0	0
Methodist & General,	1	17	10	2	6	5	3	2	0	4	6	10	6	11	9
Metropolitan, - -	2	0	5	2	9	9	3	6	4	4	12	0	7	2	10
Mutual of Australasia,	1	18	0	2	7	0	3	3	0	4	11	0	7	1	0
Mutual of New York,	1	19	1	2	8	4	3	5	6	4	16	9	7	16	2
National Mutual, -	2	0	9	2	8	4	3	3	7	4	9	6	6	6	2
Natl. Mutl. of Aust.,	1	17	7	2	6	8	3	1	6	4	7	2	6	18	10
National of Ireland, -	1	19	8	2	8	7	3	4	3	4	11	7	7	1	8
National Provident, -	2	0	3	2	10	2	3	6	3	4	11	1	6	11	10
New York, - -	1	19	3	2	8	9	3	4	5	4	17	0	7	19	10
Nth. Brit. & Mercan.,	1	19	1	2	9	10	3	6	1	4	11	11	6	16	2
Northern, - -	2	1	2	2	9	0	3	4	8	4	10	10	6	17	4
Norwich Union, -	2	3	5	2	11	9	3	6	6	4	12	5	7	2	0
Patriotic, - -	1	19	1	2	8	8	3	4	5	4	10	4	6	16	4

Find the arithmetic average, the median, and the mode of the above rates. Also draw graphs, showing how these rates increase with the age of the person when he first insures.

[In finding the mode in Ex. (*a*), (*b*), (*c*), arrange the prices in groups, rising by 3*d.*, *i.e.* rates £2. 3*s.* 0*d.* to £2. 3*s.* 2*d.* inclusive, £2. 3*s.* 3*d.* to £2. 3*s.* 5*d.*, etc.

Arrange in groups, rising by 6*d.* in Ex. (*d*), and by 1*s.* 3*d.* in (*e*).]

9. The following table represents the heights and weights of a number of boys between the ages of 13 and 15 :

No.	Height.		Weight.		No.	Height.		Weight.		No.	Height.		Weight.	
	Ft.	Ins.	St.	Lbs.		Ft.	Ins.	St.	Lbs.		Ft.	Ins.	St.	Lbs.
1	4	$11\frac{1}{2}$	6	$0\frac{3}{4}$	27	5	$0\frac{1}{4}$	5	$11\frac{3}{4}$	53	4	$11\frac{1}{2}$	6	$9\frac{3}{4}$
2	4	10	5	7	28	4	9	5	11	54	4	$5\frac{3}{4}$	4	$9\frac{1}{2}$
3	5	$5\frac{1}{4}$	7	5	29	5	$1\frac{1}{4}$	5	9	55	5	$4\frac{1}{2}$	7	0
4	5	0	6	$3\frac{3}{4}$	30	4	$9\frac{1}{2}$	4	$7\frac{1}{4}$	56	4	10	6	$2\frac{1}{2}$
5	5	$3\frac{3}{4}$	8	$0\frac{1}{2}$	31	5	1	6	$8\frac{1}{2}$	57	4	9	5	5
6	4	10	5	0	32	4	$9\frac{1}{2}$	5	$13\frac{1}{4}$	58	5	$0\frac{1}{2}$	6	4
7	4	10	6	7	33	4	$8\frac{3}{4}$	5	$8\frac{1}{2}$	59	4	7	5	$2\frac{1}{2}$
8	5	5	8	$5\frac{1}{4}$	34	5	$2\frac{1}{4}$	6	$8\frac{1}{4}$	60	4	9	5	$9\frac{3}{4}$
9	4	$9\frac{1}{2}$	5	$12\frac{1}{4}$	35	4	$11\frac{1}{2}$	5	7	61	4	$8\frac{3}{4}$	5	$5\frac{1}{4}$
10	4	$11\frac{3}{4}$	6	$11\frac{3}{4}$	36	4	11	6	5	62	4	$8\frac{1}{2}$	6	$5\frac{3}{4}$
11	4	7	5	$1\frac{1}{2}$	37	4	11	6	$0\frac{3}{4}$	63	5	$5\frac{1}{2}$	7	10
12	5	$3\frac{3}{4}$	7	$8\frac{3}{4}$	38	5	$0\frac{3}{4}$	6	2	64	4	$8\frac{3}{4}$	6	$2\frac{1}{4}$
13	4	$7\frac{3}{4}$	5	3	39	4	$11\frac{1}{2}$	6	$3\frac{1}{2}$	65	5	4	7	2
14	5	$2\frac{1}{2}$	7	$8\frac{3}{4}$	40	4	$11\frac{1}{2}$	5	7	66	4	9	5	$0\frac{1}{2}$
15	5	0	6	0	41	4	11	6	$4\frac{1}{4}$	67	4	7	5	0
16	4	9	5	6	42	4	$4\frac{1}{2}$	4	$11\frac{1}{2}$	68	4	11	6	$1\frac{1}{4}$
17	5	$2\frac{1}{2}$	7	$7\frac{1}{2}$	43	5	3	6	$7\frac{1}{2}$	69	4	$11\frac{3}{4}$	6	$4\frac{1}{4}$
18	4	$8\frac{1}{4}$	5	3	44	5	$1\frac{1}{2}$	6	$13\frac{1}{4}$	70	4	8	4	$4\frac{1}{2}$
19	4	11	6	$12\frac{3}{4}$	45	4	$8\frac{3}{4}$	6	$0\frac{3}{4}$	71	4	8	4	$4\frac{1}{2}$
20	5	1	6	9	46	5	2	7	4	72	4	$7\frac{3}{4}$	4	10
21	4	11	5	11	47	5	$2\frac{1}{2}$	6	11	73	4	11	6	5
22	4	$8\frac{3}{4}$	5	11	48	5	4	7	$4\frac{1}{2}$	74	4	$3\frac{1}{2}$	4	$1\frac{1}{2}$
23	4	$9\frac{1}{2}$	5	$8\frac{3}{4}$	49	5	$1\frac{1}{2}$	6	10	75	5	0	7	$2\frac{3}{4}$
24	5	$2\frac{1}{4}$	6	1	50	4	$8\frac{1}{4}$	5	0	76	4	$8\frac{1}{4}$	5	6
25	4	$6\frac{1}{4}$	5	$6\frac{1}{2}$	51	5	$1\frac{1}{4}$	6	7					
26	5	$3\frac{1}{2}$	7	$6\frac{1}{2}$	52	4	$11\frac{1}{2}$	7	$3\frac{1}{4}$					

Find (*a*) the median weight, also (*b*) the mode, and (*c*) the arithmetic average.

Find also the average weights of boys 4 ft. 6 in. or over, but not 4 ft. 7 in.; 4 ft. 7 in. or over, but not 4 ft. 8 in., and so on. Draw ordinary and logarithmic graphs connecting these average weights with the height.

10. Draw charts, showing the fluctuation in the proportion of the total imports and exports of manufactured articles into and from the under-mentioned countries, which were respectively derived from, or exported to the United Kingdom, in each of the years from 1890 to 1901 inclusive :

(i) IMPORTS OF MANUFACTURES.—PROPORTION (PER CENT.) DERIVED FROM THE UNITED KINGDOM.

Years.	(*a*) Germany (Estimated).	(*b*) France.	(*c*) Russia.	(*d*) Italy (Estimated).	(*e*) United States (Years ended 30th June) (Estimated).
1890	47·1	38·8	23·3	40·3	48·4
1891	47·1	36·7	25·8	...	47·3
1892	49·4	36·6	28·5	31·7	44·1
1893	49·2	38·1	26·8	33·4	44·6
1894	46·6	38·4	27·0	31·0	39·8
1895	46·5	39·9	22·8	29·5	44·5
1896	48·9	38·9	21·0	31·8	45·6
1897	48·0	36·6	21·2	26·3	42·9
1898	44·9	34·8	21·5	27·6	40·3
1899	45·2	32·3	22·3	23·9	37·8
1900	40·3	29·7	19·9	22·8	42·2
1901	34·4	28·5	...	17·6	36·5

(ii) EXPORTS OF MANUFACTURES.—PROPORTION (PER CENT.) EXPORTED TO THE UNITED KINGDOM.

Years.	(*f*) Germany (Estimated).	(*g*) France.	(*h*) Russia.	(*i*) Italy (Estimated).	(*j*) United States (Years ended 30th June).
1890	10·9	26·3	2·7	9·9	...
1891	11·7	26·8	2·6	...	...
1892	11·3	30·6	2·4	8·4	24·5
1893	11·3	29·6	2·3	7·2	23·9
1894	12·3	29·1	4·3	7·4	25·4
1895	11·2	30·2	4·0	6·7	30·5
1896	12·2	29·8	3·6	6·8	28·9
1897	13·0	30·3	3·3	7·4	30·5
1898	12·8	28·9	3·0	6·5	27·3
1899	12·2	30·9	3·6	7·1	26·0
1900	11·8	29·5	3·2	7·5	22·4
1901	13·4	30·8	...	6·6	24·8

Find also (i) the arithmetic average and (ii) the median in each case.

11. The following table shows percentage changes in the retail prices of certain articles of food of workmen's consumption, 1877-1901. (Price of each article in 1901 equals 100.)

A.—IN LONDON.

Year.	(a) Flour.	(b) Potatoes.	(c) Bread.	(d) Beef.	(e) Mutton.	(f) Bacon.	(g) Butter.	(h) Tea.	(i) Refined Sugar.
1877	200·7	181·1	158·2	141·1	127·9	97·0	97·6	120·0	160·0
1878	180·0	169·8	121·4	141·1	124·0	96·0	94·9	120·0	140·0
1879	131·4	186·6	136·8	128·0	126·0	96·0	82·4	120·0	120·0
1880	160·0	178·3	152·5	141·1	127·9	96·0	90·1	120·0	130·0
1881	148·6	163·8	148·3	138·5	131·8	104·0	80·8	120·0	130·0
1882	148·6	155·5	135·8	135·9	141·5	104·0	85·8	120·0	130·0
1883	148·6	166·4	123·4	135·9	143·4	96·0	96·5	120·0	130·0
1884	125·7	134·9	113·4	128·0	127·9	96·0	95·1	120·0	120·0
1885	102·9	134·0	118·2	122·8	104·7	96·0	85·8	120·0	80·0
1886	102·9	129·4	118·2	101·9	96·9	88·0	87·2	120·0	90·0
1887	108·6	135·1	108·5	95·4	89·8	72·0	96·3	120·0	80·0
1888	100·0	124·5	111·9	98·1	96·9	68·0	92·7	120·0	90·0
1889	111·4	124·5	130·6	96·3	102·3	72·0	87·2	120·0	90·0
1890	100·0	114·7	130·6	96·3	103·1	86·0	92·7	110·0	80·0
1891	105·7	146·4	130·6	98·1	96·1	88·0	98·2	95·0	80·0
1892	117·1	122·6	128·9	99·1	96·1	92·0	101·8	95·0	90·0
1893	85·7	120·0	104·5	97·2	90·6	106·0	103·7	95·0	80·0
1894	85·7	124·9	98·3	99·1	93·8	90·0	95·4	90·0	80·0
1895	77·1	133·8	91·8	96·3	94·5	82·0	88·1	90·0	60·0
1896	85·7	95·3	95·3	96·3	89·1	70·0	95·4	90·0	80·0
1897	102·9	98·3	115·2	97·2	92·2	90·0	95·4	90·0	60·0
1898	122·9	125·3	130·6	96·3	92·2	92·0	96·3	90·0	60·0
1899	91·4	101·9	99·3	98·1	94·5	82·0	100·9	90·0	80·0
1900	97·1	91·5	107·5	100·0	97·7	90·0	100·0	100·0	80·0
1901	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0

Draw charts showing the changes; find also (i) the mode and (ii) median in each case.

(In finding the mode, arrange the prices in suitable groups 87-89, 90-92 inclusive, and so on.)

12. The table below gives the minimum average rate of discount charged by the Bank of England for different years, beginning with 1854, and extending to 1902 inclusive:

5, $4\frac{3}{4}$, $5\frac{3}{4}$, $6\frac{3}{4}$, $3\frac{1}{4}$; $2\frac{3}{4}$, $4\frac{1}{4}$, $5\frac{1}{4}$, $2\frac{1}{2}$, $7\frac{1}{2}$; $4\frac{3}{4}$, 7, $2\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{1}{4}$; $3\frac{1}{8}$, $2\frac{7}{8}$, $4\frac{1}{8}$, $4\frac{3}{4}$, $3\frac{3}{4}$; $3\frac{1}{4}$, $2\frac{5}{8}$, $2\frac{7}{8}$, $3\frac{3}{4}$, $2\frac{3}{8}$; $2\frac{3}{4}$, $3\frac{1}{2}$, $4\frac{1}{8}$, $3\frac{9}{16}$, $2\frac{19}{20}$; 3, 3, $3\frac{1}{3}$, $3\frac{1}{3}$, $3\frac{1}{2}$; $4\frac{1}{2}$, $3\frac{1}{3}$, $3\frac{1}{3}$, $2\frac{1}{2}$, $3\frac{1}{20}$; $2\frac{1}{9}$, 2, $2\frac{1}{2}$, $2\frac{5}{8}$, $3\frac{1}{4}$; $3\frac{3}{4}$, $3\frac{15}{16}$, $3\frac{11}{16}$, $3\frac{1}{3}$.

Find (*a*) the changes in the quinquennial average (from 1855 to 1859, 1860 to 1864, etc., up to 1895-1899); find also (*b*) the arithmetic average, (*c*) the mode (dividing into groups of 0·5) and (*d*) the median.

13. (i) The average prices per £100 of the 3% Consolidated Stock of the Public Funds of the United Kingdom are given below. Find (*a*) the quinquennial averages and (*b*) the arithmetic average during the whole time taken, also (*c*) the mode (dividing into suitable groups) and (*d*) the median.

Year -	1855	1856	1857	1858	1859	1860	1861	1862	1863	1864
Price -	$90\frac{1}{2}$	$93\frac{1}{8}$	$91\frac{7}{8}$	$96\frac{7}{8}$	$95\frac{1}{8}$	94	$91\frac{1}{2}$	93	$92\frac{5}{8}$	$90\frac{1}{8}$
Year -	1865	1866	1867	1868	1869	1870	1871	1872	1873	1874
Price -	$89\frac{1}{2}$	88	93	$93\frac{7}{8}$	$92\frac{7}{8}$	$92\frac{1}{2}$	$92\frac{3}{4}$	$92\frac{1}{2}$	$92\frac{1}{2}$	$92\frac{1}{2}$
Year -	1875	1876	1877	1878	1879	1880	1881	1882	1883	1884
Price -	$93\frac{3}{4}$	95	$95\frac{3}{8}$	$95\frac{3}{16}$	$97\frac{1}{2}$	$98\frac{3}{8}$	100	$100\frac{1}{2}$	$101\frac{3}{8}$	101

(ii) Find similar averages to (i) for the value of the British coinage at the London Mint between 1884 and 1904 inclusive.

Year.	Amount.	Year.	Amount.	Year.	Amount.
1884	£2,321,000	1891	£6,723,600	1898	£5,780,600
1885	2,973,500	1892	13,907,800	1899	9,010,900
1886	*None.*	1893	9,266,300	1900	13,103,700
1887	1,908,700	1894	5,678,100	1901	2,604,000
1888	2,033,000	1895	3,811,200	1902	7,119,000
1889	7,500,800	1896	4,808,900	1903	9,928,000
1890	7,680,200	1897	1,778,300	1904	11,042,000

14. The following table shows the average annual price of bar silver, per ounce standard, in each of the undermentioned years :

Year	1835	1836	1837	1838	1839	1840	1841	1842	1843	1844
Price in pence	$59\frac{11}{16}$	60	$59\frac{9}{16}$	$59\frac{1}{2}$	$60\frac{3}{8}$	$60\frac{3}{8}$	$60\frac{1}{16}$	$59\frac{7}{16}$	$59\frac{3}{16}$	$59\frac{1}{2}$
Year	1845	1846	1847	1848	1849	1850	1851	1852	1853	1854
Price in pence	$59\frac{1}{4}$	$59\frac{5}{16}$	$59\frac{11}{16}$	$59\frac{1}{2}$	$59\frac{3}{4}$	$60\frac{1}{16}$	61	$60\frac{1}{2}$	$60\frac{1}{2}$	$61\frac{1}{2}$
Year	1855	1856	1857	1858	1859	1860	1861	1862	1863	1864
Price in pence	$61\frac{5}{11}$	$61\frac{5}{16}$	$61\frac{3}{4}$	$61\frac{5}{16}$	$62\frac{1}{16}$	$61\frac{11}{16}$	$60\frac{13}{16}$	$61\frac{7}{16}$	$61\frac{5}{8}$	$61\frac{3}{8}$
Year	1865	1866	1867	1868	1869	1870	1871	1872	1873	1874
Price in pence	$61\frac{1}{16}$	$61\frac{1}{8}$	$60\frac{9}{16}$	$60\frac{1}{2}$	$60\frac{7}{16}$	$60\frac{9}{16}$	$60\frac{1}{2}$	$60\frac{5}{16}$	$59\frac{1}{4}$	$58\frac{5}{16}$
Year	1875	1876	1877	1878	1879	1880	1881	1882	1883	1884
Price in pence	$56\frac{7}{8}$	$52\frac{3}{4}$	$54\frac{13}{16}$	$52\frac{9}{16}$	$51\frac{1}{4}$	$52\frac{3}{4}$	$57\frac{11}{16}$	$51\frac{5}{8}$	$50\frac{9}{16}$	$50\frac{5}{8}$
Year	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894
Price in pence	$48\frac{5}{8}$	$45\frac{3}{8}$	$44\frac{5}{8}$	$42\frac{7}{8}$	$42\frac{11}{16}$	$47\frac{11}{16}$	$40\frac{1}{16}$	$39\frac{13}{16}$	$35\frac{5}{8}$	$28\frac{15}{16}$
Year	1895	1896	1897	1898	1899					
Price in pence	$29\frac{7}{8}$	$30\frac{3}{4}$	$27\frac{9}{16}$	$26\frac{15}{16}$	$27\frac{7}{16}$					

Find (*a*) the quinquennial averages, also (*b*) their general arithmetic average through the whole range, (*c*) the arithmetic average between the years 1835-1874 inclusive, (*d*) the mode (when arranged in suitable groups) and (*e*) the median.

15. A number of consecutive daily barometric readings were as follows :

29·64, 29·81, 29·91, 29·87, 29·87, 30·04, 30·11, 29·99, 30·21, (missing), 30·23, 30·26 in.

The arithmetic average is known to be 30·01, with a possible error of 0·005. What is the missing reading? What errors are possible? Find also a probable value by means of a graph.

16. Calculate the (*a*) mode, (*b*) median and (*c*) arithmetic average of the annual rainfall in the Thames basin during the period 1883-1902.

(In calculating the mode, arrange in groups ranging within 1 inch, *i.e.* 26-27 in., 27-28 in., etc.)

Year	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892
Rainfall in inches	28·4	22·9	29·1	31·1	21·3	28·4	25·6	22·8	33·3	23·0

Year	1893	1894	1895	1896	1897	1898	1899	1900	1901	1902
Rainfall in inches	22·1	32·3	26·3	25·8	27·8	22·1	24·8	27·9	23·5	21·9

17. A number of cubes are weighed first in air, then in water, and the loss of weight found to be 2·537 gr. When placed in a row, face to face, the total length is found to be 6·333 cm.

State (*a*) what you think the number of cubes probably is, also (*b*) the average edge.

18. Calculate probable values for the following :

(*a*) A length, different measurements of which are

32·03, 32·04, 32·02, 32·05, 32·01, 32·06, 32·02.

(*b*) The wave length of a light from the following readings :

$4x = 1{\cdot}80,\ 1{\cdot}81,\ 1{\cdot}81,\ 1{\cdot}79,\ 1{\cdot}78$ mm.,

where the wave length $= x\left(\frac{c}{a}\right)$

and $a = 112{\cdot}0$ cm.,

while readings for c are 0·092, 0·091, 0·093, 0·093, 0·092 cm.

(*c*) The number of centimetres a body falls through from rest in the first second of its motion, from the following readings :

490, 491, 489, 489, 491, 492, 498.

19. The table below gives the average yearly prices of wheat in the United Kingdom, with remarks as to cases where the price seems exceptional:

Year.	Average Price Per Imperial Qr.	Year.	Average Price Per Imperial Qr.	Year.	Average Price Per Imperial Qr.
	s. *d.*		*s.* *d.*		*s.* *d.*
1870	46 11	1881	45 4	1892	30 3
1871	56 8	1882	45 1	1893	26 4
1872	57 0	1883	41 7	1894	22 10
1873	58 8	1884	35 8	1895	23 1
1874	55 9	1885	32 10	1896	26 2
1875	45 2	1886	31 0	1897	30 2
1876	46 2	1887	32 6	1898	34 0
1877	56 9	1888	31 10	1899	25 8
1878	46 5	1889	29 9	1900	26 11
1879	43 10	1890	31 11	1901	26 9
1880	44 4	1891	37 0	1902	28 1

Remarks. 1876—Bad harvest.

1891—Failure of rye harvest in Russia.

1897-1898—Wheat crops generally short.

1898—'Leiter corner.'

1891-1895—Series of abundant harvests.

Draw a chart showing the fluctuation in price; calculate also the arithmetic average for

(*a*) 10 years 1872-1881,

(*b*) „ 1882-1891,

(*c*) „ 1892-1901,

and express the average prices as percentages of the price in 1871.

Compare also the fall in price in wheat with the fall in commodities generally, as shown by the following table:

Price of Commodities	1871	1872-1881	1882-1891	1892-1901
As % of Prices in 1871	100	104·8	84·7	75·8

Permutations and Combinations.

EXAMPLE 1.—*Find the number of ways in which the vowels* a, e, i, o, u *may be arranged in groups of* 3.

Here we may say there are 3 vacant places to be filled by the vowels

□ □ □.

For the 1st place we have 5 choices *a*, *e*, *i*, *o*, *u*;
∴ it may be filled up in 5 ways;
for the 2nd place we have 4 choices, each one of which may be associated with each of the 1st;
for the 3rd place, we have 3 choices;
∴ the total number of ways is $5 \times 4 \times 3$ or 60.

EXAMPLE 2.—*In how many ways could all the letters of the word 'vowels' be arranged?*

Here there are 6 different letters, and the number of arrangements is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

EXAMPLE 3.—*In how many ways could* 4 *three-quarter backs be picked from* 7 *possible players, disregarding any question as to their positions?*

If the position of the men on the field were taken into account, say

Left Inside left Inside right Right,

the number of possible arrangements would be

$$7 \times 6 \times 5 \times 4,$$

but any one choice of four players could be arranged in

$$4 \times 3 \times 2 \times 1 \text{ ways.}$$

$$\text{The number of choices} = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 35.$$

The four players could therefore be chosen in 35 ways.

Probabilities.

Important applications of averages are the calculation of probabilities, chances, expectations, etc.

From the examination of many mortality returns it is found that out of a large number of persons alive, say at 55 years,

a certain number would be alive, say at 60 years. Thus, in one case, out of 424,677 persons alive at fifty-five, 365,011 were alive at sixty.

The **probability** that a person alive at 55 would be alive at 60 is $\frac{365011}{424677}$, and it is evident that "probability" is really an "average probability."

Similarly, if there is a large number of equally possible events N, and in n of them a certain result happens, then the probability or chance of the result happening is $\frac{n}{N}$.

The "odds" that the event happens is

$$\frac{\text{No. of favourable cases}}{\text{No. of unfavourable cases}} = \frac{n}{N - n}.$$

Thus, if a coin be thrown up, the chance that it falls with 'heads' uppermost is $\frac{1}{2}$.

If a die is thrown the chance that a 3 turns up is $\frac{1}{6}$ and so on, and the "odds" against it, 5 to 1.

EXAMPLE 1.—*Two dice are thrown together; what is the chance that the throw is* 9?

The total number of possible throws is 6×6 or 36.

In these throws 9 can occur by the following arrangements:

1st die.	2nd die.
6	3
5	4
4	5
3	6

i.e. in 4 ways.

The number of favourable cases = 4.

Total number = 36.

$\therefore$ the chance is $\frac{4}{36}$ or $\frac{1}{9}$.

EXAMPLE 2.—*Find the chance that a man throws a* 5 *once in* 3 *throws of a single die, stopping directly the* 5 *has been thrown.*

The chance of turning up a 5 in the 1st throw $= \frac{1}{6}$.

„ „ „ having a 2nd throw = chance of missing in the 1st throw.

$\therefore$ chance of turning up a 5 in the 2nd throw $= \frac{5}{6} \cdot \frac{1}{6}$.

The chance of having a 3rd throw = chance of missing in 1st two throws $= (\frac{5}{6})^2$.

The chance of turning up a 5 in the 3rd throw $= (\frac{5}{6})^2 \cdot \frac{1}{6}$.

$$\text{Total chance} = \frac{1}{6} + \frac{1}{6} \cdot \frac{5}{6} + \frac{1}{6}(\frac{5}{6})^2$$
$$= \frac{91}{216}.$$

EXAMPLE 3. *A purse contains* 7 *similarly marked counters: one marked to represent* 10*s., two others to represent* 5*s., the rest to represent* 1*s.*

If a person be allowed to pick a counter at random, and thereby becomes entitled to the sum of money marked on the counter, find the expectation value of the drawing.

Here the chance of drawing a

10*s.* counter $= \frac{1}{7}$; $\therefore$ expectation $= \frac{10}{7}$ shillings.

5*s.* „ $= \frac{2}{7}$ „ $= \frac{2 \times 5}{7}$ „

1*s.* „ $= \frac{4}{7}$ „ $= \frac{4}{7}$ „

$\therefore$ total expectation value is $= \frac{10+10+4}{7} = 3\frac{3}{7}$ „

EXAMPLE 4.—*The following are turf quotations for a horse race, taken from a daily paper:*

2 *to* 1	*against horse*		A.
9 *to* 2	„	„	B.
10 *to* 1	„	„	C.
100 *to* 9	„	„	D.
100 *to* 8	„	„	E.
100 *to* 7	„	„	F.
33 *to* 1	„	„	G.

*Assuming that the above represent the chances of the respective horses winning, find the expectation of a person who bets £*1 *on each horse.*

Chance of A winning is $\frac{1}{2+1} = \frac{1}{3}$, since odds are 2 : 1 against ;
$\therefore$ expectation $= \frac{1}{3}$ of £2.

So for the others, whence the total expectation

$$= £\{\tfrac{1\times 2}{3} + \tfrac{2}{11} \times \tfrac{9}{2} + \tfrac{1}{11} \times 10 + \tfrac{100}{109} + \tfrac{100}{102} + \tfrac{100}{107} + \tfrac{33}{34}\}$$
$$= £6.\ 2s.\ 10d.,$$

for which the person has paid $£\{1+1+1+1+1+1+1\} = £7$.

EXERCISES C.

1. In how many ways could you arrange the letters of the words (*a*) Four, (*b*) While, (*c*) Result?

2. In how many ways could you arrange three letters chosen from the words (*a*) Ways, (*b*) Newmark, (*c*) Chosen?

3. In how many ways could a group of four letters be chosen (not arranged) from the English Alphabet?

4. Two dice are thrown ; find the chance that the throw is (*a*) 10 exactly, (*b*) less than 10, (*c*) greater than 10.

5. A cricket eleven is to be picked from fifteen players ; in how many ways can this be done?

6. A football fifteen has to be chosen from

10 eligible forwards	(8 forwards to be chosen).
4 half-backs	(2 to be chosen).
6 three-quarters	(4 to be chosen).
3 full backs	(1 to be chosen).

In how many ways might the team be made up?

7. Find the chance that if three cards be taken in succession from the same pack, they shall be king, queen and knave of the same suit?

Expectation of Life.

The table below gives the expectation of life at different ages. The numbers in the first three columns are the results

of observations, those in columns four and five are obtained by calculation founded on the death-rates of 1871-80:

Age.	Of 1,000,000 Born, the Number Surviving at the end of each Year of Life.		Mean After-lifetime (Expectation of Life).	
	Male.	Female.	Male.	Female.
12	703,595	732,697	45·96	48·13
17	690,746	718,993	41·76	44·00
18	687,507	715,622	40·96	43·21
19	683,941	711,946	40·17	42·43
26	651,998	679,822	34·96	37·26
27	646,757	674,661	34·24	36·54
33	611,827	641,045	30·01	32·30
36	592,107	622,554	27·96	30·21
37	585,167	616,144	27·29	29·52
40	563,077	596,113	25·30	27·46
41	555,254	589,167	24·65	26·78
48	495,761	537,043	20·18	22·03
49	684,794	529,048	19·55	21·36
50	476,980	520,901	18·93	20·68
51	467,254	512,607	18·31	20·01
55	424,677	477,440	15·95	17·33
56	413,351	467,443	15·37	16·69
60	365,011	422,835	13·14	14·24
61	352,071	410,477	12·60	13·65
62	338,820	397,644	12·07	13·08
63	325,256	384,319	11·56	12·51
64	311,368	370,495	11·05	11·96
70	222,056	277,225	8·27	8·95
71	206,539	260,207	7·85	8·50
72	190,971	242,934	7·45	8·07
73	175,449	225,497	7·07	7·65
74	160,074	208,003	6·70	7·25
75	144,960	190,566	6·34	6·87
76	130,227	173,316	6·00	6·51
77	115,986	156,392	5·68	6·16
78	102,359	139,927	5·37	5·82
79	89,449	124,065	5·07	5·50
80	77,354	108,935	4·79	5·20
81	66,153	94,662	4·51	4·90
82	55,842	81,305	4·26	4·63
83	46,489	68,966	4·01	4·37
84	38,132	57,723	3·58	4·12

EXAMPLE 1.—*Employ the table on p.* 525 *to evaluate the value of life annuities at different ages; for example, to find the value of a life annuity of* £1 *at the age of* 50, *the insurer being male.*

From the table the expectation of life is 18·93 years, and the value of the annuity is very nearly the present value of 19 yearly payments of £1.

Assume 3 % per annum, Compound Interest, payable yearly.

$$\text{The present value in £} = 1 + \frac{1}{1{\cdot}03} + \frac{1}{(1{\cdot}03)^2} + \ldots + \frac{1}{(1{\cdot}03)^{18}}$$

$$= \left\{1 - \frac{1}{(1{\cdot}03)^{19}}\right\} \div \left\{1 - \frac{1}{1{\cdot}03}\right\}.$$

$$\text{But, } \left(\frac{1}{1{\cdot}03}\right)^{19} = 0{\cdot}57029 \text{ and } \frac{1}{1{\cdot}03} = 0{\cdot}97087 \Bigg\} \text{ (cf. tables);}$$

$\therefore$ the value of £1 annuity

$$= \frac{1 - 0{\cdot}57029}{1 - 0{\cdot}97087} = \frac{0{\cdot}42971}{0{\cdot}02913} = £14.\ 15s. \text{ approx.},$$

while £100 would buy a yearly annuity of

$$£\frac{2{\cdot}913}{0{\cdot}42971} = £6{\cdot}778 = £6.\ 15s.\ 9d. \text{ approx.}$$

Similarly, the tables may be employed in calculating the value of a Reversion.

Consider the following:

EXAMPLE 2.—*What is the value of a Reversion to* £2750 *on the death of a lady, aged* 77 *years?* (*Compound Interest, at* 3 % *per annum.*)

When the lady dies, the sum of £2750 will be in the possession of the holder of the reversion.

The expectation to the nearest year of after life in the case of a lady, aged 77, is 6 years.

The present value is therefore required of £2750, due in 6 years.

∴ the present value

$$= \frac{£2750}{(1{\cdot}03)^6} = £(2750) \times (0{\cdot}83748) = £2303{\cdot}07$$

$$= £2303.\ 1s.\ \text{(to the nearest } 1s.\text{)}.$$

EXAMPLE 3.—*An Insurance Company charges a certain premium to a male client who begins insuring at 26 years, the calculation employed in fixing the premium is based on money earning interest at* $2\frac{1}{2}\%$ *per annum. What is the price of the premium (disregarding expenses of management, etc.) on each* £100 *policy?*

The expectation of life, at 26, for a male is 35 years. Now £1 per annum will amount in 35 years to £54·9282 (see tables) at $2\frac{1}{2}\%$.

∴ the premium per £100 $= \frac{£100}{54{\cdot}9282} = £1{\cdot}839 = £1.\ 16s.\ 9d.$

EXAMPLE 4.—*If of every* 1,000,000 *male persons born,* 77354 *survive at* 80 *years of age, and if of every* 1,000,000 *male persons born* 66153 *survive at* 81 *years of age, and if the expectation of life at* 81 *years is given as* 4·51 *years, find the expectation of life at* 80 *years.*

Out of 77354 people 11201 (*i.e.* 77354 − 66153) die during the ensuing year. Hence the probability of dying within the year is $\frac{11201}{77354} = 0{\cdot}145$ approx., and the probability of living until the end of the year $= (1 - 0{\cdot}145) = 0{\cdot}855$.

For the men who live through the year the expectation is 4·51 years at the end of the year, or 5·51 years at the beginning of the year.

For the men who die, assume that the average life was 0·5 yr., whence the expectation at 80

$$= 0{\cdot}5(0{\cdot}145) + (5{\cdot}51)(0{\cdot}855)$$

$$= 4{\cdot}79 \text{ years.}$$

EXERCISES CI.

Use the table on page 525 to find, by calculation, the value of the immediate annuity that may be purchased by the payment of £100 at the following ages and rates of interest, the purchaser being (*a*) male, (*b*) female. (Take expectation of life to the nearest year, and state answer to nearest penny.)

1. Age 55, rate of interest $2\frac{1}{2}\%$.
2. „ 55, „ „ 3%.
3. „ 60, „ „ 3%.
4. „ 40, „ „ $2\frac{1}{2}\%$.
5. „ 70, „ „ 4%.

Employ the mortality table on page 525 to find the insurance premium on £100 policy (without profits) at $2\frac{1}{2}\%$ per cent. per annum, Compound Interest.

6. When the insurer is male, aged 12 years.
7. „ „ „ „ 19 „
8. „ „ „ „ 33 „
9. „ „ „ „ 36 „
10. „ „ female, „ 48 „
11. „ „ „ „ 17 „
12. „ „ „ „ 51 „

13. Of 1,000,000 persons born the numbers surviving (census returns 1871-1880) at the end of each of the given years of life, are as follows :

Age.	Male.	Female.
50	476,880	520,901
51	467,254	512,607
52	457,022	504,188
53	446,510	495,645
54	435,729	486,973

The expectation of life at 54 is 16·53 years for a male.
" " " " 17·98 " female.
Find the expectations at (*a*) 53, (*b*) 52, (*c*) 51, (*d*) 50 for a male, (*e*) 53, (*f*) 52, (*g*) 51, (*h*) 50 for a female.

14. Find the values of Reversions to the following:

(*a*) £800, on death of lady, aged 72 years.
(*b*) £1200, " " man, " 83 "
(*c*) £1500, " " " " 79 "
(*d*) £7575, " " lady, " 64 "
(*e*) £1250, " " " " 70 "

(Take expectation of life to nearest year.) Assume Compound Interest at 3% per annum.

REVISION EXERCISES CII.

Miscellaneous: Mainly from Examination Papers.

A 1. Explain the rule for finding the G.C.M. of two numbers, also any methods with which you are acquainted for simplifying the operations.

Find the G.C.M. of 622932 and 2087664.

Also the L.C.M. of 3663, 16280 and 9768.

2. (i) Reduce 0·63 of £1. 0*s.* 4*d.* to the decimal of 0·5625 of £4. 10*s.*

(ii) Find the value of 0·18637 of £4, true to the nearest penny.

3. The sides of a rectangular piece of paper are measured both in centimetres and in inches, and the length of a metre in inches is determined from each set of measurements. Find, to 2 decimal places, the average of the two results if the length of one side of the rectangle is 16 inches or 40·6 cm., and that of the other 12·75 inches or 32·5 cm.

4. An observatory building consists of a cylindrical base of 12 feet radius and 13 feet high, surmounted by a hemisphere of the same radius. Find the cost of covering the outside with water-proofing material at 4*s.* 6*d.* per square yard.

5. Two men begin at noon to fill a cistern holding 200 gallons. The first empties a three-gallon cask into the cistern at the end of every two minutes, while the second empties a two-gallon cask into it at the end of every three minutes. When will the cistern be full?

6. A man, who can row three miles per hour in still water, rows to a place 9 miles down stream and back again. How long does he take if the stream flow at the rate of 2 miles per hour?

7. Find the rate per cent. at which a customer pays interest, in buying the following on credit terms: "We send a cycle on receipt of 1*s.* deposit and payment of 1*s.* per week for 89 weeks, making a total of £4. 10*s.* in all, or our net price is only £4."

8. The weight of 1000 sovereigns is 7·988 kilo., that of 1000 twenty-mark pieces is 7·962.

If £1 = 20·43 marks at the mint par of exchange, find the fineness of the twenty-mark piece.

9. A capitalist has £27,900 which he invests, part in 4 per cent. debenture stock at $114\frac{7}{8}$, and part in 3 per cent. stock at $96\frac{3}{8}$. If the resulting income be £873, find how much is invested in each stock, allowing as brokerage, 2*s.* 6*d.* for each £100 stock and £3 for stamp fees on the total amount invested.

10. AC is a straight railway and B a place 5 miles from the nearest point C of the railway. Suppose goods to be sent from A to a siding S by rail and thence by road to B, S being x miles from C. The cost per mile being 1 shilling by rail, and the cost per mile by road 2 shillings and the distance AC 12 miles, give an expression for the total cost of carriage. Draw a graph showing the cost of carriage for different values of x, and determine where the siding S must be placed in order to make the cost as small as possible. For the graph represent 1 shilling and also 1 mile by 2 centimetres.

B 1. Obtain the least perfect cube which is exactly divisible by 42, 56, 154, 231 and 484. (Factorize the numbers.)

2. A cubic foot of water weighs approximately $62\frac{1}{2}$ lb. A litre of water weighs a kilogram. An inch = 2·54 cm. From these data calculate to one decimal place the number of grams in 1 lb.

3. How far does a roller, width 5 ft. 3 inches, have to be pulled in rolling a rectangular field of area $7\frac{1}{4}$ acres? Allow 10 per cent. extra for turning, and assume that the whole field can be covered without one crease overlapping another.

4. The side wall of a lean-to shed slopes from a height of 5 feet to a height of 20 feet, and 17 upright slips of wood are fastened to the wall at equal intervals, each reaching from the ground to the top of the wall. Find the total length of the wood.

5. Three persons, A, B, C, enter into partnership with sums of £1800, £1500 and £1400 respectively. A withdraws £600 at the end of 3 months, and the deficit in the capital is made up by B subscribing £300 and C £300. The profits at the end of a year's business amount to £840; how should it be divided?

6. A bicycle was sold at a loss of 40 per cent. on the cost price, and a second bicycle was bought with the proceeds when 5 guineas had been added. This second bicycle was sold at a loss of 35 per cent., and a third bicycle was bought with the proceeds after the addition of £5. 15*s*. 3*d*. This third bicycle cost £16; what was the price of the first?

7. A hollow ball, of external diameter 6 centimetres and of thickness half a centimetre, weighs 162 grams when it is filled with liquid. Find to the nearest hundredth of a gram the weight of a cubic centimetre of the liquid, a cubic centimetre of the substance of the ball weighing 2 grams. (If a sphere is inscribed in a cube, then volume of sphere : volume of cube : : 11 : 21.)

8. Tithe rent charge in 1901 is of such value that every nominal £100 is really worth only £66. 10*s*. $9\frac{1}{4}$*d*. Find the amount of tithe rent charge which is worth £1. 14*s*. 4*d*. less than its nominal value.

9. A member of a building society obtains a loan of £400 for the purchase of a house, and agrees to repay it with compound interest at the rate of 5 per cent. in five equal annual instalments, the first payment to be made at the end of a year. What should each instalment be?

10. In an experiment with a screw jack the following results were obtained:

Load W lifted in pounds	Power P applied in pounds.
100	12·6
120	13·8
140	15·7
160	17·6
180	19·6
200	21·5

Plot these observations, draw a fair line through them, and from your diagram read off the values of a and b which satisfy the equation

$$W = a + b \,.\, P.$$

C 1. In subtraction how do you get over the difficulty of taking a greater digit from a less? Illustrate your answer by taking 863 from 952.

By what factor less than 1000 must 7683 be multiplied so that the last three figures of the product may be 768?

2. Find the area of the walls of a room 23 ft. 8 in. long, 18 ft. 4 in. wide and 12 ft. high, and the expense of covering them with paper 3 ft. 6 in. wide costing 4*s.* 6*d.* per dozen yards.

3. Obtain the square root of 11·0405067, exact to four places of decimals.

4. A sum of £371 is due to me at the end of 18 months; what sum of money paid down *now* would discharge the debt, reckoning simple interest at 4 per cent.?

5. Explain the following extract with reference to the British coinage:

Denomination.	Standard Weight.	Least Current Weight.	Remedy of Weight.
Gold.	*Grains.*	*Grains.*	*Grains.*
Sovereign	123·27447	122·500	0·20000
Silver.			
Shilling	87·27272		0·578

A mining company extracted 19744 tons of gold ore, yielding 2275 oz. of gold. What number of sovereigns would the mint supply from this? [Mint gold is of $\frac{11}{12}$ fineness.]

6. The squares of the times of revolution of the planets of the Solar system are as the cubes of their distances from the Sun. Find, to the nearest day, the time Jupiter takes to go round the Sun, given that its distance from the Sun = 5·2 times the Earth's distance.

7. A person bought the materials of an old house for £120, upon condition that they should be removed within 12 days under a penalty of £1 a day for every day over the 12 days. He paid £3. 10s. a day for labour, and having sold the materials for £214. 10*s*., he found that he had made a profit of £30. How many days did he take to remove the materials?

8. A shallow tinplate dish is in the form of a frustum of a cone. Its depth is 3 inches, its base a circle of 6-inch radius, its upper margin a circle of 8-inch radius. Find its total surface.

9. If pure gold be worth £3. 17*s*. 6*d*. per oz. and silver be worth 2*s*. 8*d*. per oz., what will be the value of an alloy consisting of 3 parts of the former to 5 of the latter?

If this alloy be sold at £1. 13*s*. $2\frac{1}{4}$*d*. per oz., what is the gain per cent.?

10. The distance, s feet, of a moving point from a fixed point at time, t seconds, is given in the form $s = 10 + 8t + 6t^2$. Find

the average velocity during the first 6 seconds. Find also the velocity at the time $t=3$ (1) by drawing the distance-time curve, (2) by determining the average velocity during the interval between $t=3$ and $t=3+x$, when x is very small indeed.

D 1. The numbers 4242, 2903, when divided by a certain number which contains three digits, leave the same remainder; find that remainder.

2. Of the children attending school in Switzerland, 71·7 per cent. speak German only, 22·4 speak French only, 4·5 speak Italian only, and the remainder consists of 5,838 children who speak only Romansch; find the total number of children attending school.

3. Given that one metre = 39·371 inches, and 1 kilogram = 2·2046 lb., and if 1 cubic metre of water weigh 1,000 kilograms, find the weight of a cubic foot of water in lb., correct to the nearest tenth of an ounce. Use contracted methods of multiplication and division.

4. Three persons contribute sums of £250, £500 and £750 respectively towards a venture, on the understanding that the profits shall be divided in such a way that the *rate* of interest which each receives shall be in proportion to the amount of his contribution. If the profits for a year amount to £245, how much will each of them receive?

5. Find the cost of discounting on Sept. 1, 1904, a bill of £580. 12*s*., maturing on Oct. 18, 1904. (Mercantile Discount at the rate of 4 per cent. per annum.)

6. Express $0\cdot\dot{2}9\dot{6}$ as an infinite geometrical series, find its sum, and show that it is equal to $(0\cdot\dot{6})^3$.

7. On a map is shown a triangular piece of land bounded by three roads which measure 4 in., 4 in., 3 in. The whole map is 18 inches long and 12 inches wide, and represents 216 sq. miles. What area does the triangle represent? Use a carefully drawn figure to find the area of the triangle.

8. The radii of the circular faces of a frustum of a right cone are 12 and 8 feet, and the area of its curved surface is $20\pi\sqrt{241}$ square feet; find the thickness of the frustum.

9. By buying 3 per cent. consols at a certain price I find I obtain $3\frac{3}{8}$ per cent. for my money, and derive a net income therefrom, after paying an income-tax of 6*d.* in the £, of £421. 4*s.* Find the amount of stock and the price at which I bought it.

10. The gas-service pipe to a house 75 feet from the main is $\frac{1}{8}$ in. in diameter; for how many burners, each taking five cubic feet of gas per hour, will this serve? The number of cubic feet per hour delivered by a pipe on that main is $1000\,\frac{d^5}{0{\cdot}45\,\mathrm{L}}$, where d is the diameter of the pipe in inches, and L is the length of the pipe in yards.

E 1. (*a*) Add together

$$3\tfrac{7}{15},\ \tfrac{27}{35},\ 26\tfrac{5}{7},\ 5\tfrac{29}{70},\ 20\tfrac{1}{21}.$$

(*b*) A room is 45 feet 6 inches long and 23 feet 10 inches broad. Could it be covered exactly with tiles 13 inches square? How many would be required?

2. Find the prime factors of 1066, 1815 and 1896; and hence write down the factors of their L.C.M.

3. Reduce 35469099 cub. in. to cub. ft. by short division, explaining how the remainder is obtained, and the reason of the rule.

4. A customer obtains from his banker the loan of £1250 on June 2nd; he pays £450 on July 2nd; £600 on October 1st, and finally pays in £800 on November 1st.

How does the customer stand with regard to the bank on December 31st, assuming the bank charges 5 per cent. per annum interest on all sums that the customer owes it, but allows the customer interest at $2\frac{1}{2}$ per cent. per annum on all sums standing to the credit of the customer.

5. Find the change in a man's income, occasioned by selling £10,000 Two and three quarters per cent. consols at $101\frac{13}{16}$, and investing the proceeds in Corporation of London $3\frac{1}{2}$ per cent. Debentures at 100. Brokerage $\frac{1}{16}$ per cent. on Consols, 1 per cent. on the Debentures.

6. A silver bowl weighs $67\frac{1}{2}$ oz. of 480 grains each. What is its weight in French kilograms, etc., to the nearest gram [a gram = 15·45 grains].

7. The times of departure from Euston, and arrival at Birmingham, of passenger trains (starting between 6 a.m. and 6 p.m.) are as follows :

	Euston.	Birmingham.
a.m.	7.10	10.12
	7.20	10.30
	9.20	11.30
	10.15	1.20
p.m.	12.5	2.50
	12.15	3.45
	1.30	4.0
	2.35	4.45
	2.45	5.37
	4.30	7.0
	5.35	8.25

What is the arithmetic average time of the journey? What is the difference between it and the actual time which most trains take in doing the journey? Which time do you regard as the more useful for the purpose of an average (using the term "average" in a general sense)? State your reasons.

8. A concentric disc is removed from a circular disc 24 inches in diameter so as to leave only two-thirds of the original area. Find the radius of the disc removed.

9. For a certain book it costs a publisher £100 to prepare the type and 2*s.* to print each copy. Find an expression for the total cost in pounds of x copies.

Also make a diagram on the scale of 1 inch to 1000 copies and 1 inch to £100, to show the total cost of any number of copies up to 5000. Read off the cost of 2500 copies, and the number of copies costing £525.

10. A very long strip of paper is rolled on a cylinder of wood 1 inch in diameter, and the total diameter of the roll is 2 inches. Find what would be the total diameter of the new roll if the same strip of paper were coiled on a cylinder of wood 2 inches in diameter.

F 1. Employ an abbreviated method to divide 33·82289 by 66·4684 to six places of decimals (so that the error is less than 0·0000005).

2. Write down the nine numbers which immediately precede 81 and the nine which immediately follow, and show that their sum is exactly divisible by 9. Would the sum still be divisible by 9, if instead of a group of nine numbers on either side of 81, a group of five, six, or any other amount were chosen? Give your reasons.

3. Find the greatest number which will exactly divide both 867,502 and 1,000,369; find also the smallest number which is exactly divisible by both the numbers.

4. The surface of a sphere of radius r inches is $4\pi r^2$ square inches, where $\pi = 3{\cdot}14159$. If π is taken, instead, to be equal to $\frac{22}{7}$, find, roughly in square miles, the difference that this will make in the calculated area of the earth's surface, the earth being a sphere of radius 4000 miles.

5. How many moons would it take to form a body the size of the Earth; the Moon's diameter being 2180 miles, and the Earth's diameter 7957·75 miles?

6. Genuine milk contains 88·75 per cent. by bulk of water, 2·75 of fat and 8·5 of non-fatty solids. A purchaser buys 7 gallons of milk at 3*d.* a quart; the milk on being analysed is found to contain 90·84 per cent. of water, 2·24 of fat, the residue being non-fatty solids. Find whether anything besides water has been added to it; and find the sum of which the purchaser has been defrauded.

7. A right circular wooden cylinder has a length of 38 inches and a radius of 1 foot. At one end a hollow in the shape of a hemisphere, of radius 1 foot, is scooped out, and at the other end is scooped out a hollow in the shape of a cone, two feet high, and the base of which is one foot in radius; find the volume of the remaining solid. Also find the weight of this solid, given that $\pi = 3{\cdot}1416$, that the density of the wood is nine-tenths of the density of water, and that a cubic foot of water weighs 1000 oz.

8. A person repays the principal of a debt by annual instalments, paying £120 the first year, and each succeeding year 10 per cent. more than the year before: how much will he have repaid in 10 years?

9. A farmer bought 6 oxen and 100 sheep for £336; of the sheep, 4 died, and the rest were sold at £2. 7*s*. 6*d*. each; and 2 of the oxen fetched £15 each. At what price must the remaining 4 oxen have been sold if the profit on the whole transaction amounted to 5 per cent.?

10. From the table given below plot curves showing for the given period (1) the hour of sunrise, (2) the hour of sunset, (3) the duration of sunshine. Deduce (1) the latest sunrise, (2) the earliest sunset, (3) the date of the shortest day.

	Sunrise.		Sunset.	
	h.	m.	h.	m.
Nov. 26	7	37 a.m.	3	57 p.m.
Dec. 3	7	48 „	3	51 „
„ 10	7	58 „	3	47 „
„ 17	8	4 „	3	47 „
„ 24	8	8 „	3	50 „
„ 31	8	9 „	3	56 „
Jan. 7	8	7 „	4	4 „
„ 14	8	4 „	4	14 „
„ 21	7	56 „	4	26 „

G 1. (i) A walk two yards broad taken off a square field all round it reduces its area by one quarter; what was its area in square yards?

(ii) Is it possible for two numbers to have a 100 for their sum and 3000 for their product? State the reasons for your answer.

2. A left £15,325 between B, C and D. To B he left such a sum that, after paying legacy duty at the rate of 10 per cent., B actually received £600. C's legacy was to D's legacy as 3 to 2. What net sums did C and D receive if they paid legacy duty at the rate of 3 per cent.?

3. Find the square root of 87798284·89146121; and find the area of a face of a cube whose contents are 13634·789869 cubic feet.

4. Express $0{\cdot}5\dot{7}\dot{3}$ as a series involving geometrical progression, and thence show by summation that its value is $\frac{284}{495}$.

5. Describe any two methods by means of which a London merchant might settle a debt incurred with a merchant abroad.

Find the cost of drawing at London a bill of 48,725 francs due Paris, at 25·22 cheque rate in 4 months.

Bill Stamp - - - - -	0·5 per mille.
Brokerage - - - - -	1 per mille.
Rate of Discount in Paris - -	$2\frac{3}{4}$ per cent.

6. Find, by using the tables, in what time a sum of money will treble itself at 5 per cent. compound interest.

7. Supposing the quantity of land under barley in England to be this year half as much again as that under wheat, and the quantity under oats to be equal to the other two together; if the quantity under wheat next year be reduced by 25 per cent., and the quantity under barley increased by 5 per cent., the whole quantity remaining the same as before, by how much per cent. will the quantity under oats be increased?

8. A right circular cylinder $10\frac{1}{2}$ ft. long has a volume 528 cubic feet. Find the cost of gilding its convex surface at $9\frac{3}{4}d.$ per square foot. (Use $\pi = 3\frac{1}{7}$.)

9. The length of a room is to its breadth as 5 to 4. The flooring costs £46. 17*s.* 6*d.* at 7*s.* 6*d.* per square yard, the walls cost £30. 7*s.* 6*d.* to colour at 4½*d.* per square foot. Find the dimensions of the room, neglecting windows and door.

H 1. If 0·917 of a sovereign be pure gold, how much pure gold is there in 23·27 lb. of standard gold? (Standard gold is the gold from which sovereigns are made.)

2. If 1 cub. ft. of brass weigh 551·25 lb., how many cubic feet will there be in a block weighing 1290·95 lb.? (State your answer to five significant figures.)

3. Express as £, and decimals of a £, (*a*) £173. 3*s.* 1¼*d.*, (*b*) £18. 17*s.* 9¾*d.* (True to the third decimal figure.)

4. Find the value of 547⅜ cwt. at £1. 2*s.* 8½*d.* per cwt.

5. Employ the given table to express the following lengths in English measure: (*a*) 1·24 metres, (*b*) 0·5321 metre; state the answer true to three significant figures:

1 metre	=	39·37 in.
2 „	=	78·74 „
3 „	=	118·11 „
4 „	=	157·48 „
5 „	=	196·85 „

6. Coal is sold in England "free on board" a vessel for 17*s.* a ton, and in France the price realised is 30 francs per tonneau (1000 kilo.). What is total amount in francs per tonneau of freight and profit? (£1 = 25·23 francs.)

7. An estate consists of house property of gross rental of £150, but on which the following payments are made:

Insurance - - -	£3	15	0
Land Tax - - -	3	10	0
Sewers Tax - -	1	15	0

The losses on account of bad tenants, etc., amount to 12½ per cent. of the rental, ordinary repairs and expenses to 17½ per cent., and

collection of rent 2½ per cent. Find the value of the freehold, calculated at 4 per cent. simple interest.

8. Which is the more advantageous of the following methods, which a London merchant might employ to settle in debt in Paris?

(i) Buying French 4 months Bills at 25·76.

(ii) Buying Vienna Bills at 24·30, and selling these in Paris at 1·05.

Brokerage 1 per mille.

9. Construct an isosceles triangle ABC, base BC 8 cm. and height AD 9 cm. Imagine a point O to start from D and to travel along the line DA to A (see Fig. 108). From measurements give a table of values for OA+OB+OC corresponding to values 0, 1, 2 ... 9 cm. of DO, and from these data draw a graph, representing DO by its actual length and OA+OB+OC by 5 times its actual length. Make the scales used and the position of the axes clear; the zero may be allowed to fall beyond the paper.

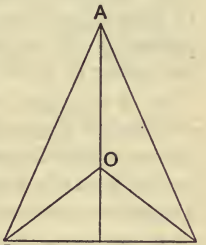


FIG. 108.

Determine from your graph how far O has travelled when the sum of its distances from A, B, C is the least possible. Show this position in your figure, and from measurement give the value of the angles AOB, BOC, COA.

I 1. Find how many different integers, besides unity, divide 9009 without remainder.

2. Simplify $\sqrt{\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}}$,

and calculate it to three places of decimals.

3. In 1875 there were 10,803,030 acres of cotton fields in the United States, and they produced 4,632,313 bales of cotton. In 1898 the acreage was 24,967,295, and the production 11,189,205 bales. Calculate, for each year, the average number of bales produced per 100 acres. (Give the nearest whole number.)

4. A person bought £3000 worth of stock in $2\frac{1}{2}$ per cents at 88. He sold out when these funds had fallen to 85, and invested the proceeds in the 6 per cent. railway preference shares at £180 per £100 share. Find the alteration in his income, brokerage being neglected.

5. Find the rate of interest incurred by a person who buys the piano player below by monthly payments:

"The piano player. £29 cash, or 36 monthly payments of 20*s.*"

6. Find the commercial discount at $3\frac{3}{4}$ per cent. on £787. 13*s.* 5*d.* due six months hence.

7. A hollow spherical shell of metal has an internal diameter of 10 inches, and the thickness of the metal is 1 inch: find the number of cubic inches of metal in the shell, and the cost of gilding the surface at 3*s.* 6*d.* per square foot, taking $\pi = \frac{22}{7}$.

8. A wine glass has its interior in the form of an inverted cone of 2 inches diameter and 2 inches depth. Compare the internal volume of such a glass with that of a tankard, the interior of which has the form of a frustum of a cone, the diameters of the ends being 3 inches and 4 inches respectively, and the depth 4·5 inches. ($\pi = \frac{22}{7}$.)

9. A commences business with a capital of £4000, and after 4 months takes B into partnership with a capital of £300. Two months later they take C into partnership with a capital of £5000. At the end of the year their net profits amount to 16 per cent. on the whole capital invested. What should each receive of the profits?

10. Draw a *plan* of the field, and find its *area* in acres, roods and perches from the following field-book entries in links:

	AD	
0	1120	0
C 225	920	
	780	180 E
	520	60 F
B 175	220	
	80	120 G
0	0	0

From A go East.

J 1. The following work is sent up by a person in answer to the question: "Find the L.C.M. of 165, 216, 209, 192, 200."

5	165, 216, 209, 192, 200	
6	33, 216, 209, 192, 40	L.C.M. $= 5 \times 6 \times 8 \times 11 \times 36 \times 19 \times 5$
8	33, 36, 209, 32, 40	$= 9{,}028{,}800.$
11	33, 36, 209, 4, 5	
	3, 36, 19, 4, 5	

Show by an independent method of working the question that the answer is correct.

The method being, nevertheless, fundamentally wrong, point out the errors, and explain why none of them affects the result. Also construct a simple example wherein an error of the same type does affect the answer, making it, *e.g.*, three times as large as it should be.

2. Simplify $\frac{23}{165} - \frac{15}{253} - \frac{11}{345}$.

3. Express $0{\cdot}0\dot{4}1\dot{6}$ as a vulgar fraction in its lowest terms. Make use of this result in reducing $0{\cdot}375\dot{4}1\dot{6}$ to a fraction.

4. A workman is to be paid 1*s.* for his first day's work, 1*s.* 1*d.* for the second day, 1*s.* 2*d.* for the third day, and so on increasing at the rate of a penny each day that he works. Find how much more he earns in the second week than in the first, if he works six days in the week.

5. A closed cubical cistern, made of metal 1 centimetre thick, has an internal capacity of 1 cubic metre. Assuming the metal to be 7 times as heavy as water, find the weight of the cistern in kilograms.

6. How much stock at $97\frac{1}{2}$ must I sell out in order to realise £1300. And if, by investing this £1300 in $4\frac{1}{4}$ per cent. stock at 102, I increase my income by £8. 6*s.* 8*d.* per annum, what rate of interest was I receiving on the former stock?

7. A hollow sphere of iron is of uniform thickness. Its external diameter is 9 inches and its weight is 72 lb. Find the volume of the internal cavity. The iron weighs 480 lb. per cubic foot, and the volume of a sphere of diameter d is $\frac{1}{6}\pi d^3$.

If the diameter may be wrong by 0·1 inch, the weight of the

sphere by an ounce, and the weight of a cubic foot of the iron by 10 lb., estimate the possible error in your result.

8. Find to the nearest penny the amount of £453 in 3 years at $2\frac{1}{2}$ per cent. compound interest.

9. A retail bookseller is supplied by a wholesale firm, which allows discount off the published price of certain books (Class A) at 40 per cent.; off all others (Class B) at $33\frac{1}{3}$ per cent. At the end of a year he finds that he would have paid $2\frac{1}{2}$ per cent. less than he actually did, had he accepted the tender of another wholesale firm, which offered a uniform discount of $37\frac{1}{2}$ per cent. What percentage of his purchases (reckoned at published price) belonged to each class? Give your answer to the nearest integer.

10. Draw, as accurately as possible, a fair-sized plan of a field from the following field-book reading. Then find the area in acres, roods and poles.

The Field-Book.

	Links.	
	To D	
	3250	
	2660	900 to C
to E 400	2300	
	1980	300 to B
to F 1600	1700	

From A go North.

K 1. Compute the value of

$$\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{2}-1}$$

correct to two places of decimals.

Compute the sum of the infinite series of terms

$$1+\frac{1}{1}+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ldots$$

to six places of decimals.

2. The basin of the Severn is 4350 square miles in area, an[illegible] the yearly rainfall 29 inches. Half of the rain that falls is agai[illegible]

evaporated and the rest flows to the sea. Find, to the nearest million gallons, how much water flows to the sea per hour. (A cubic foot of water is 6·23 gallons.)

3. If the assessable value of the County of London is £40,142,274, find what rate per pound must be levied in order to raise £2,500,000. (Give your answer in current coin.)

Also, by how much must the assessable value increase that an increase in the rate of 1*d.* in the pound should add £250,000 to the amount raised?

4. A solid iron cube, the edge of which is two feet in length, and a solid iron sphere, the radius of which is one foot, are thrown into a cubical tank, which is six feet across and is half filled with water. Find the rise of the surface of the water in inches, to five places of decimals, it being taken for granted that the cube and the sphere are both completely submerged.

5. Find the percentage error in the following approximations occasionally used in practical work:

(i) Circumference of a circle is equal to three times the diameter added to one-seventh the diameter.

(ii) To get the area of a circle: "Square the diameter, add the one-twentieth part of the result, and take three-quarters of the sum for the area."

State your result true to two important figures, assuming that the true rules are:

$$\text{Circumference} = \pi(\text{diameter}).$$

$$\text{Area} = \frac{\pi(\text{diameter})^2}{4},$$

where $\pi = 3{\cdot}14159$ true to the fifth decimal figure.

6. The breaking weight W in tons for a long iron column of length L feet, and diameter D inches, is sometimes calculated by the formula

$$W = 14{\cdot}9\,\frac{D^{3{\cdot}76}}{L^{1{\cdot}7}}.$$

What is the diameter of a column 60 feet long, for which the breaking weight is 226 tons?

7. The difference between the simple and compound interest on a certain sum, for three years at four per cent. per annum, is £12. 13*s*. 4*d*.; what is the sum?

8. There are four vessels of equal capacity; the first is filled with spirit to an extent of $\frac{1}{5}$th, the second to $\frac{1}{6}$th, the third to $\frac{1}{4}$th and the last to $\frac{1}{3}$rd. The first is then filled up with water, and from this mixture the second is filled up, again from this second mixture the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

9. The population of a country increases regularly by 1 per cent. every year; find, to three significant figures, what will be the total percentage of increase in 10 years.

10. The railway rate per ton for a certain class of goods is specified as follows: For any distance up to 20 miles, 1 penny per mile; additional for the next 30 miles 0·8 penny per mile; additional for the next 50 miles 0·5 penny per mile; additional for any further distance 0·4 penny per mile. There is also a fixed terminal charge of 3 shillings per ton irrespective of distance. Find the rate for 25, 75 and 145 miles, in pence per ton, and plot a curve showing the rate for all distances up to 200 miles.

L 1. Some of the following statements are obviously wrong; indicate any errors that you can find, explaining your reasons. You need not correct the mistakes. The last figure in each answer is approximate only:

(i) $(4{\cdot}816)^3 = 111{\cdot}7$.

(ii) $\sqrt{0{\cdot}4816} = 0{\cdot}2194$.

(iii) The area of a circle of radius 81·90 feet is 5268 square feet.

2. Work three of the following (*a*, *b*, *c*, *d*) in the most intelligent way you can:

(*a*) 30 miles an hour is how many feet per second?

(*b*) If 15 per cent. of a certain amount is £27. 10*s*. 6*d*., what would $16\frac{1}{2}$ per cent. of the same amount be?

(*c*) $3{\cdot}142(7{\cdot}86^2 - 5{\cdot}93^2)$.

(*d*) $(1{\cdot}23456)^2$ to five significant figures.

3. Write down, or obtain expressions for the volume of (i) a pyramid on a square base; (ii) a truncated pyramid on a square base.

Find the volume of air contained in a street lamp with a flat top and bottom, the four slanting faces of the lamp being equal trapeziums with sides 6, $9\frac{1}{2}$, 12 and $9\frac{1}{2}$ inches long.

4. In the game of dominoes a number of blocks are used. Each block is marked off into two parts, and on each of these parts a number of pips, between 0 and 9 inclusive, are marked. These are arranged in all possible ways, and the pips on the two parts of a block may be the same or different, and no two blocks are alike. How many blocks are there?

5. The formula $\frac{nr^{-1} - r^{-n}}{n-1}$ is used by engineers. Find its value to two decimal places when $n = 1{\cdot}05$ and $r = 2$.

6. A man invests £1980 in the $3\frac{1}{2}$ per cents at 99 and £3220 in the $4\frac{1}{2}$ per cents at 105; find the average rate of interest on his whole investment.

7. A cubic centimetre of an alloy of gold and silver weighs $13\frac{1}{3}$ grams; assuming that a cubic centimetre of gold weighs 19·6 grams and a cubic centimetre of silver 10·2 grams, and that the volume of the alloy is equal to the sum of the separate volumes of the gold and the silver contained in it, find the proportions by weight of gold and silver in the alloy.

8. Find the radius of a sphere, the volume of which is equal to the volume of a solid circular cylinder of height one foot and radius nine inches.

9. A merchant who sold his goods at a profit of 10 per cent. found that, when he allowed $2\frac{1}{2}$ per cent. discount off his selling price, his business increased by one-third. Find whether his

total profits were increased or diminished by adopting this plan, and in what proportion.

10. Make a list or table of the integral powers of 2 from index -2 to index $+4$. Use your values to draw a curve, the ordinates of which show the logarithms of numbers between 0·3 and 16·0 to base 2; use as unit for the numbers 1 cm. From your curve determine as accurately as you can $\log_2 1{\cdot}7$ and $\log_2 5{\cdot}4$.

Use your curve to graduate the two pieces of gummed paper for use as a slide rule and fix the two pieces to a page of your book so as to show the product of 1·7 and 5·4.

M 1. Express as a decimal:

$$\frac{1}{[1.2]^2}+\frac{1}{[1.2.3]^2}+\frac{1}{[1.2.3.4]^2}+\frac{1}{[1.2.3.4.5]^2}+\ldots.$$

2. A person writes down all the numbers from 21 to 40, both included, and proceeds to form the product of every two of them. Without attempting to form the products, determine what percentage of the whole number of them will be divisible by 6.

3. What is the saleable value per ton of the sweepings of a manufacturing goldsmith, in whose workshop the quantity of metal amounts to 0·02 per cent. of the sweepings; assuming that the metal is 10 carat gold, and that 30 per cent. of the value of the gold recovered is absorbed in the expenses of extraction, etc.

Standard gold is 916·6 millièmes (p. 483) fine, and worth £3. 17*s.* 6*d.* per oz.

4. There are four stations on a line of railway. How many different kinds of single third-class tickets must be supplied for use on the line?

5. Three places, A, B, C, on a railway are passed by a train at 9, 9.40 and 10.32; another train passes A and B at 9.50 and 10.16, both trains travelling uniformly. At what time should the second train pass C? If the first is travelling 24 miles an hour, what is the rate of the second? [You may work by squared paper if you choose.]

6. A man invests £7000 in the 4 per cents at $109\frac{3}{8}$ and has to pay an income tax of 1*s.* in the £; what amount of tax does he pay and what income has he left?

7. An ordinary pail is 10 inches wide at the bottom and 14 inches at the top. If its height be 18 inches, find to the nearest tenth of a gallon how much water it will hold. [A cubic foot of water weighs 1000 oz., and a gallon of water weighs 10 lb.]

8. Almost all we know of Diophantus, the only algebraist Greece produced, is contained in the following epitaph—from it find how long he lived: Diophantus passed $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ in youth and $\frac{1}{7}$ more as a bachelor; five years after his marriage was born a son who died four years before his father at half his father's age at death.

9. The table below gives the relation between pressure and volume for 1 lb. of saturated steam, between certain limits of pressure. Plot a graph which will show this relation, and by counting squares on sectional paper, determine the area bounded by the curve, the horizontal axis or line of zero pressure, and the limiting ordinates (parallel to the line of zero volume).

If for any small change of volume of the steam, the product of pressure in lb. per square foot and the change of volume in cubic feet represents the work done in foot lb., find how many foot lb. of work will be done in compressing the steam from a volume of 4·29 cubic feet to a volume of 1·53 cubic feet.

Pressure in lb. per square inch.	Volume in cubic feet.
101·9	4·29
115·1	3·82
129·8	3·42
145·8	3·07
163·3	2·76
182·4	2·48
203·3	2·24
225·9	2·03
250·3	1·84
276·9	1·68
305·5	1·53

10. ABCD is a quadrilateral such that

AB = 8 inches.
BC = 7 „
CD = 9 „
DA = 6 „
AC = 7 „

Find the ratio of the two segments into which the diagonal BD is cut by the diagonal AC.

N 1. Given that one foot = 30·48 centimetres, find, correct to the nearest foot, the side of a square field of area 15,600 square metres.

2. Find the weight in kilograms of an iron girder which is 5·4 metres long, 0·54 metre wide and 0·45 metre thick, having given that a cubic centimetre of iron weighs 7·76 grams.

3. An investor secures an annual income of £755 from a capital of £30,000 after paying income tax at 8*d.* in the pound. What sum should be invested in like securities to yield the same income after paying income tax at 1*s.* in the pound?

4. Define a *sector*.

Two adjacent sides of a rectilineal field contain an angle of 67° 30′. A cow is tethered at the angular point. Find to the nearest foot the length of the tether so that the cow may graze on 3 acres of land.

5. The following is an advertisement from a daily paper:

"New and well built house for sale, price £265 cash down, or £2. 8*s.* monthly for 15 years. To what rate per cent. simple interest is this equivalent?

6. A bicycle wheel 28 inches in diameter makes 132 revolutions per minute. Express its speed in miles per hour. ($\pi = \frac{22}{7}$.)

A cyclist with a 28-inch wheel says that he can find his speed in miles per hour by simply counting the number of "clicks" made by his cyclometer in 5 seconds, each click indicating a revolution of the wheel. Test this assertion.

7. The annual average depth of rainfall for the three years 1879, 1880, 1881 at a certain place was 24·98 inches; the succeeding

three years it was 29·62. The year 1883 was the rainiest, when there fell 4·8 inches more than in 1882, 6·36 inches more than in 1884, and 7·47 more than in 1880. The year 1881 was short of the preceding year by only 0·17 inch. Find the depth of rain that fell in each of the six years.

8. A fundholder directs his broker to purchase eight £100 shares in a certain mine, quoted at 272$\frac{3}{8}$ per share. To accomplish this he authorises him to sell out £850 stock in the 3 per cents at 95$\frac{5}{8}$, and £1300 stock in the 4$\frac{1}{2}$ at 105$\frac{1}{8}$. The broker's charge on each of the three transactions is $\frac{1}{8}$th per cent. What had the broker to receive upon the whole?

9. A person C buys an estate, and some time afterwards disposes of it to another D for £5400. At this price it yields D 2$\frac{3}{4}$ per cent. C previously obtained 3 per cent. for his investment, but his income, owing to less careful management, was £36 less than D's. What did C originally pay for it?

10. Show by a graph (unit 2·5 cm.) how the sum of a number n and its reciprocal $\frac{1}{n}$ varies as the number changes from 0·2 to 6·5; use the graph for solving the equation $n+\frac{1}{n}=a$, (i) to find n when $a=4·8$, (ii) to find a when $n=3·24$.

O 1. What is the difference between the compound interest on £100 for two years, according as the interest is paid yearly or half-yearly, 4 per cent.?

2. The legal weight of a 20-yen piece is 257·21 grains, the sterling value £2. 0*s.* 11$\frac{3}{4}$*d.*, the weight of an English sovereign being 123·27 grains. What is the fineness of the 20-yen piece?

3. A person buys £6000 Argentine 5 per cent. stock when at 93. On the price rising to 95, he sells out, reinvesting the proceeds in Chinese 6 per cents. He finds that his new income is £30 more than his old. What was the price of the Chinese 6 per cents? Brokerage $\frac{1}{2}$ per cent. on the Argentines and $\frac{1}{4}$ per cent. on the Chinese.

4. The salary of a clerk begins at £50 a year, and every year he receives a rise of £5. What is his salary in his 30th year of service, and what has he received altogether at the end of the 29th year?

5. A gasometer is in the form of a cylinder with a rounded top, and the following dimensions are given: diameter = 28 feet, height at edges 14 feet, height in the middle 16 feet. Draw a section of the gasometer on the scale of 1 inch to 10 feet, and calculate the number of cubic feet of gas that the gasometer will hold. Take the volume of the portion at the top (above 14 feet) as half the volume of a cylinder of the same base and height, and the area of a circle as 3·14 times the square on its radius.

6. I know that the time of the half-swing of a pendulum l feet long is either $2\pi\sqrt{\frac{l}{32\cdot2}}$ seconds or $\pi\sqrt{\frac{l}{32\cdot2}}$ seconds, where $\pi = 3\cdot1416$; I am uncertain which of these is correct, but I know that the half-swing of a pendulum a yard long takes about a second. Find from these data, with as little work as possible, which is the correct expression.

7. A platinum bar expands $\frac{1}{11\,81\,00}$ of its length at 0° C. for a rise of temperature of 1° C., and copper expands $\frac{1}{58\,20\,0}$ for the same. Find the lengths at 0° C. of two bars, one of platinum and the other of copper, such that, whatever be the temperature, the difference of their lengths may be always 50 centimetres.

8. The populations of two towns are 107509 and 189160; their birth-rates per thousand are 27·9 and 25·7. Find to the same degree of exactness the birth-rate for the two towns taken together.

9. Evaluate $\dfrac{2\pi k(t_2 - t_1)}{\log r_2 - \log r_1}$,

where $\pi = 3\cdot142$, $k = 0\cdot74$,
$t_1 = 69\cdot4$, $t_2 = 82\cdot3$,
$r_1 = 1\cdot25$, $r_2 = 1\cdot55$.

10. A person changes English money into French at the rate of 25 francs for £1, and then changes the money back into English at the rate of $9\frac{1}{2}d.$ per franc. Does he gain or lose, and how much does the difference amount to on £100?

P 1. In 1901 the population of the United Kingdom was 41,800,000, and 34,500,000 cwt. of sugar were imported. How many pounds (to the nearest whole number) did we use per head in that year?

2. Each member of a society subscribes as many pence as there are members in the society; the total subscription is £29. 8*s.*: how many members are there?

3. Express 0·7962 as a vulgar fraction in its lowest terms, without assuming any rule in the process.

Find the sum of all numbers between one and a million that are divisible by 119.

4. A refiner has 10 lb. of gold 20 carats fine, and melts it with 16 lb. of gold 18 carats fine: how much pure gold must he put into it to make the whole 22 carats fine? [24 carats fine is pure gold, 20 carats is 20 parts gold and 4 parts alloy.]

5. The sides of a rectangle are measured as a inches and b inches respectively, and its area calculated. If the true lengths are within x per cent. of these measurements, express in sq. inches the greatest possible error in the calculated area. Give this error to the nearest 100th of a sq. inch when $a=9{\cdot}7$, $b=8{\cdot}6$, $x=1$.

6. Give an expression for the number of bricks measuring 8 by 4 by 2 inches that it takes to build a wall a yards long, b feet high and c inches thick. Give the result to the nearest thousand when $a=70$, $b=6$, $c=16$. (Space occupied by mortar is to be disregarded.)

7. A and B would take 15 days to do a piece of work. They work together for 5 days and then call in C, and the three finish in 6 days more. C does in 2 days what A does in 3. Find their times separately for doing the work and thus compare them as workmen.

8. Equal sums are invested in 3 per cent. stock at 80, 4 per cent. at 90 and 5 per cent. at 100. The price of each stock rises £10, so the 1st and 3rd are sold and the proceeds invested in the 2nd. If the rise in income be £1. 10*s.*, find the whole sum invested. (No brokerage is to be reckoned.)

9. Electric trams start from a terminus every 4 minutes and when in motion run at 10 miles an hour. Stations are $\frac{1}{3}$ mile apart and there is a stop of $\frac{1}{2}$ a minute at each. Find the greatest and least distance apart of two successive trams. You may treat the question by arithmetic or graphic methods.

10. A length of 10 feet of a water pipe, $\frac{3}{4}$-inch in diameter, freezes. In freezing any volume of water expands to 1·087 times that volume. Sometimes this freezing bursts the pipe, sometimes the ice occupies an additional length of pipe. What additional length of pipe is necessary in the present case? If no additional length of pipe is available, to what diameter does the water expand in freezing?

TYPICAL EXAMINATION PAPERS.

PART I.

Cambridge Local (Preliminary). I.

1. Divide one hundred and two million three hundred and twelve thousand seven hundred and eighty-three by eighty-seven, and express your answer in words.

2. Add together £7389. 7*s.* 11½*d.*, £306. 13*s.* 8½*d.*, £94. 19*s.* 4d., £2765. 14*s.* 9½*d.*, 18*s.* 7*d.*, £784. 4*s.* 10*d.*, and £3065. 9*s.* 7½*d.*

3. Find the cost of 796 pints of spirit at 12*s.* 7*d.* per gallon.

4. If 3 cwts. 1 qr. 11 lbs. cost £1. 5*s.*, what would one ton cost at the same rate?

5. Find the greatest common measure of 42889 and 56903.

6. Reduce to their simplest forms

(1) $1\frac{7}{15} - 2\frac{1}{24} + 1\frac{9}{20}$;

(2) $(1\frac{1}{36} \times 6\frac{1}{4}) \div (11\frac{9}{16} \times \frac{20}{21})$.

7. Divide 0·566 by 0·0807 to four places of decimals.

8. Express ·891 of £1 in shillings and pence correctly to the nearest penny.

9. Find, correct to the nearest centime, the cost of 287 metres 7 centimetres at 5 francs 70 centimes per metre.

10. Find the simple interest on £985. 10*s.* for 150 days at 3 per cent. per annum.

11. By selling a horse for £57. 12*s.* a dealer gains $6\frac{2}{3}$ per cent. What was the original price of the horse?

12. An estate is let to three tenants, of whom the first occupies two-fifths of the whole, the second three-eighths, and the third the remainder, which contains 25 acres 2 roods 24 poles. What is the size of the whole estate?

Cambridge Local (Preliminary). II.

1. Divide three hundred and fifty million three hundred and one thousand three hundred and two by seven hundred and nine.

2. Add together £479. 18*s.* 8½*d.*, £7036. 9*s.* 10*d.*, £87. 12*s.* 7½*d.*, £3874. 7*s.* 11½*d.*, 19*s.* 5*d.*, £4287. 5*s.* 6*d.*, £39. 14*s.* 5½*d.*, and £176. 13*s.*

3. A man spends on the average 4*s.* 9½*d.* per day. How much does he spend in a year of 365 days?

4. A plot of building land containing 3 acres 3 roods 25 poles is sold for £1312. 10*s.* What is the price per acre?

5. Reduce to their simplest forms

(1) $1\frac{52}{21} \times \frac{42}{87} \frac{5}{5}$;

(2) $(\frac{1}{2} - \frac{2}{3} + \frac{3}{4}) \div (\frac{1}{4} + \frac{2}{5} + \frac{1}{6})$.

6. Simplify, without reducing to vulgar fractions,

$$(0{\cdot}437 \times 4{\cdot}8) + (0{\cdot}056 \times 0{\cdot}4).$$

7. Express 0·7187 of a mile in yards correctly to the nearest yard.

8. A merchant buys 235 metres 50 centimetres of silk for 659 francs 25 centimes. Find, correct to the nearest centime, the price per metre.

9. Find the simple interest on £375 for 2½ years at 2½ per cent. per annum.

10. A grocer mixes 28 lbs. of tea costing 1*s.* 4¼*d.* per lb. with 11 lbs. costing 2*s.* 2*d.* per lb. Find the cost of the mixture per lb.

Find also, correct to one place of decimals, what is the grocer's gain per cent. if he sells the mixture at 1*s.* 10½*d.* per lb.

11. Two bicyclists, A and B, start at the same instant to ride from Cambridge to London, a distance of 52½ miles, and travel at uniform speeds of 12¼ and 11⅜ miles per hour respectively. How far will B be behind when A has reached London?

Cambridge Local (Preliminary). III.

1. Divide sixty millions eight hundred and forty-two thousands five hundred and thirty-five, by five hundred and seven; and express the answer in words.

2. Add together 5*s.* 7½*d.*, £2. 6*s.* 4½*d.*, £3041. 7*s.* 11¼*d.*, £253. 4*s.* 9*d.*, £476. 12*s.* 5*d.*, £219. 15*s.* 10*d.*, and £4006. 6*s.* 8¾*d.*

3. The workmen employed in a certain factory earn on the average 5*s.* $2\frac{1}{2}$*d.* each per day, and 234 men are employed. Find the total sum paid in wages for a week of six days.

4. Find the greatest common measure of 39483 and 138993.

5. Find the amount of tax on £5354. 13*s.* 4*d.*, at the rate of 1*s.* $7\frac{1}{2}$*d.* in the pound.

6. Simplify (1) $1\frac{2}{3} \times 3\frac{3}{8} \div 1\frac{5}{16}$;

(2) $(3\frac{1}{10} - 1\frac{4}{1\frac{1}{5}}) \div (3\frac{4}{5} + 1\frac{1}{3})$.

7. Divide 2789·44 by 0·0368, and 2·78944 by 368.

8. Find the interest for three months on £720 at the rate of $3\frac{3}{4}$ per cent. per annum.

9. Express 0·5347 of an acre correctly to the nearest square yard. Find it also correctly to the nearest square foot.

10. Given that a metre contains 39·37 inches, express five miles in kilometres and metres, correct to the nearest metre.

11. If oranges can be bought at the rate of three for twopence, how many must be sold for a shilling to gain eighty per cent. profit on the cost?

Cambridge Local (Junior).

A.

1. Find the highest common factor of 1881, 1976, and 4332.

2. How many seconds are there between 2 p.m. on Monday, December 10, and 4.30 p.m. on the following Saturday?

3. Simplify $\dfrac{\frac{5}{3}+\frac{3}{4}}{1\frac{3}{4}+2\frac{5}{12}} + \dfrac{1}{3\frac{1}{11}}$.

Divide 230·1052 by 0·00137.

4. Add together 4·15 of £2. 15*s.* and 1·4 of half-a-crown.

5. Find the cost of 21 cwts. 2 qrs. 10 lbs. of cocoa at £3. 7*s.* 8*d.* per cwt.

6. Gold wire, of a certain thickness, is worth 1 franc 56 centimes per centimetre: what would be the cost of 2 metres 9 centimetres of this wire?

7. Find the simple interest on £1049. 7*s.* 6*d.* for 30 days at $4\frac{1}{2}$ per cent. per annum.

B.

1. A cubic foot of water weighs 1000 oz., and a gallon of water weighs 10 lbs. Find the number of cubic inches in a pint.

2. A contractor engaged to construct 4 miles of railway in 184 days; but 77 men working for 52 days made only one mile. How many additional men had he to employ, supposing all the men worked at the same rate as before, in order to complete his contract at the time specified?

3. In a cricket match between two schools A and B, A made 85 more runs than B in the first innings; but in the second innings B made 64 per cent. more runs than A, and won the match by 43 runs. What were the scores in the second innings?

4. A hare makes 9 leaps in the same time as a dog makes 4, but the dog's leap is 7 feet while the hare's is only 3 feet; how many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 16 yards on a straight road?

5. A merchant in New York buys 1000 metres of silk in Brussels at 13 francs 54 centimes the metre: what is the cost, to the nearest dollar, assuming that 25 francs 20 centimes are worth 4·839 dollars?

6. A person invests £299 in consols; on the price of £100 consols being increased by £3. 10*s.* he sells out, making a profit of £11. 7*s.* 6*d.* At what price did he buy the consols?

Oxford Local (*Preliminary*). *I.*

1. Divide £7351. 16*s.* $6\frac{3}{4}$*d.* by 71.

2. Find the Lowest Common Multiple of 48, 66, 27, and 72.

3. Simplify $3\frac{5}{17} \times 4\frac{6}{7} \div 2\frac{2}{15}$.

4. Multiply 34·725 by 0·0128.

5. Express $\frac{27}{32}$ as a decimal.

6. Express 3*s.* $4\frac{1}{2}$*d.* as a fraction of £3.

7. Find the rent of a farm 25 acres 3 roods 16 perches in extent at £1. 16*s.* 8*d.* an acre.

8. If 5 cwt. 3 qrs. 20 lb. of tea cost £44. 5*s*. 4*d*., what is the value of 2 cwt. 16 lb.?

9. At what rate per cent. per annum, simple interest, will £66. 13*s*. 4*d*. amount to £68. 15*s*. in 1 year 3 months?

Oxford Local (*Preliminary*). *II.*

1. Multiply £28. 11*s*. $7\frac{1}{4}d$. by 79.

2. Find the Greatest Common Measure of 5434 and 8436.

3. Add together $2\frac{1}{3}$ and $4\frac{3}{14}$. Also multiply $2\frac{1}{2}$ by $15\frac{5}{7}$.

4. Find the value of $\frac{4}{7}$ of £7. 5*s*. 11*d*.

5. Divide 15·2559 by 50·6.

6. Reduce 3*s*. 9*d*. to the decimal of £5.

7. Find the cost of 6 tons 7 cwt. 21 lb. at £2. 6*s*. 8*d*. per ton.

8. If 4 acres 3 roods 14 perches of land cost £516, what should 9 acres 1 rood 20 perches cost?

9. Find, correct to the nearest farthing, the simple interest on £650. 4*s*. 9*d*. for 5 years at $4\frac{1}{2}$ per cent. per annum.

Oxford Local (*Preliminary*). *III.*

1. If £1028. 17*s*. $4\frac{1}{2}d$. is divided equally among 67 people, how much will each receive?

2. Find all the common measures of 364 and 546.

3. Simplify $(2\frac{1}{3}+4\frac{1}{5})\div(2\frac{5}{8}-1\frac{1}{5})$.

4. What fraction of £2. 5*s*. 6*d*. is 2*s*. 2*d*.?

5. Multiply 23·625 by 0·0034, and prove your answer by division.

6. Express 0·8125 as a vulgar fraction.

7. What is the cost of 6 cwt. 3 qrs. 20 lb. at two guineas per cwt.?

8. A field of 4 acres 2 roods 16 poles is rented at £13. 16*s*.; what is the rent of a similar field of 5 acres 3 roods 12 poles?

9. A housekeeper takes three half-pints of milk each week-day and one pint on Sunday, and her bill for the week comes to 1*s*. $5\frac{1}{2}d$. What is the price of milk per quart?

Oxford Local (*Preliminary*).

HIGHER ARITHMETIC.

1. Find the square root of 0·00034969.

2. Linoleum is laid on a floor whose dimensions are 18 feet by 14 feet, so as to leave everywhere a margin of $2\frac{1}{2}$ feet. What will the linoleum cost at 2*s.* 6*d.* per square yard?

3. In what time will £175 amount to £197. 15*s.* at 4 per cent per annum simple interest?

4. What is the cost of 157 centimetres of a material sold at 2 marks 30 pfennigs per decimetre?

5. A man leaves £6400 to be divided among 4 sons, 3 daughters, and 3 nephews. If each daughter receives three times as much as each nephew, and each son five times as much as each nephew, how much does each nephew receive?

6. A man holding £3000 of a 3 per cent. stock sells out when the stock is standing at 95, and invests the proceeds in a 5 per cent. stock at 114. What is the change in his income?

7. A man travels from A to B to buy goods which he can get 10 per cent. cheaper in B than in A. If the expenses of the journey are 15 shillings and he makes a clear saving of ten shillings, what does he pay for the goods?

Oxford Local (*Junior*).

1. Simplify $(2\frac{1}{5}+4\frac{1}{3}-1\frac{1}{6})\div(4\frac{1}{10}-2\frac{1}{5}+1\frac{1}{6})$.

2. What fraction of £1. 0*s.* $6\frac{1}{2}$*d.* is 4*s.* 10*d.*?

3. Find (correct to the nearest penny) the rent of a field of 16 acres 1 rood 34 poles at £5. 17*s.* 6*d.* an acre.

4. Show that to four places of decimals $1\frac{61}{72}$ is a correct approximation to the square root of 5.

5. If 25 francs = £1, and a gram = 0·002204 lb. avoirdupois, find (in French money) the price of a kilogram of an article which costs a shilling an ounce.

6. A builder undertakes to complete a contract in 40 days. He employs 15 men, and at the end of 26 days only half the work is done. What is the least number of extra men that he must engage, in order to complete the work in the specified time?

7. Find (correct to the nearest penny) the compound interest on £1500 for 3 years at $3\frac{1}{2}$ per cent. per annum.

8. The internal dimensions of a box without a lid are as follows: length 4 feet 2 inches, breadth 2 feet 4 inches, depth 2 feet 1 inch. Find, correct to the nearest penny, the cost of lining it with a material costing 6*d.* per square foot.

9. A bankrupt's liabilities are £1350. His assets are nominally £970, part of which consists of debts owing to him. He only succeeds in recovering 60 per cent. of his debts, and pays his creditors 12 shillings in the pound. What is the amount of the debts originally owing to him?

10. Which is the better investment: a 5 per cent. stock at 132 or $3\frac{1}{2}$ per cent. stock at 94? Give reasons for your answer.

College of Preceptors (*Third Class*). *I.*

1. (i) Write down the number three hundred and forty-one million three hundred and eighty thousand three hundred and fifty-four.

(ii) Divide 9876543 by 567 (using factors), and explain how you find the remainder.

(iii) Express the quotient in words.

2. A train is made up of an engine and nine carriages. If the weight of the engine be 36 tons 2 cwt. 3 qr. 21 lb., and that of each of the carriages 11 tons 15 cwt. 3 qr. 19 lb., find the whole weight of the train.

3. A grass field contains 10 acres 2 roods 36 poles. If 162936 square feet be ploughed, find how many acres, roods, square yds., etc., remain under grass.

4. A landlord's net rental, after paying income tax at the rate of 1*s.* in the £, is £7101. 14*s.* 6*d.* Find his gross rental.

5. Add together $3\frac{3}{4}$, $4\frac{4}{5}$, $5\frac{5}{6}$, $6\frac{7}{8}$, $7\frac{9}{10}$; and subtract $\frac{13}{85}$ from $\frac{19}{51}$.

6. Simplify $\dfrac{3\frac{2}{3} \text{ of } \frac{1}{8} - \frac{5}{12}}{(\frac{1}{3} \text{ of } \frac{1}{16}) + (\frac{3}{16} - \frac{1}{6})}$.

7. What do you mean by a decimal? What do the 3 and 6 respectively mean in 38·046?

Express as vulgar fractions in their lowest terms: 0·02, 0·018, 2·25, 1·005.

8. Multiply 36·9 by 0·0058; and divide 1·392 by 0·00116.

9. If 3 tons 18 cwt. of lead cost £100. 5*s*. 3*d*., what is the value of 3 cwt. 2 qr. of the same?

10. Find by Practice the rent of 69 acres 2 roods 28 poles at £2. 5*s*. per acre.

11. At an election where only seven-eighths of the whole number of voters voted, one of the two candidates received a fourth as many votes again as the other, and beat him by 896 votes. What was the whole number of voters?

College of Preceptors (*Professional Preliminary*).

1. A light was seen at intervals of 13 sec. It was seen the first time at 1 hr. 54 min. 50 sec. A.M., and the last time at 3 hr. 17 min. 49 sec. A.M. How many times was the light seen?

2. If 257 sheep and 37 oxen were bought for £1187. 9*s*., and the average price of an ox was £18. 7*s*. 6*d*., what was the average price of a sheep?

3. Explain how to obtain quantities of sand weighing 2 lb., 13 lb., 23 lb., and 39 lb. respectively from a large heap of sand, using scales sufficiently large to weigh 7 lb. of sand at a time, and only two weights, one of 4 lb., the other of 7 lb.

4. Simplify $\{5\frac{1}{35} + 3\frac{11}{63} + 2\frac{17}{105}\} \div \{8\frac{4}{9} - 3\frac{7}{11} - 2\frac{2}{3}\}$.

5. Find the value of $0{\cdot}6037 + \frac{5}{19} + 0{\cdot}023 - \frac{13}{457}$, in decimal form, correct to the nearest figure in the fifth decimal place.

6. Express the sum of 2 tons 2 lb. and $\frac{11}{17}$ of 6 tons 5 cwt 3 qr. 9 lb. as a decimal of 25 tons.

7. What is the rent, to the nearest centime, of 27·3 ares of land, at the rate of 5830 francs for 89·8 hectares?

[1 franc = 100 centimes.]

8. Calculate, to the nearest penny, the sum to be paid to a creditor whose debt is £183. 13*s.* 10*d.* when his debtor pays 14*s.* 7¼*d.* in the £.

9. The base of a rectangular tank is horizontal, and the area of the base *inside* is 37 square metres. What is the depth of the water, to the nearest centimetre, when the tank contains 17 kilolitres of water? [A litre is a cubic decimetre.]

10. A square field whose area is 128634 sq. yd. is to be enclosed with barbed wire placed at heights of 1, 2, 3, and 4 feet above the ground. What length of wire, to the nearest yard, will be wanted if the length of wire required for each circuit is 3 per cent. greater than the perimeter of the field?

11. Eleven persons contributed a certain sum. Nine of them gave £5 apiece, and the other two persons gave £5 and £10 more respectively than the average subscription of the eleven subscribers. What sums did the two persons give?

12. If 46 per cent. of a regiment containing 1150 men are recruits, how many recruits must be taken out in order that 25 per. cent of the remainder may be recruits?

College of Preceptors (*Second Class or Junior*). *I.*

1. How much will a million and a quarter of oranges cost at 4½*d.* a dozen?

2. How many sheep worth £3. 17*s.* 6*d.* each must I give in exchange for 7 oxen worth £27. 2*s.* 6*d.* each?

3. I design a room 18 ft. square and 11 ft. high. Its windows, doors, and fireplace will take up 10 sq. yds. How many pieces (each 12 yds. long) of paper, 2¼ ft. wide, will give me what I require for the walls; and how many yards of carpet, 22 in. wide, shall I need for the floor, allowing a space 7 ft. by 1½ ft. for the hearthstone?

4. My lad has gone to the post, distant 121 yds. less than half a mile away. If he walks down to it at the rate of 3 miles an hour, and returns up from it at the rate of 2 miles an hour, find how long, to the nearest second, he will be gone.

5. Simplify—

(i) $(47\frac{7}{9}+16\frac{2}{3}-19\frac{3}{10})\times 15\frac{3}{7}$.

(ii) $(4\frac{1}{3}-3\frac{3}{4})\times(7\frac{2}{5}-5\frac{3}{14})\div(\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{7}{9}$ of $2\frac{19}{175})$.

6. Add together 40, 0·04, 0·0004, 4000·4; and divide the result by 400.

7. Obtain the square root of 1787·5984; and square 154·305.

8. What will the Simple Interest on £14,500 amount to in $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.?

9. If I give £10,500 for 741 acres of land, how much do I pay per are? [1 hectare = 2·47 acres.]

10. Write down $3\frac{1}{2}$ cub. millimetres as a decimal of a cub. decimetre; and $2\frac{1}{2}$ square centimetres as a decimal of a square metre.

11. The "Victorious" left Malta on Saturday, May 13th, 1905, at 2 p.m., steaming 10 knots an hour. The Royal Yacht, with the Queen on board, caught her up and steamed past her at the rate of 15 knots an hour, at 8 p.m. on the Monday following. At what time and date did the latter leave Malta; and at what distance from Malta were the two ships abreast, their speeds being uniform throughout?

College of Preceptors (*Third Class*). *II.*

1. The average daily supply of water within the area of the Metropolitan Water Board during the month of January last was two hundred and four millions nine hundred and ninety-four thousand gallons. The estimated population within the same area being six millions seven hundred and seventy-four thousand eight hundred and thirty-nine, calculate to the nearest gallon the supply per head per day.

2. A and B set out walking from the same place in opposite directions at the rate of $3\frac{1}{2}$ miles an hour and $3\frac{3}{4}$ miles an hour respectively. In what time will they be 58 miles apart?

3. A sum of £25. 14*s.* 3*d.* is divided among 3 men and 5 women so that each man may have twice as much as a woman. What sum will each man receive?

4. Find, by Practice, the value of 159 cwt. 3 qr. 22 lb. at £2. 12*s.* 6*d.* per cwt.

5. What fraction added to the sum of $1\frac{2}{3}+4\frac{5}{8}+7\frac{8}{9}$ will make a sum total of 15?

6. Simplify $\dfrac{8\frac{1}{4}-5\frac{7}{8}}{1\frac{1}{3}\text{ of }3\frac{4}{5}} \div \dfrac{9\frac{1}{2}\text{ of }6\frac{1}{4}}{\frac{19}{20}}$.

7. Reduce 0·0375 and 0·0025 to vulgar fractions in their lowest terms; and find the value of 0·018 of £4. 3*s.* 4*d.*

8. Multiply 0·59324 by 0·00675; and divide 5·084976 by 16·86.

9. If 2 acres 1 rood 27 poles of land cost £258, what should 18 acres 3 roods of the same sort of land cost?

10. The debts of a bankrupt amount to £4269. 1*s.* 0*d.* He has assets amounting to £2846. 0*s.* 8*d.* How much in the pound can he pay his creditors?

11. Calculate in tons the weight of water in a reservoir 1 acre in area, and having a depth of 18 inches, a cubic yard of water weighing 15 cwt.

College of Preceptors (*Second Class or Junior*). *II.*

1. Find in millimetres, correct to the nearest millimetre, the circumference of a carriage wheel which makes 59,173 turns in travelling 170 kilometres.

2. Express in pounds, shillings, pence, and farthings the least exact sum of money greater than 1541 half-crowns, and the greatest exact sum of money less than 1957 florins. How many different exact sums of money are there which are both greater than 1541 half-crowns and less than 1957 florins?

3. What is the greatest common measure of 12,936 and 20,328; also what numbers between 20 and 50 will exactly divide both of them?

4. Simplify $\dfrac{(2\frac{11}{36}-\frac{5}{14})\text{ of }9\frac{1}{3}-1\frac{4}{3}}{2\frac{11}{36}+\frac{15}{14}\text{ of }9\frac{1}{3}}$.

5. Obtain, in decimal form, the values of

$$0{\cdot}3\times0{\cdot}2+0{\cdot}21\times0{\cdot}4 \text{ and } 0{\cdot}4\times0{\cdot}2-0{\cdot}21\times0{\cdot}3;$$

and divide the *sum* of the squares of the two values by 14·5. What is the square root of that sum?

6. Determine the difference, correct to the nearest second, between 29·5306 days and 27·3217 days.

7. If 457 quarters of wheat cost £109. 4*s.* 8*d.* more than 659 quarters of oats at 16*s.* 4*d.* per quarter, what was the price of 1 quarter of wheat?

8. Find, by Practice, correct to the nearest penny, the value of 13 cwt. 3 qr. 20 lb. at £59 per ton.

9. Assuming that A spends £30,000 in 365 days, and that B spends 24*s.* in 7 days, express B's daily expenditure, (i) as a fraction, and (ii) as a decimal, of A's average daily expenditure—in the latter case, correct to the nearest figure in the *sixth* decimal place.

10. A dealer bought some pears at 9*s.* per hundred and sold them for £2. 5*s.* 10*d.* If he obtained an average price of 1*s.* 4½*d.* per dozen, what whole profit, and what rate per cent. of profit, did he make?

11. If a sheet of lead 10·9 metres long and 19 decimetres wide weighs 1747 kilogrammes, and 1 cubic centimetre of lead weighs 11·4 grammes, what is the thickness of the sheet, correct to the nearest tenth of a millimetre?

College of Preceptors (*Third Class*). *III.*

1. Divide eight million one hundred and fifty-three thousand and seventy-eight by sixty-three, using factors, by short division; and give the method by which you find the remainder.

2. £840. 9*s.* 3¼*d.* is to be divided equally among 65 persons. How much does each receive?

3. A plot of land 45 ft. by 16 ft. produces 32 stones of potatoes. What is the produce per acre?

4. Find, by Practice, the cost of 11 tons 12 cwt. 2 qrs. 14 lb. of goods at £5. 6*s.* 8*d.* per ton.

5. What fraction must be subtracted from the sum of

$$2\tfrac{2}{3}+3\tfrac{3}{4}+4\tfrac{5}{6}+6\tfrac{7}{8}$$

to leave 18 exactly?

6. Simplify $(3\frac{2}{5}+4\frac{1}{12}\text{ of }1\frac{5}{7})\div(5\frac{2}{21}-3\frac{12}{28})$.

7. What is meant by the *local value* of a digit? Write down the local value of each figure in the number 78·543.

Express $\frac{4}{5}$ of half-a-crown as a fraction of a guinea.

8. Multiply 0·00806 by 7·5, and divide 17·175 by 0·0125.

9. A train travels 30 miles an hour. Express this speed in feet per second.

10. A ship worth £20,000 belonging to four persons, A, B, C, and D, became a total loss. A owned ⅜, B ¼, C ⅛, D ¼. She was insured for £15,000. What will each receive, and what will he lose?

11. A brick measures 9 in. long, 4·5 in. broad, and 2·75 in. deep. How many such bricks are contained in a wall 16·5 ft. long, 13·5 in. broad, and 4·5 ft. high?

Pupil Teacher Candidates (Junior Grade).

1. A man borrows £300 from one person at 3 per cent. and also £750 from another person at 4½ per cent. At what rate does the man pay interest on the whole sum of money borrowed?

2. How many lengths of wire 0·85 inch long can be cut from a piece 37·076 inches long? What is the length of the remainder?

3. A cistern open at the top is 13 ft. long, 8 ft. broad, and 5½ ft. deep. What will it cost to paint the inside at 3½*d.* per square yard?

4. A man after spending three-quarters of his money and two-fifths of the remainder had 8*s.* 9*d.* left. What did he spend?

5. A grocer adds to a chest of tea worth 2*s.* 6*d.* a lb., twenty per cent. of its weight of another kind of tea worth 1*s.* 9*d.* a lb. What is the mixture worth?

Pupil Teacher Candidates (*Senior Grade*).

(*Questions* 1 *to* 5 *were not arithmetical questions.*)

6. The heights of the barometer at certain times on March 16th are given in the following table:

Time of observation	2 p.m.	4.15	8.0	10.30	Midnight
Height of barometer in inches - -}	29·4	29·3	29·1	28·95	29·05

Construct on squared paper a graph showing this variation.

7. Three trucks contain 25 tons 16 cwt. of coal. The first truck contains 0·411 and the second 0·214 of the whole. What weight of coal is there in the third?

8. A can mow a field in $4\frac{1}{2}$ days, B in $5\frac{1}{2}$ days. If they work together for two days, what fraction of the field will be left unmown?

9. How many times can a cup containing $\frac{3}{8}$ of a pint be filled from a can containing 2 galls. 1 quart of milk?

10. A rectangular room measuring 16 ft. by 14 ft. 6 inches is carpeted so as to leave a width of 1 ft. 3 inches of bare board on every side. What is the area of the carpet?

11. A shopkeeper sells an article so as to make 25 per cent. on the cost price. If he had sold it for sixpence more he would have made thirty per cent. What is the selling price?

12. A man went to market with 8*s.* 9*d.* in his pocket. He sold 45 sheep, and then bought 3 cows for £18. 5*s.* each. He then had £25. 6*s.* 9*d.* What sum did he get for the sheep?

Preliminary Examination for the Certificate.

PART I.

1. Annie was born on August 3rd, 1903, while her baby brother was 106 days old on last Christmas Day. How old was Annie in years and days when the baby was born?

2. I have 12 reels of cotton each containing a single piece of cotton 220 yards in length. Explain clearly how many pieces of cotton can be cut off, so that each may be 75 feet in length and without a join.

3. Continue the decimal fraction 7·3 by writing in the second decimal place a number that means one-sixth as much as the 3, and give reasons for your answer.

4. What is the nearest sum to $\frac{2}{17}$ of £12 that can be paid in our coinage—

(*a*) if farthings are allowable,

(*b*) without the use of farthings?

5. Write out so as to be understandable by a child acquainted with multiplication, division, and fractions but not with ratio, the solution to the following sum:

If it cost £3 to carry a weight for 144 miles, what will it cost to carry the same weight for 117 miles at the same rate per mile?

6. A florist buys flowers at the rate of 1*s.* for 14; he sells them at 1*s.* 5*d.* a dozen. How many must he sell to make a profit of 11*s.* 9*d.*?

7. A brick measures 9 inches, by $4\frac{1}{2}$ inches, by 3 inches. How many will be required for a solid wall 21 feet 9 inches long, $13\frac{1}{2}$ inches thick, and 4 feet 6 inches high, when the thickness of the mortar is neglected?

8. Assuming that the product of 376038 and 294876 is 110884581288, find the product of 376038 and $294879\frac{1}{2}$.

9. Write out fully *in words* the work involved in dividing 4731 by 3, being careful to state *fully* the *true value* of every dividend, quotient, and remainder.

10. A person who has 0·3 of a mine, sells $\frac{2}{7}$ of his share for £1275. What is the value of all the shares in the mine?

11. State clearly why the suggested answers to the following questions are obviously wrong:

(1) Multiply $9\frac{3}{5}\frac{1}{7}$ by $6\frac{1}{1}\frac{2}{7}$. Answer 69.

(2) How much tea at 1*s.* 7*d.* a lb. must be mixed with a cwt. of tea at 2*s.* 6*d.* a lb. in order that the mixture may be worth 1*s.* $11\frac{1}{2}$*d.* a lb.? Answer 49 lb.

(3) Multiply 573 by 963. Answer 550799.

(4) Which is the greater—26 cubic feet or a cubic metre? Answer 26 cubic feet.

12. A coal merchant bought 44 tons of coal at an average price of 12*s.* 6*d.* a ton. When doing this, he gave 20*s.* a ton for 9 tons, 9*s.* 6*d.* a ton for 13 tons, and 11*s.* 7*d.* a ton for 12 tons. What was the price per ton that he gave for the remainder?

13. A man borrowed £20 and agreed to pay 1*s.* a month for the use of the money. At what rate per cent. did he pay interest?

14. A grocer weighed out what he thought were 1 lb. packets of tea. He then discovered that his lb. weight was really $15\frac{3}{4}$ oz. How much tea was there in 100 packets, and how many complete packets did he make out of 1 cwt.?

15. Work as clearly as possible, by the shortest method you can think of, the following:

(*a*) 1993 ÷ ·1993.

(*b*) Find the cost of 2400 articles at 19*s.* $11\frac{1}{2}$*d.* each.

(*c*) $(9\frac{1}{2} - 3\frac{1}{5}) \times \frac{1}{3\frac{1}{6} - 1\frac{1}{9}}$.

(*d*) The cost of 8250 bricks at 10*s.* 6*d.* a thousand.

PART II

Civil Service Commission.

1. A sample of petroleum weighed about 864 ounces per cub. foot. Find how many grams it weighed per litre. (1 kilo = 2·206 lb., 1 metre = 3·28 ft.)

Show that for any substance the number of ounces per cubic foot is nearly equal to the number of grams per litre.

2. Fig. 1 is drawn on a scale of 10 feet to the inch, and represents the end view and side view of a shed, the roof and side walls of which are made of corrugated iron. Find the cost of the roof and side walls at $1\frac{3}{4}d.$ per square foot.

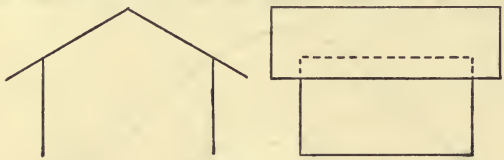


FIG. 1.

3. The following statistics are given in the Census of 1901 :

District.	Number of Acres.	Population.
London - -	74,839	4,536,541
Westmoreland -	505,330	64,409
England - -	32,611,033	30,829,695

Find, in each district, the population per acre correct to three decimal places, that is, to the nearest thousandth. And calculate to two significant figures the ratio of the density of London to that of England and the ratio of the density of Westmoreland to that of England.

4. The pressure of the atmosphere is 15 pounds per square inch. When a bicycle tyre, of capacity A cubic inches, is inflated by a pump of capacity B cubic inches, the pressure of air in the tyre, after n strokes of the pump, exceeds the atmospheric pressure by

$n \times B \div A$ times the atmospheric pressure. In a certain case the pump is 10 inches long and $\frac{3}{4}$-inch in diameter; the tyre may be treated as a cylinder 88 inches long and $1\frac{3}{4}$ inches in diameter; and the capacity of a cylinder a inches long and b inches in diameter is $0{\cdot}79 \times a \times b \times b$ cubic inches.

Find the capacity of the tyre and of the pump, and the pressure of the air in the tyre after 40 strokes, each to two significant figures.

5. A book of gold leaf contains 25 leaves, $3\frac{1}{3}$ inches square and $\frac{1}{282000}$ of an inch thick. If a cubic inch of gold weighs 0·697 lb., and 1 lb. of pure gold is worth £61. 19*s.*, find the value of the gold in the book to the nearest penny.

6. Fig. 2 shows on a scale of 1 to 2000 a level field ABCD, containing a pond E, whose average depth is 60 cm. below the level

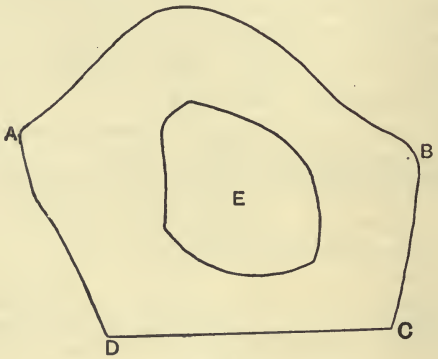


FIG. 2.

of the field. It is proposed to remove the water from E and fill up with earth taken from the surface of the rest of the field. Find by pricking the diagram through on to your book and counting squares, what fraction of the whole field is covered by the pond and hence calculate how many centimetres the surface of the field will have to be lowered in order to fill up the pond.

If the work costs 2*s.* per cubic metre moved, what is the total cost?

Civil Service Commission.

HIGHER ARITHMETIC.

1. A man buys a house for £1150. There is a ground rent of £25 a year, and annual repairs come, on an average, to $1\frac{1}{2}$ per cent. on the purchase price of the house. At what rent must he let the house to clear 8 per cent. per annum on the purchase price?

2. By pricking off Fig. 3 (shown only half-size) on to your book, and counting squares, find its area in square centimetres. Hence

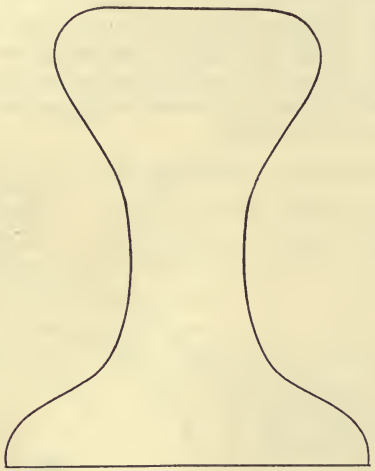


FIG. 3.

find the weight of steel rail, of this section, for a single line railway 17 kilometres long. Answer in tonnes.

(1 cub. centimetre of steel weighs 7·8 grams.)

3. A man takes a boat and rows up stream. When he has gone $1\frac{1}{2}$ miles up stream he passes a bottle floating down with the stream. He goes on up the river for another 18 minutes and then turns. When he gets back to the starting-place he finds that the

bottle has just arrived there. Assuming that his speed through the water and the speed of the stream are both uniform, find the speed of the stream in miles per hour.

4. A wine merchant buys in Bordeaux a cask of claret, and the freight to London is £1. It contains 208 litres of wine at 75 centimes the litre, and for the cask itself he has to pay 12 francs. He has to pay duty 1*s.* 3*d.* per gallon, and bottles the wine at a cost of 2*s.* a dozen bottles. If he sells it at 18*s.* a dozen, what percentage of this price is profit?

(£1 = 25 francs = 2500 centimes; 6 bottles = 1 gallon = 4·546 litres.)

5. From the following table find (to the nearest shilling) the value per cwt. of coffee in 1903, in 1904, and in 1905. Find also the percentage falling off in 1905 in quantity and in total value as compared with 1904, each percentage to the nearest integer and reckoned on the 1904 numbers:

	Quantity Imported. Cwts.	Value in £.
1903 - - -	1,143,526	3,134,924
1904 - - -	1,055,866	3,329,598
1905 - - -	956,226	2,707,169

6. In Fig. 4 ABC represents the circumference of a new grindstone weighing 102 lbs. Find its weight when it is worn down to DEF. Ignore the hole through the stone.

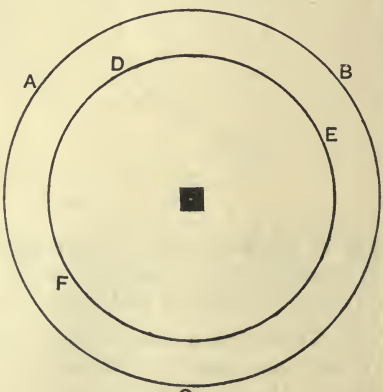


FIG. 4.

7. If a water-pipe is L yards long, d inches in diameter, and one end is H feet higher than the other, then $\sqrt{(3d)^5 \times H \div L}$ gallons of water will flow through the pipe in a minute. Use this formula to find how many gallons per minute will flow through a pipe a mile long, $4\frac{1}{4}$ inches in diameter, one end being 38 feet higher than the other.

8. Two crews, A and B, row a race. For the first 4 minutes A row 33 strokes a minute and their boat travels 36 feet through the water at each stroke. B, who have been rowing 32 strokes a minute, are then 80 ft. behind. A then drop to 31 strokes a minute, but their boat moves 37 ft. through the water at each stroke, while B quicken to 33 strokes a minute, but their boat moves at each stroke $1\frac{1}{2}$ ft. less than it did before. If they go on like that, how long will it be before the boats are level, and how far will they then have rowed?

Board of Education (*Certificate Examination*).

(Questions 7, 9, 10, 11, 12 were Algebraical.)

1. Evaluate as shortly as you can the three most important figures in the product of 4680·13576 and 286423·5791, stating shortly the reasons for your procedure.

2. How would you teach children who are acquainted with the mensuration of a circle to find the volume of a cylinder?

Three cylinders are of the same height and of the same material. Prove that a triangle whose sides are equal to the diameters of the cylinders will be right-angled if one cylinder weighs as much as the other two together.

3. Describe a method of practically demonstrating to children the operations performed in the sum usually set down as follows:

	£.	s.	d.	
3)	15 .	16 .	2	
5)	5 .	5 .	4	and 2 over.
	1 .	1 .	0	and 4 over.

Explain the method by which we can calculate the value of the remainder.

4. Two trains are travelling in opposite directions at uniform speeds. They each pass a telegraph post in the same time, and they pass each other in five seconds. If the lengths of the trains added together be 264 yards, and the faster train's speed exceed that of the slower by 25 %, find the length and speed of each train.

5. A rectangular board measuring 6 decimetres by half a metre is just large enough to display 120 circular medals, all of the same size, arranged edge to edge in 12 rows.

Find—

(*a*) The area of the board that is left uncovered by the medals;

(*b*) The area of a face of one medal.

[Take $\pi = 3{\cdot}1416$.]

6. Simplify without using unnecessarily lengthy methods—

(*a*) $(4{\cdot}68)^3 + 3 \times 1{\cdot}32 \times (4{\cdot}68)^2 + 3 \times (1{\cdot}32)^2 \times 4{\cdot}68 + (1{\cdot}32)^3$.

(*b*) $$\frac{5 - 6 \times \frac{9}{13} + \left(\frac{9}{13}\right)^2}{15 + 2 \times \frac{9}{13} - \left(\frac{9}{13}\right)^2}.$$

(*c*) $2117 \times 1883 - 1113 \times 887$.

8. Two trains, moving at the rates of 45 and $47\frac{1}{2}$ miles an hour pass simultaneously but in opposite directions through a station at 9 a.m. Draw a graph to show their distance apart at any time up to noon. Use the graph to find their distance apart at 10.48 a.m., and the time at which they are 200 miles apart.

College of Preceptors (First Class).

MENSURATION.

1. Write down the tables (*a*) for lineal measure, (*b*) for superficial measure.

2. The base of the largest of the Egyptian Pyramids is a square. A side of this square is 693 feet. What extent of ground in acres, roods, and perches is covered by this Pyramid?

3. A rectangular court is 120 feet long and 90 feet broad, and has a path of 10 feet round it. Find the cost of covering the path with flag-stones at 4*s.* 6*d.* per square yard and turfing the court itself at 9*s.* 6*d.* per 100 square feet.

4. The area of a triangular field is 7 acres 3 roods 14 poles. The length of its base is 16½ chains. Find the length of its perpendicular in chains from the vertex to the base.

5. The sides of a triangular field measure: AB, 1400; BC, 1760; CA, 3000 links. It is let at £2 per acre. Find the annual rent.

6. The cost of fencing a circular piece of land was £59. 11*s.* 8*d.* at 2*s.* 1*d.* per yard. Find the length of a straight path running from side to side through the centre.

7. The circumference of a wheel is 2 metres 7 centimetres. Find the number of revolutions it will make in travelling 2 kilometres 33 decametres 91 decimetres.

8. The diameter of a circular well is 9 feet 4 inches, and the depth of the well is 30 feet. Find the cost of sinking it at the rate of 13*s.* 6*d.* per cubic yard.

9. Give the definition of a *prism*, also of a *prismoid*, with figures; and find the solidity in cubic inches of a prismoid whose greater end is 18 and its breadth 8 inches, the length of the less end being 12 and its breadth 6 inches, the perpendicular height being 18 inches.

10. Work done in digging, etc., is estimated (English) by the number of pounds weight raised 1 foot, or (French) by the number of kilogrammes raised 1 metre, called "metre-kilogrammes." Express a metre-kilogramme in foot-pounds.

[Given 1 metre = 39·37 inches.
1 kilogramme = 15430 grains.
1 foot = 12 inches.
1 pound = 7000 grains.]

11. A field is in the form of an irregular pentagon. It is required from the particulars given below—

(*a*) To draw a diagram of the field on *one* sheet of paper.

(*b*) To draw up the Field-Book on *one* sheet.

(*c*) To find the area of the field ABCDE.

A horizontal line AD is first measured = 351 links.

At 71 links along AD from A a perpendicular measuring 84 links falls to B.

At 205 links along AD a perpendicular is drawn on the opposite side of the horizontal line, measuring 129 links.

At 305 links along AD a perpendicular is drawn to it, ending in C on the same side as the first perpendicular, measuring 84 links.

Cambridge Local (Junior).

MENSURATION AND SURVEYING.

1. A building is in the form of a cylinder 40 ft. in diameter and 25 ft. high, surmounted by a hemispherical dome of equal diameter. Estimate the cost of painting its whole external surface, at 10½*d.* per square yard.

2. A rectangular cottage measures 28 ft. 6 in. by 24 ft. 0 in. The rain which falls on its roof is led in equal shares into three equal water-butts, whose depth is 3 ft. 4 in., and average diameter 2 ft. 6 in. Find how many inches of rainfall will fill the butts.

3. Draw on the scale of one chain to the inch the outline of an irregular pond about half an acre in extent, and explain briefly how you would make a chain survey to determine its actual area.

4. On the scale of two chains to the inch draw a plan of the roughly triangular field surveyed according to the following field book :

⊙ C	0	1311 ↘ Line 3
	58	1100
	103	900
	157	700
		672 ⊙ T
	188	500
	115	200
⊙ B	0	0 ↙ Line 1

Line 2. B to C.

⊙ A	0	738
⊙ T		0

Tie line. T to A.

⊙ A	0	856 ↘ Line 1
	33	700
		551 0
		500 22
		400 37
	0	245
	28	80
⊙ C	0	0 ↙ Line 2

Line 3. C to A.

⊙ B	0	1108 ↗ Line 2
	84	1000
	116	800
	98	600
	56	200
⊙ A	0	0 ↙ Line 3

Line 1. A to B bearing N.N.W.

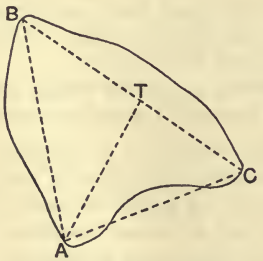


FIG. 5.—Diagram of field.

5. Find the area of the property whose plan, on the scale of 4 chains to 1 inch, is here given.

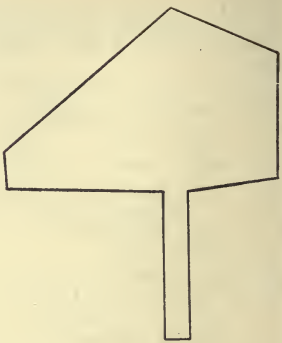


FIG. 6.

The London Chamber of Commerce.

1. The weight of a rectangular block of iron is 865·432 grams, to the nearest milligram, the weight of each cubic centimetre being 7·734 grams, also true to the nearest milligram. On being measured with an instrument reading to the nearest hundredth of a centimetre, the length is found to be 7·63 cm., and the breadth 5·48 cm. Find the thickness of the slab, *as accurately as the data allows.*

2. The following is an advertisement taken from a weekly paper:

"Handsome Cuckoo Clock offered at 12*s.* 6*d.* post free for full cash with order, or post free on receipt of 5*s.* and your promise to pay 2*s.* 6*d.* monthly until 15*s.* is paid."

If money be reckoned as earning simple interest, what rate per cent. per annum of interest is the above equivalent to?

3. A steam engine employed in pumping water from a coal mine is found to consume an amount of coal proportional to the product of the amount in gallons of water pumped up, and the depth in feet from which the water is raised. If 112 lbs. of coal is burnt in pumping 9600 gallons of water from a depth of 600 feet, how much coal will be burnt when 12,000 gallons are raised from a depth of 750 feet?

4. Prove the truth of the following rule for expressing sums of money in pounds and decimals of a pound, "Count each shilling as 0·05 of a £, express the pence as farthings, increasing the number by one twenty-fourth of itself, regard each modified farthing as 0·001 £." What would be the corresponding rule for converting decimalized sums into £ *s.* *d.* within a farthing?

(*a*) Express £35. 6*s.* 4½*d.* as a decimal of a £.

(*b*) Express £0·8732 in shillings and pence within a farthing.

(*c*) Find the value of 7856 articles at £6. 2*s.* 8*d.* each.

5. A person instructs a stockbroker to buy 3000 shares at £0. 17*s.* 6*d.* per share, the settlement with his broker is deferred however for 7 months, at the end of which time the price of the shares having risen to £1. 2*s.* 9*d.*, the person instructs his broker to sell. If the broker charges 8 per cent. per annum for the money value of the shares, what payment is due to or from the broker? (brokerage 3*d.* per share on each transaction).

6. Find the average temperature at Barnstaple for the twelve months stated, also the average temperature at Oxford during July, 1903:

	Barnstaple.	Oxford.
1903.		
July	60·4°	?
August	58·6°	58·7°
September	56·7°	56·8°
October	53·4°	52·2°
November	47·3°	44·1°
December	41·4°	38·6°
1904.		
January	42·4°	39·4°
February	41·0°	39·1°
March	42·3°	41·0°
April	48·9°	49·3°
May	52·3°	52·7°
June	57·4°	57·2°
Average for 12 months,	?	49·2°

If in each case the data be known to the nearest tenth of a degree, state the maximum possible error in each answer.

7. Average limestone consists of 112 parts of carbonate of lime, to 9 of clay and 29 of sand. Find its percentage composition. If on burning, 56 per cent. of the carbonate yields lime, what quantity of limestone is required to yield 1 ton 8 cwt. of lime?

8. An iron rod is known to weigh 17·346 lb. per foot run. What would be the corresponding figures in grams per centimetre? State clearly the assumptions you make.

9. What sum of money must be invested at 4 per cent. per annum compound interest (interest payable yearly), so as to gain £37. 5*s.* 6*d.* as interest in three years?

Oxford Local (*Senior*).

1. Resolve 462 into prime factors, and hence find all the factors of 462 which are greater than 50.

2. Use decimal fractions to find the value of

$$\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\frac{1}{1.2.3.4.5}+\frac{1}{1.2.3.4.5.6}.$$

3. If 1 metre = 3·2809 feet, find, correct to the nearest foot, the difference between 161 kilometres and 100 miles.

4. Find how many cubical blocks, each of whose edges is 62·5 centimetres long, could be made out of 75 cubic metres of clay: and find, also, to the nearest cubic centimetre, how many cubic centimetres of clay would remain over.

5. (1) How many decimal places must generally be retained in working a sum in compound interest in which money is expressed as the decimal of a £, in order that the result may be correct to the nearest penny?

(2) Find to the nearest penny the compound interest on £4387. 12*s.* 6*d.* for 3 years at 4 per cent. per annum.

6. A slow train leaves A at 1.30 p.m. and reaches B, 110 miles away, at 7 p.m., while an express leaving B at 4 p.m. reaches A at 6 p.m. Assuming that each train travels uniformly, find the time at which the trains will meet.

7. A tea-dealer allows 10 per cent. discount off the marked price of his tea. On tea marked 1*s.* 8*d.* per lb. he makes a profit of $12\frac{1}{2}$ per cent. The duty on tea is reduced one penny per lb., and the dealer then marks his tea at 1*s.* 7*d.* per lb. (discount being allowed at the same rate as before). Find his percentage of profit now.

8. A chemist has two bottles containing mixtures of nitric acid and water. The liquid in the first bottle contains 25 per cent. of acid, and the liquid in the second bottle contains 37·5 per cent. of acid. How much liquid must be taken from each bottle in order to obtain a third mixture containing 5 litres of acid and 9 litres of water?

9. Two brothers receive equal money legacies. One of them invests all his money in India $3\frac{1}{2}$ % Stock at $103\frac{1}{2}$. The other invests part of his money in $2\frac{1}{2}$ % Consols at $86\frac{1}{4}$ and the rest of it in Railway 4 % Preference Stock at $112\frac{1}{8}$. If the two brothers receive equal incomes from their investments, find the ratio of the money invested in Consols to the money invested in the Railway Stock.

Cambridge Local (*Senior*).

1. Find the cost of 8 tons 14 cwt. 3 qr. 12 lb. at £3. 15*s.* 10*d.* per ton.

2. Find the Greatest Common Measure of 78913 and 501866.

3. Simplify $\dfrac{5\frac{9}{14}+1\frac{17}{24}}{17\frac{4}{15}-7\frac{6}{7}} \div (2-\frac{3}{64})$.

4. Find to the nearest foot the value of 0·04 of 5·745 miles; and reduce £9. 14*s.* 3*d.* to a decimal of 10 guineas.

5. If 29 metres 75 centimetres of cloth cost 190 francs 40 centimes, what is the cost of 22 metres 5 centimetres?

6. A square field has an area of $5\frac{1}{2}$ acres. Find to the nearest penny the cost of a fence round it at 3*s.* 9*d.* per yard.

7. Find to the nearest penny the compound interest on £1600 for 3 years at 3 per cent. per annum.

8. A grocer mixes 60 lbs. of tea, which cost 1*s.* 9*d.* a pound, with 80 lbs. of tea, which cost 2*s.* 4*d.* a pound, and sells the mixture at 2*s.* 7*d.* a pound. What is his profit per cent. on his outlay?

9. Find the quarterly income derived from the investment of £4256 in a $3\frac{3}{4}$ per cent. stock at 95.

10. The railway line between two towns A and B is $55\frac{1}{4}$ miles long. The 4.42 p.m. express train from A arrives at B at 6.3 p.m.; and the 4.30 p.m. express train from B arrives at A at 5.42 p.m. Assuming that each train travels at a uniform rate, find the time at which they meet one another and the distances of the meeting place from the two towns.

TABLES.

DESCRIPTION OF STANDARDS OF WEIGHT AND MEASURE.

(*Based on a Table exhibited in the Victoria and Albert Museum, South Kensington.*)

British Standards.

The **yard** of 36 inches is the unit or standard measure of extension, from which all other imperial measures of extension, whether linear, superficial, or solid, are ascertained.

The **pound** is the unit of the measure of weight from which all other imperial weights, or measures having reference to weight, are ascertained. The one seven thousandth part of the pound is a **grain.** One sixteenth part (437½ grains) of the imperial standard pound is an **ounce avoirdupois.**

The **ounce troy** consists of 480 grains and is now the only legal standard troy weight.

All articles sold by weight must be sold by avoirdupois weight; except that—

(1) Gold and silver, and articles made thereof, including gold and silver thread, lace or fringe, also platinum, diamonds, and other precious metals or stones, may be sold by the ounce troy or by any decimal parts of such ounce—and

(2) Drugs, when sold by retail, may be sold by apothecaries' weight.

[The "apothecaries'" ounce is of the same weight as the troy ounce of 480 imperial grains.]

The unit or standard measure of capacity from which all other measures of capacity, as well for liquids as for dry goods, are derived, is the **gallon** containing ten imperial standard pounds weight of distilled water (weighed in air against brass weights) at the temperature of 62° Fahrenheit and with the barometer at 30 inches.

Metric Standards.

The third Schedule to the Weights and Measures Act, 1878, sets forth the equivalents of imperial weights and measures and of the weights and measures therein expressed in terms of the metric system, and Sect. 21 of the Act provides that a contract shall not be invalid if weights or measures

of the metric system, or decimal subdivisions of imperial weights and measures, are used in it.

The unit of Length is the **Metre** = 39·3708... inches.
,, ,, Surface ,, **Are** = 100 sq. metres = 119·6033 sq. yds.
,, ,, Capacity ,, **Litre** = $\frac{1}{1000}$ cub. metre = 1·76077 pints.
,, ,, Weight ,, **Gram** = 15·4323487 grains.

(A gram is the weight of a cubic centimetre of distilled water at 4° centigrade.)

BRITISH TABLES OF WEIGHT AND MEASURE.

Linear Measure.

12 inches (in.) = 1 foot (ft.).
3 feet = 1 **yard** (yd.).
1760 yards = 1 **mile** (mi.).

Gunter's Chain Measure (Linear).

100 links or 4 poles = 1 **chain** = 22 yards.
10 chains = 1 furlong.
8 furlongs = 1 mile.

Measures Used for Special Purposes.

1 **Geographical mile** or Nautical mile = 1·15 ordinary or Statute mile.
1 League = 3 miles.
1 **Fathom** = 6 feet.
1 Pace (Military) = 2 feet 6 inches.
1 Pace (Geometrical) = 5 feet.
1 Ell = $1\frac{1}{4}$ yards.
1 Cubit = 18 inches.
1 Quarter or Span = 9 inches.
1 **Hand** = 4 inches.
1 Palm = 3 inches.
1 Nail = 2·25 inches.
12 Lines = 72 points = 1 inch.

Square Measure.

12^2 = 144 **square inches** (sq. in.) = 1 **square foot** (sq. ft.).
3^2 = 9 square feet = 1 **square yard** (sq. yd.).
10,000 sq. links = 1 sq. chain.
10 sq. chains = 1 **acre**.
640 acres = 1 sq. mile.
40 sq. poles = 1 rood.
4 roods = 1 acre.
10,000 sq. links = 484 sq. yards = 16 sq. poles = 1 sq. chain.

Cubic Measure.

12^3 = 1728 cubic inches (cub. in.) = 1 **cubic foot** (cub. ft.).
3^3 = 27 cubic feet = 1 **cubic yard** (cub. yd.).

Measures of Capacity.

4 gills = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 **gallon** (gal.).
2 gallons = 1 peck (pk.).
4 pecks (8 gal.) = 1 bushel (bus.).
8 bushels = 1 quarter (qr.).

Measures Used for Special Purposes.

4½ gallons = 1 pin.
9 gallons = 1 firkin.
36 gallons = 1 barrel.
54 gallons = 1 hogshead.

---

1 hogshead of Ale = 54 gallons.
1 hogshead of Claret = 46 gallons.
1 hogshead of Port = 57 gallons.

Apothecaries' Measure.

60 minims = 1 fluid drachm (f℥).
8 drachms = 1 fluid ounce (f℥).
20 ounces = 1 **pint** (pt.).
8 pints (160 f℥) = 1 **gallon** (gal.).

Avoirdupois Weight.

(The grain has the same weight in each of the three tables below, and is the seven-thousandth part of the Standard Pound.)

16 drams (dr.) = 1 ounce (oz.).
16 ounces (7000 **grains**) = 1 **pound** (lb.).
14 pounds = 1 stone (st.).
28 pounds (2 st.) = 1 quarter (qr.).
4 quarters (112 lbs.) = 1 hundredweight (cwt.).
20 hundredweights (2240 lbs.) = 1 ton (t.).

Troy Weight.

24 grains (gr.) = 1 pennyweight (dwt.).
20 pennyweights (480 grains) = 1 ounce (oz.).
12 ounces (5760 grains) = 1 pound (lb.).

Apothecaries' Weight.

20 grains (gr.) = 1 scruple (℈).
3 scruples = 1 drachm (ʒ).
8 drachms (480 grains) = 1 ounce (℥).
12 ounces (5760 grains) = 1 pound (lb.).

METRIC TABLES OF WEIGHT AND MEASURE.

Linear Measure.

10 millimetres (mm.) = 1 centimetre (cm.).
10 centimetres = 1 decimetre (dm.).
10 decimetres = 1 **metre** (m.).
10 metres = 1 decametre.
10 decametres = 1 hectometre.
10 hectometres = 1 **kilometre** (km.).

Square Measure.

10^2 = 100 square millimetres = 1 square centimetre.
10^2 = 100 square centimetres = 1 square decimetre.
10^2 = 100 square decimetres = 1 **square metre**
= 1 **centiare.**
10^2 = 100 square metres = 1 square decametre
= 1 **are.**
10^2 = 100 square decametres = 1 square hectometre
= 1 **hectare.**
10^2 = 100 square hectometres = 1 square kilometre.

The terms **centiare, are** and **hectare** are used principally in dealing with land measures.

Cubic Measure.

10^3 = 1000 cubic millimetres = 1 **cubic centimetre (c.c.).**
10^3 = 1000 cubic centimetres = 1 cubic decimetre.
10^3 = 1000 cubic decimetres = 1 **cubic metre**
= 1 **stere.**

Measures of Capacity.

10 centilitres = 1 decilitre.
10 decilitres = 1 cubic decimetre = 1 **litre.**
10 litres = 1 decalitre.
10 decalitres = 1 hectolitre.
10 hectolitres = 1 kilolitre = 1 cubic metre.

Weight.

10 milligrams = 1 centigram.
10 centigrams = 1 decigram.
10 decigrams = 1 **gram** (gr.).
10 grams = 1 decagram.
10 decagrams = 1 hectogram.
10 hectograms = 1 **kilogram** (kilo.).

(The gram is the weight of 1 cubic centimetre of water at the temperature of 4° C.)

Summary of Metric Measures.

Fraction or Multiple of Standard Measure	Length.	Volume.	Weight.
0·001	Millimetre	Millilitre	Milligram
0·01	Centimetre	Centilitre	Centigram
0·1	Decimetre	Decilitre	Decigram
1	**Metre**	**Litre (1000 c.c.)**	**Gram**
10	Decametre	Decalitre	Decagram
100	Hectometre	Hectolitre	Hectogram
1000	Kilometre	Kilolitre	Kilogram

APPROXIMATE EQUIVALENTS OF IMPERIAL WEIGHTS AND MEASURES IN TERMS OF METRIC UNITS.

Imperial to Metric.

Linear Measure:

1 inch - - - - - - - = 2·54 centimetres.
1 foot (12 inches) - - - - - = 0·30 metre.
1 yard (3 feet) - - - - - - = **0·914 metre.**

Square Measure:

1 square inch - - - - - - = 6·45 sq. centimetres.
1 square foot (144 square inches) - - = 929 sq. centimetres.
1 square yard (9 square feet) - - - = 0·836 sq. metre.

Cubic Measure:

1 cubic inch - - - - - - = 16·387 cub. centimetres.
1 cubic foot (1728 cubic inches) - - = 0·028 cub. metre.
1 cubic yard (27 cubic feet) - - - = 0·764 cub. metre

Measures of Capacity:

1 gill	= 1·42 decilitres.
1 pint (4 gills)	= 0·568 litre.
1 quart (2 pints)	= 1·136 litres.
1 gallon (4 quarts)	**= 4·546 litres.**

Apothecaries' Measure:

1 minim	= 0·059 millilitre.
1 fluid scruple	= 1·184 millilitres.
1 fluid drachm (60 minims)	= 3·552 millilitres.
1 fluid ounce (8 drachms)	= 2·841 centilitres.
1 pint	= 0·568 litre.
1 gallon (8 pints or 160 fluid ounces)	**= 4·546 litres.**

Avoirdupois Weight:

1 grain	= 0·065 gram.
1 dram	= 1·77 grams.
1 ounce (16 drams)	= 28·35 grams.
1 pound (16 ounces or 7000 grains)	**= 0·4536 kilogram.**

Troy Weight:

1 grain	= 0·065 gram.
1 pennyweight (24 grains)	= 1·55 grams.
1 troy ounce (20 pennyweights)	= 31·10 grams.

Apothecaries' Weight:

1 grain	= 0·065 gram.
1 scruple (20 grains)	= 1·296 grams.
1 drachm (3 scruples)	= 3·888 grams.
1 ounce (8 drachms)	= 31·10 grams.

APPROXIMATE EQUIVALENTS OF METRIC WEIGHTS AND MEASURES IN TERMS OF IMPERIAL UNITS.

Metric to Imperial.

Linear Measure:

1 millimetre (mm.) (0·001 m.)	= 0·03937 inch.
1 centimetre (0·01 m.)	= 0·3937 inch.
1 decimetre (0·1 m.)	= 3·937 inches.

1 metre (m.) = **39·37 inches. / 3·28 feet. / 1·09 yards.**

1 decametre (10 m.) = 10·936 yards.

1 hectometre (100 m.) = 109·36 yards.

1 kilometre (1000 m.) = 0·62 mile.

Square Measure:

1 square centimetre = 0·155 square inch.

1 square decimetre (100 square centimetres) = 15·50 square inches.

1 square metre (100 square decimetres) = 10·76 square feet. / 1·19 square yards.

Cubic Measure:

1 cubic centimetre = 0·06 cubic inch.

1 cubic decimetre (c.d.) (1000 cubic centimetres) = 61·02 cubic inches.

1 cubic metre (1000 cubic decimetres) = 35·31 cubic feet. / 1·31 cubic yards.

Measure of Capacity:

1 centilitre (0·01 litre) = 0·070 gill.

1 decilitre (0·1 litre) = 0·176 pint.

1 litre = **1·76 pints.**

Weight:

Avoirdupois.

1 milligram (0·001 grm.) = 0·015 grain.

1 centigram (0·01 grm.) = 0·154 grain.

1 decigram (0·1 grm.) = 1·543 grains.

1 gramme (1 gm.) = 15·432 grains.

1 decagram (10 gm.) = 5·644 drams.

1 hectogram (100 gm.) = 3·527 oz.

1 kilogram (1000 gms.) = **2·20 lb. or 15432·3 grains.**

Troy.

1 gram (1 gm.) = 0·032 oz. troy. / 15·43 grains.

Apothecaries.

1 gram (1 gm.) = 0·257 drachm. / 0·771 scruple. / 15·43 grains.

MONEY TABLES.

British.

4 farthings ($\frac{1}{4}d.$)	= 1 penny ($d.$).
12 pence	= 1 shilling ($s.$).
2 shillings	= 1 florin.
5 shillings	= 1 crown.
20 shillings	= 1 sovereign or pound (£).
21 shillings	= 1 guinea.

France and Other Countries.

100 { centimes, centesimos, centimos, leptas } = 1 { franc (France, Switzerland, Belgium, etc.), lire (Italy), peseta (Spain), drachma (Greece) }

1 franc, lire, peseta, or drachma has a value of about $9\frac{1}{2}d.$

Germany.

100 pfennigs	= 1 mark.
3 marks	= 1 thaler.
10 marks	= 1 krone.

1 mark has a value of about $1s.$

United States.

10 cents	= 1 dime.
10 dimes (100 cents)	= 1 dollar.
10 dollars	= 1 eagle.

1 dollar has a value of about $4s.\ 2d.$

MEASUREMENT OF TIME.

60 **seconds** (sec. or ″)	= 1 **minute** (min. or ′).
60 minutes	= 1 **hour** (h.).
24 hours	= 1 **day** (d.).
7 days	= 1 **week** (wk.).
28 days or 4 weeks	= 1 **lunar month** (mon.).
365 days	= 1 ordinary or civil year (**yr.**).
366 days	= 1 leap year.

SQUARE ROOTS AND RECIPROCALS.

Numbers (n) from 1 to 100.

n	$\sqrt{n}$	$1/n$	n	$\sqrt{n}$	$1/n$	n	$\sqrt{n}$	$1/n$
1	1·0000	1·0000	**36**	6·0000	0·02778	**71**	8·4621	0·01408
2	1·4142	0·5	**37**	6·0828	0·02703	**72**	8·4853	0·01389
3	1·7321	0·33333	**38**	6·1644	0·02632	**73**	8·5440	0·01370
4	2·0000	0·25	**39**	6·2450	0·02564	**74**	8·6023	0·01351
5	2·2361	0·2	**40**	6·3246	0·025	**75**	8·6603	0·01333
6	2·4495	0·16667	**41**	6·4031	0·02439	**76**	8·7178	0·01316
7	2·6458	0·14286	**42**	6·4807	0·02381	**77**	8·7750	0·01299
8	2·8284	0·125	**43**	6·5574	0·02326	**78**	8·8318	0·01282
9	3·0000	0·11111	**44**	6·6332	0·02273	**79**	8·8882	0·01266
10	3·1623	0·1	**45**	6·7082	0·02222	**80**	8·9443	0·0125
11	3·3166	0·09091	**46**	6·7823	0·02174	**81**	9·0000	0·01235
12	3·4641	0·08333	**47**	6·8557	0·02128	**82**	9·0554	0·01220
13	3·6056	0·07692	**48**	6·9282	0·02083	**83**	9·1104	0·01205
14	3·7417	0·07143	**49**	7·0000	0·02041	**84**	9·1652	0·01190
15	3·8730	0·06667	**50**	7·0711	0·02	**85**	9·2195	0·01176
16	4·0000	0·0625	**51**	7·1414	0·01961	**86**	9·2736	0·01163
17	4·1231	0·05882	**52**	7·2111	0·01923	**87**	9·3274	0·01149
18	4·2426	0·05556	**53**	7·2801	0·01887	**88**	9·3808	0·01136
19	4·3589	0·05263	**54**	7·3485	0·01852	**89**	9·4340	0·01124
20	4·4721	0·05	**55**	7·4162	0·01818	**90**	9·4868	0·01111
21	4·5826	0·04762	**56**	7·4833	0·01786	**91**	9·5394	0·01099
22	4·6904	0·04545	**57**	7·5498	0·01754	**92**	9·5917	0·01087
23	4·7958	0·04348	**58**	7·6158	0·01724	**93**	9·6437	0·01075
24	4·8990	0·04167	**59**	7·6811	0·01695	**94**	9·6954	0·01064
25	5·0000	0·04	**60**	7·7460	0·01667	**95**	9·7468	0·01053
26	5·0990	0·03846	**61**	7·8102	0·01639	**96**	9·7980	0·01042
27	5·1962	0·03704	**62**	7·8740	0·01613	**97**	9·8489	0·01031
28	5·2915	0·03571	**63**	7·9373	0·01587	**98**	9·8995	0·01020
29	5·3852	0·03448	**64**	8·0000	0·015625	**99**	9·9499	0·01010
30	5·4772	0·03333	**65**	8·0623	0·01538	**100**	10·0000	0·01
31	5·5678	0·03226	**66**	8·1240	0·01515			
32	5·6569	0·03125	**67**	8·1854	0·01493			
33	5·7446	0·03030	**68**	8·2462	0·01471			
34	5·8310	0·02941	**69**	8·3066	0·01449			
35	5·9161	0·02857	**70**	8·3666	0·01429			

N.B.—The use of these tables can be extended by the methods suggested on pp. 310 and 320.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170						4 9 13	17 21 26	30 34 38
						0212	0253	0294	0334	0374	4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569						4 8 12	15 19 23	27 31 35
						0607	0645	0682	0719	0755	4 7 11	15 19 22	26 30 33
12	0792	0828	0864	0899	0934						3 7 11	14 18 21	25 28 32
						0969	1004	1038	1072	1106	3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271						3 7 10	13 16 20	23 26 30
						1303	1335	1367	1399	1430	3 7 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584						3 6 9	12 15 19	22 25 28
						1614	1644	1673	1703	1732	3 6 9	12 15 17	20 23 26
15	1761	1790	1818	1847	1875						3 6 9	11 14 17	20 23 26
						1903	1931	1959	1987	2014	3 6 8	11 14 17	19 22 25
16	2041	2068	2095	2122	2148						3 5 8	11 14 16	19 22 24
						2175	2201	2227	2253	2279	3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405						3 5 8	10 13 15	18 20 23
						2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648						2 5 7	9 12 14	16 19 21
						2672	2695	2718	2742	2765	2 5 7	9 11 14	16 18 21
19	2788	2810	2833	2856	2878						2 4 7	9 11 13	16 18 20
						2900	2923	2945	2967	2989	2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

*From Castle's *Practical Mathematics for Beginners.* (Macmillan.)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE OF DAYS.

Day of the Month.	Day of the Year.											
	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	—	88	119	149	180	210	241	272	302	333	363
30	30	—	89	120	150	181	211	242	273	303	334	364
31	31	—	90	—	151	—	212	243	—	304	—	365

COMPOUND INTEREST TABLE

(showing the sum to which (a) £1, (b) £1 yearly, will amount at Compound Interest).

No. of Years.	Rates per Cent.	2·5	3	3·5	4	4·5	5
1	(a)	1·02500	1·03000	1·03500	1·04000	1·04500	1·05000
	(b)	1·	1·	1·	1·	1·	1·
2	(a)	1·05063	1·06090	1·07123	1·08160	1·09203	1·10250
	(b)	2·025	2·03	2·035	2·04	2·045	2·05
3	(a)	1·07689	1·09273	1·10872	1·12486	1·14117	1·15763
	(b)	3·07563	3·0909	3·10623	3·1216	3·13703	3·1525
4	(a)	1·10381	1·12551	1·14752	1·16986	1·19252	1·21551
	(b)	4·15252	4·18363	4·21494	4·24646	4·27819	4·31013
5	(a)	1·13141	1·15927	1·18769	1·21665	1·24618	1·27628
	(b)	5·25633	5·30914	5·36247	5·41632	5·47071	5·52563
6	(a)	1·15969	1·19405	1·22926	1·26532	1·30226	1·34010
	(b)	6·38774	6·46841	6·55015	6·63298	6·71689	6·80191
7	(a)	1·18869	1·22987	1·27228	1·31593	1·36086	1·40710
	(b)	7·54743	7·66246	7·77941	7·89829	8·01915	8·14201
8	(a)	1·21840	1·26677	1·31681	1·36857	1·42210	1·47746
	(b)	8·73612	8·89234	9·05169	9·21423	9·38001	9·54911
9	(a)	1·24886	1·30477	1·36290	1·42331	1·48610	1·55133
	(b)	9·95452	10·1591	10·3685	10·5828	10·8021	11·0266
10	(a)	1·28008	1·34392	1·41060	1·48024	1·55297	1·62889
	(b)	11·2034	11·4639	11·7314	12·0061	12·2882	12·5779
15	(a)	1·44830	1·55797	1·67535	1·80094	1·93528	2·07893
	(b)	17·9319	18·5989	19·2957	20·0236	20·7841	21·5786
20	(a)	1·63862	1·80611	1·98979	2·19112	2·41171	2·65330
	(b)	25·5447	26·8704	28·2797	29·7781	31·3714	33·0660
25	(a)	1·85394	2·09378	2·36324	2·66584	3·00543	3·38635
	(b)	34·1578	36·4593	38·9499	41·6459	44·5652	47·7271
30	(a)	2·09757	2·42726	2·80679	3·24340	3·74532	4·32194
	(b)	43·9027	47·5754	51·6227	56·0849	61·0071	66·4388
35	(a)	2·37321	2·81386	3·33359	3·94609	4·66735	5·51602
	(b)	54·9282	60·4621	66·6740	73·6522	81·4966	90·3203
40	(a)	2·68506	3·26204	3·95926	4·80102	5·81636	7·03999
	(b)	67·4026	75·4013	84·5503	95·0255	107·030	120·800
45	(a)	3·03790	3·78160	4·70236	5·84118	7·24825	8·98501
	(b)	81·5161	92·7199	105·782	121·029	138·850	159·700
50	(a)	3·43711	4·38391	5·58493	7·10668	9·03264	11·46740
	(b)	97·4843	112·797	130·998	152·667	178·503	209·348

log of amount of £1 in 1 year			
	0·0107238654 at 2·5 %.	0·0128372247 at 3 %.	0·01494703498 at 3·5 %.
	0·0170333393 at 4 %.	0·0191162904 at 4·5 %.	1·0211892991 at 5 %.

PRESENT VALUE TABLE

(showing the present value of future payments of (a) £1, (b) £1 yearly).

Time in Years.	Rates per Cent.					
	2·5	3	3·5	4	4·5	5
1 (*a*)	0·97561	0·97087	0·96618	0·96154	0·95694	0·95238
(*b*)	0·97561	0·97087	0·96618	0·96154	0·95694	0·95238
2 (*a*)	0·95181	0·94260	0·93351	0·92456	0·91573	0·90703
(*b*)	1·92742	1·91348	1·89969	1·88609	1·87267	1·85941
3 (*a*)	0·92860	0·91514	0·90194	0·88900	0·87630	0·86384
(*b*)	2·85602	2·82862	2·80164	2·77509	2·74897	2·72325
4 (*a*)	0·90595	0·88849	0·87144	0·85480	0·83856	0·82270
(*b*)	3·76198	3·71711	3·67308	3·62990	3·58753	3·54595
5 (*a*)	0·88385	0·86261	0·84197	0·82193	0·80245	0·78353
(*b*)	4·64583	4·57971	4·51505	4·45182	4·38998	4·32948
6 (*a*)	0·86230	0·83748	0·81350	0·79031	0·76790	0·74622
(*b*)	5·50813	5·41720	5·32855	5·24214	5·15788	5·07569
7 (*a*)	0·84127	0·81309	0·78599	0·75992	0·73483	0·71068
(*b*)	6·34939	6·23029	6·11454	6·00205	5·89271	5·78637
8 (*a*)	0·82075	0·78941	0·75941	0·73069	0·70319	0·67684
(*b*)	7·17014	7·01969	6·87396	6·73274	6·59589	6·46321
9 (*a*)	0·80073	0·76642	0·73373	0·70259	0·67290	0·64461
(*b*)	7·97086	7·78611	7·60769	7·43533	7·26879	7·10782
10 (*a*)	0·78120	0·74409	0·70892	0·67556	0·64393	0·61391
(*b*)	8·75206	8·53020	8·31661	8·11090	7·91273	7·72173
15 (*a*)	0·69047	0·64186	0·59689	0·55526	0·51672	0·48102
(*b*)	12·3814	11·9379	11·5174	11·1184	10·7395	10·3797
20 (*a*)	0·61027	0·55368	0·50257	0·45639	0·41464	0·37689
(*b*)	15·5892	14·8775	14·2124	13·5903	13·0079	14·4622
25 (*a*)	0·53939	0·47761	0·42315	0·37512	0·33273	0·29530
(*b*)	18·4244	17·4131	16·4815	15·6221	14·8282	14·0939
30 (*a*)	0·47674	0·41199	0·35628	0·30832	0·26700	0·23138
(*b*)	20·9303	19·6004	18·3920	17·2920	16·2889	15·3725
35 (*a*)	0·42137	0·35538	0·29998	0·25342	0·21425	0·18129
(*b*)	23·1452	21·4872	20·0007	18·6646	17·4610	16·3742
40 (*a*)	0·37243	0·30656	0·25257	0·20829	0·17193	0·14205
(*b*)	25·1028	23·1148	21·3551	19·7928	18·4016	17·1591
45 (*a*)	0·32917	0·26444	0·21266	0·17120	0·13796	0·11130
(*b*)	26·8330	24·5187	22·4955	20·7200	19·1563	17·7741
50 (*a*)	0·29094	0·22811	0·17905	0·14071	0·11071	0·08720
(*b*)	28·3623	25·7298	23·4556	21·4822	19·7620	18·2559

log of present value of £1 at the end of 1 year	$\bar{1}$·9892761346 at 2·5%.	$\bar{1}$·9871627753 at 3%.	$\bar{1}$·985096503 at 3·5%.
	$\bar{1}$·9829666607 at 4%.	$\bar{1}$·98088370096 at 4·5%.	$\bar{1}$·9788107009 at 5%.

ANSWERS

PART I

Exercises I. P. 4.

2. (*a*) 20, (*b*) 23, (*c*) 15, (*d*) 10, (*e*) 18, (*f*) 62.
3. (*a*) 86, (*b*) 100, (*c*) 101, (*d*) 110, (*e*) 111.
4. (*a*) 128, (*b*) 182, (*c*) 108, (*d*) 180.
5. (*a*) 303, (*b*) 330, (*c*) 333.
6. (*a*) 2006, (*b*) 6002, (*c*) 6020, (*d*) 6200.
7. (*a*) 8706, (*b*) 8076, (*c*) 876.
8. (*a*) three, (*b*) eight, (*c*) eighteen, (*d*) eighty-one, (*e*) twenty-five, (*f*) fifty-two, (*g*) five hundred and twenty, (*h*) three hundred, (*i*) three thousand, (*j*) three thousand and five, (*k*) three thousand and fifty; (*l*) three thousand five hundred, (*m*) five thousand and three, (*n*) one hundred and eighty-six, (*o*) one hundred and sixty-eight, (*p*) eight hundred and sixty-one, (*q*) eight hundred and sixteen, (*r*) seventy-one, (*s*) seven hundred and one, (*t*) seven thousand and one.

Exercises II. P. 6.

1. 1914.	**2.** 3087.	**3.** 2748.	**4.** 2869.	**5.** 71.
6. 463.	**7.** 131.	**8.** 417.	**9.** 889.	
10. 939.	**11.** 681.	**12.** 1453.	**13.** 1599.	
14. 1911.	**15.** 517.	**16.** 1072.	**17.** 1950.	
18. 2713.	**19.** 2002.	**20.** 1687.	**21.** 2814.	

Exercises III. P. 7.

1. Eighty.	**2.** One thousand.	**3.** Sixty.	**4.** Eight hundred.
5. Seven hundred.	**6.** One.	**7.** One thousand.	**8.** Eight hundred.
9. Twenty.	**10.** Thirty.	**11.** Thirty.	**12.** Two.
13. Seven hundred.	**14.** Three hundred.		**15.** Ten.
16. One.	**17.** Nought.	**18.** 8814.	**19.** 2312.
20. 2282.	**21.** 5016.	**22.** 765.	**23.** 508.
24. 1236.	**25.** 172.	**26.** 111.	**27.** 712.

Exercises IV. P. 7.

1. 1311. **2.** 1276. **3.** 405. **4.** 1719. **5.** 660.
6. 1311. **7.** 1172. **8.** 1721. **9.** 503. **10.** 12377.
11. 2359. **12.** 9972. **13.** 3330. **14.** 2121. **15.** 7632.
16. 7529. **17.** 2710.

Exercises V. P. 9.

1. 450. **2.** 411. **3.** 149. **4.** 417. **5.** 334.

Exercises VI. P. 11.

1. 1310. **2.** 1655. **3.** 1872. **4.** 1562. **5.** 2657.
6. 4950. **7.** 13154. **8.** 3172. **9.** 5053. **10.** 4684.

Exercises VII. a. P. 14.

1. (*a*) 9, (*b*) 16, (*c*) 22, (*d*) 27. **2.** (*a*) 9, (*b*) 8, (*c*) 10, (*d*) 11.
3. 7. **4.** 9. **5.** 5. **6.** 13 inches. **7.** 4 yards. **8.** 12 lbs.

Exercises VII. b. P. 15.

1. (*a*) 917, (*b*) 956, (*c*) 149, (*d*) 1124, (*e*) 763, (*f*) 692, (*g*) 1147, (*h*) 8508, (*i*) 2.

2. (*a*) 53, (*b*) 16, (*c*) 144, (*d*) 203, (*e*) 75, (*f*) 688, (*g*) 534, (*h*) 397, (*i*) 1286, (*j*) 1693, (*k*) 1087, (*l*) 714, (*m*) 72, (*n*) 98, (*o*) 997.

3. 4892. **4.** 4221. **5.** 802. **6.** 154. **7.** 1978.
8. 929. **9.** 1999. **10.** 1468. **11.** 3834. **12.** 1926.

Exercises VII. c. P. 16.

1. 24, 87, 4, 16, 4, 1, 1, 7, 13, 2, 280.
2. 20, 150, 13, 1, 5, 7, 6.
3. 2, 24, 104, 40, 2, 172.
4. 44, 6, 10, 10, 52, 94.
5. 25, 140, 36, 13, 4, 2, 180.

Exercises VII. d. P. 17.

1. 4. **2.** 18. **3.** 3. **4.** 21. **5.** 14. **6.** 12.
7. 7. **8.** 13. **9.** 6. **10.** 14. **11.** 7. **12.** 14.
13. 6. **14.** 8. **15.** 5. **16.** 15.

Exercises VIII. c. P. 21.

1. (*a*) 2×24, 4×12, 3×16, 6×8; (*b*) 2×14, 7×4; (*c*) 5×7; (*d*) 9×10, 3×30, 2×45, 6×15, 5×18; (*e*) 2×44, 4×22, 8×11; (*f*) 4×8, 2×16.

2. (i) 14, (ii) 22, (iii) 99, (iv) 96, (v) 96, (vi) 91.

Exercises IX. a. P. 23.

1. (*a*) 12, (*b*) 9, (*c*) 18, (*d*) 15, (*e*) 24.
2. (*a*) 96, (*b*) 72, (*c*) 48, (*d*) 60, (*e*) 108.
3. (*a*) 24, (*b*) 30, (*c*) 42, (*d*) 12, (*e*) 48.
4. (*a*) 28, (*b*) 84, (*c*) 49, (*d*) 63. **5.** 96. **6.** 54.
7. 180. **8.** 1120. **9.** (*a*) 50, (*b*) 100, (*c*) 150, (*d*) 200.
10. (*a*) 75, (*b*) 125, (*c*) 175.
11. (*a*) 500, (*b*) 1000, (*c*) 375, (*d*) 625, (*e*) 875.
12. (*a*) 432, (*b*) 576, (*c*) 1728, (*d*) 720, (*e*) 864.
13. (*a*) 472, (*b*) 1180, (*c*) 1652, (*d*) 1888, (*e*) 2124.
14. (*a*) 1331, (*b*) 363, (*c*) 726, (*d*) 605.
15. (*a*) 111, (*b*) 222, (*c*) 333, (*d*) 148.
16. (*a*) 112, (*b*) 224, (*c*) 84. **17.** (*a*) 560, (*b*) 896, (*c*) 1008.
18. (*a*) 3×8, 4×6, 2×12; (*b*) 2×16, 4×8; (*c*) 3×15, 5×9.

Exercises IX. b. P. 24.

1. 405. **2.** 324. **3.** 1380. **4.** 1224. **5.** 13440.
6. 3836. **7.** 45760. **8.** 2660. **9.** 4183. **10.** 1311.
11. 8091. **12.** 6144. **13.** 7992. **14.** 23180. **15.** 120015.
16. 4838400. **17.** 241902. **18.** 491337. **19.** 515080. **20.** 330165.
21. 501160. **22.** 383358. **23.** 229914. **24.** 242782. **25.** 130400.
26. (*a*) 25740, (*b*) 42250, (*c*) 31916, (*d*) 560, (*e*) 29000.
27. (*a*) 388, (*b*) 227, (*c*) 214, (*d*) 894, (*e*) 3510, (*f*) 3079, (*g*) 3561, (*h*) 176, (*i*) 5173, (*j*) 3708, (*k*) 3048, (*l*) 2181, (*m*) 70, (*n*) 135, (*o*) 13, (*p*) 366, (*q*) 715.
28. 512616 lbs. **29.** 104390 miles. **30.** 11 ozs. **31.** 24 inches.

Exercises IX. c. P. 26.

1. (*a*) 9360, (*b*) 936, (*c*) 468000, (*d*) 163800.
2. (*a*) 2679, (*b*) 267900, (*c*) 2679000.
3. (*a*) 4000, (*b*) 35000, (*c*) 400000, (*d*) 6000, (*e*) 81000, (*f*) 2800.

Exercises X. P. 27.

1. (*a*) 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024;
   (*b*) 3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049.
2. 5, 25, 125, 625, 3125.
3. (*a*) 2^3, 2^6, 2^9, 2^{12}, 2^{15}; (*b*) 2^4, 2^8, 2^{12}, 2^{16}, 2^{20}.
4. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400.

5. (*a*) 3, (*b*) 4, (*c*) 5, (*d*) 6, (*e*) 7, (*f*) 8, (*g*) 9, (*h*) 10, (*i*) 11, (*j*) 12.
6. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728.
7. (*a*) 4, 9, 6, 5, 0, 1; (*b*) 8, 7, 4, 5, 6, 3, 2, 9, 0, 1.
8. (*a*) a^5, (*b*) b^6, (*c*) x^3, (*d*) y^2, (*e*) z^8.

Exercises XI. P. 28.

6. (*a*) 225, 625, 1225, 2025, 3025; (*b*) 289, 484, 1089, 8281, 6724; (*c*) 10404, 41209, 93025.
7. (*a*) 9, (*b*) 16, 25, 36, 49, 64, 81, (*c*) 100.
8. 169. 9. (*a*) $17^2 - 16^2$; (*b*) $15^2 - 14^2$; (*c*) $16^2 - 15^2$.
10. 5 rows 25 pennies. 11. 576.

Exercises XII. a. P. 31.

1. 6, 10, 18, 24. 2. 7, 13, 32, 8. 3. 9, 18, 14, 32.
4. 15 boys, 4 apples. 5. 7, 10, 4, 40.

Exercises XII. b. P. 31.

1. (*a*) Quotient 791, remainder 2; (*b*) quotient 475, remainder 0; (*c*) quotient 339, remainder 2.
2. 2001. 3. 149. 4. 250 feet. 2 inches over.
5. 178 times. 6. 546. 7. 7.

Exercises XII. c. P. 32.

1. (*a*) 673, rem. 2; (*b*) 538, rem. 4; (*c*) 449, rem. 0; (*d*) 384, rem. 6; (*e*) 336, ,, 6; (*f*) 299, ,, 3; (*g*) 244, rem. 10; (*h*) 224, ,, 6.
2. (*a*) 2029, rem. 1; (*b*) 1893, rem. 12; (*c*) 1775, rem. 7; (*d*) 1578, ,, 3; (*e*) 1352, ,, 15; (*f*) 1183, ,, 15.
3. (*a*) 1121, ,, 2; (*b*) 945, ,, 29; (*c*) 864, ,, 29; (*d*) 840, ,, 29; (*e*) 756, ,, 29; (*f*) 687, ,, 41.
4. (*a*) 25, ,, 0; (*b*) 23, ,, 21; (*c*) 22, ,, 47; (*d*) 20, ,, 25; (*e*) 18, ,, 45; (*f*) 17, ,, 37.
5. (*a*) 38, ,, 48; (*b*) 33, ,, 12; (*c*) 29, ,, 0; (*d*) 26, ,, 54; (*e*) 25, ,, 34; (*f*) 24, ,, 96.
6. (*a*) 24, ,, 29; (*b*) 22, ,, 35; (*c*) 19, ,, 77; (*d*) 18, ,, 29; (*e*) 17, ,, 5; (*f*) 15, ,, 119.

Exercises XII. d. P. 33.

1. (*a*) 11, rem. 5; (*b*) 4, rem. 14; (*c*) 6, rem. 4.
2. 6 groups, rem. 7. 3. (*a*) 5, rem. 11; (*b*) 3, rem. 14. 4. 5.

Exercises XII. e. P. 36.

(*a*) (i) 2710, rem. 11; (ii) 2451, rem. 2; (iii) 3835, rem. 11;
(iv) 3046, ,, 4; (v) 1502, ,, 16; (vi) 709, ,, 17;
(vii) 3613, ,, 11.

(*b*) (i) 1113, rem. 13; (ii) 1006, rem. 39; (iii) 1575, rem. 16;
(iv) 1251, ,, 6; (v) 617, ,, 10; (vi) 291, ,, 28;
(vii) 1484, ,, 6.

(*c*) (i) 506, rem. 103; (ii) 458, rem. 41; (iii) 717, rem. 25;
(iv) 569, ,, 75; (v) 280, ,, 122; (vi) 132, ,, 88;
(vii) 675, ,, 85.

(*d*) (i) 130, rem. 201; (ii) 117, rem. 449; (iii) 184, rem. 264;
(iv) 146, ,, 274; (v) 72, ,, 146; (vi) 34, ,, 72;
(vii) 173, ,, 416.

(*e*) (i) 118, rem. 391; (ii) 107, rem. 200; (iii) 168, rem. 16;
(iv) 133, ,, 237; (v) 65, ,, 437; (vi) 31, ,, 49;
(vii) 158, ,, 160.

(*f*) (i) 149, rem. 208; (ii) 135, rem. 80; (iii) 211, rem. 229;
(iv) 168, ,, 6; (v) 82, ,, 368; (vi) 39, ,, 61;
(vii) 199, ,, 127.

(*g*) (i) 85, rem. 206; (ii) 77, rem. 88; (iii) 120, rem. 496;
(iv) 95, ,, 617; (v) 47, ,, 205; (vi) 22, ,, 242;
(vii) 113, ,, 507.

(*h*) (i) 67, rem. 567; (ii) 61, rem. 133; (iii) 95, rem. 626;
(iv) 75, ,, 912; (v) 37, ,, 448; (vi) 17, ,, 650;
(vii) 90, ,, 130.

Exercises XII. f. P. 36.

1. (*a*) 4, (*b*) 4, (*c*) 6. **2.** (*a*) 5, (*b*) 4, (*c*) 4, (*d*) 2.
3. (*a*) 28, (*b*) quotient 25, rem. 7, (*c*) 866, (*d*) 1095, (*e*) 28, (*f*) 35.
4. 5 or 0. **5.** 25, 50, 75. **6.** 0, 0, 25, 50, 75.
7. 5, 5, 12, 7, 8, 8, 3, 1, 16. **9.** 3150, 450, 3600, 1200, 8425.
11. 92, 15, 35, 77, 188.

Exercises XIII. P. 38.

1. 2176, 51232, 6328; 1836, 3024. **2.** (*a*) 0, 1, (*b*) 8, 1, 21, (*c*) 7.
3. 59, and 182 lbs. over. **4.** 62nd. **5.** $b=6$, $a=1$.
6. 12852 and 17856. **7.** Dividend = 83618, quotient = 352.
8. 28 and 62 inches over. **9.** 29. **10.** 8.

Exercises XIV. P. 41.

1. 8. **2.** 7 (to nearest unit). **3.** 12. **4.** 7811. **5.** 55, 2 approx.
6. Between 63546 and 63545. **7.** (*a*) 19980, (*b*) 3092, (*c*) 8643.
8. 46°. **9.** Apr. 16, Sept. 20. **10.** (*a*) 175706, (*b*) 591370, (*c*) 1074.

Exercises XV. P. 45.

3. 45. **4.** (*a*) 36, (*b*) 28. **5.** (*a*) 36, (*b*) 70, (*c*) 49, (*d*) 92, (*e*) 78, (*f*) 63.

Exercises XVI. P. 46.

1. 6338, 3071. **2.** 85, and 27 over. **3.** 79712. **4.** 123.
5. 28348. **6.** 9359. **7.** £3465. **8.** 12288.
9. 49. **10.** 4616. **11.** 4035230. **12.** 19664.
13. 56, £28 over. **14.** 620. **15.** 1512000. **16.** 2019.
17. One hundred thousand and two hundred millions, two hundred thousands one hundred.
18. 7256. **19.** 18. **20.** 8620. **22.** 120886698962.
23. (*a*) 69801000, (*b*) 64791000, (*c*) 75416000, (*d*) 72326000.

24.

	Imports.	Exports.
1892	39326	37200
1893	38383	29685
1894	41923	36933
1895	44086	41247
1896	46444	34507
1897	53820	36731
1898	53521	44197
1899	55778	49433

25.

	Imports.	Exports.
1892	253076	243967
1893	239698	192628
1894	265134	265713
1895	261212	307787
1896	275799	247410
1897	276901	295728
1898	302223	285769
1899	319848	346415
1900	323818	242317
1901	336118	266841

26. (*a*) 224089, (*b*) 35308, (*c*) 21197, (*d*) 18198, (*e*) 15563, (*f*) 21410.
27. (*a*) (1900) 1659586, (1901) 1459091, (1902) 1611979, (*b*) 242154 (*c*) 16701, (*d*) 98092, (*e*) 9788.
28. (*a*) £11225954, (*b*) £254334, (*c*) £1445210, (*d*) £398119, (*e*) £10489990, (*f*) £5767056, (*g*) £9185720.
29. (*a*) £19125355, (*b*) £18178835, (*c*) £15016677.
30. (*a*) £3741985 to nearest £, (*b*) £481737 to nearest £, (*c*) £4223721 (*d*) £1922352.

Exercises XVII. b. P. 55.

1. (*a*) 30*d.*, (*b*) 40*d.*, (*c*) 50*d.*, (*d*) 60*d.*, (*e*) 72*d.*, (*f*) 84*d.*, (*g*) 96*d.*, (*h*) 108*d.*, (*i*) 120*d.*, (*j*) 132*d.*, (*k*) 144*d.*, (*l*) 156*d.*, (*m*) 168*d.*, (*n*) 180*d.*, (*o*) 192*d.*, (*p*) 204*d.*, (*q*) 216*d.*, (*r*) 228*d.*, (*s*) 240*d.*
2. (*a*) £1. 5*s.*, (*b*) £1, (*c*) £2. 1*s.* 8*d.*, (*d*) £2. 10*s.*, (*e*) £3, (*f*) £3. 10*s.*, (*g*) £3. 15*s.*, (*h*) £4, (*i*) £4. 3*s.* 4*d.*

Exercises XVII. c. P. 55.

1. (*a*) 895*d.*, (*b*) 1902*d.*, (*c*) 11954*d.*, (*d*) 141799*d.*, (*e*) 168965*d.*, (*f*) 2321917*d.*, (*g*) 1681348*d.*, (*h*) 4085743*d.*
2. (*a*) 2920, (*b*) 575, (*c*) 8227, (*d*) 282143, (*e*) 118521, (*f*) 77365, (*g*) 733883, (*h*) 412199.
3. (*a*) 1383, (*b*) 2345, (*c*) 23550, (*d*) 386723, (*e*) 16001, (*f*) 77274, (*g*) 243607, (*h*) 47800.
4. (*a*) 20, (*b*) 30, (*c*) 45, (*d*) 91, (*e*) 147, (*f*) £141. 15*s.*, (*g*) £774. 18*s.*, (*h*) £281. 18*s.* 6*d.*, (*i*) £1653. 4*s.* 6*d.*
5. (*a*) 15*s.* 9$\frac{3}{4}$*d.*, (*b*) £3. 5*s.* 9$\frac{3}{4}$*d.*, (*c*) £13. 2*s.* 0$\frac{1}{4}$*d.*, (*d*) £14. 18*s.* 0*d.*, (*e*) £79. 7*s.* 9*d.*, (*f*) £74. 18*s.* 8*d.*, (*g*) £9. 18*s.* 3*d.*, (*h*) £236. 6*s.* 0*d.*

Exercises XVIII. a. P. 56.

1. (*a*) 1*s.* 11*d.*, (*b*) 3*s.*, (*c*) 2*s.*, (*d*) 4*s.* 3*d.*, (*e*) 8*s.*, (*f*) 38*s.* 6*d.*
2. (*a*) 16*s.*, (*b*) 20*s.*, (*c*) 4*s.* 6*d.*, (*d*) 16*s.*, (*e*) 20*s.*, (*f*) 7*s.*

Exercises XVIII. b. P. 57.

1. £415. 4*s.* 1$\frac{1}{2}$*d.*
2. £3073. 12*s.* 1$\frac{1}{2}$*d.*
3. £12030. 2*s.* 1$\frac{1}{2}$*d.*
4. £78534. 16*s.* 9$\frac{1}{2}$*d.*
5. £26861. 7*s.* 8*d.*
6. £18891. 7*s.* 0*d.*
7. £23352. 3*s.* 6$\frac{3}{4}$*d.*
8. £4081. 4*s.* 0$\frac{3}{4}$*d.*
9. £37. 11*s.* 1*d.*
10. £573. 10*s.* 3$\frac{1}{2}$*d.*
11. £68. 16*s.* 0*d.*
12. £106. 17*s.* 1$\frac{1}{2}$*d.*
13. £38. 8*s.* 11$\frac{1}{2}$*d.*
14. £467. 12*s.* 6*d.*, 18705.
15. £119. 0*s.* 2*d.*
16. £4557. 14*s.* 0*d.*

Exercises XIX. a. P. 59.

1. (*a*) 3$\frac{1}{4}$*d.*, (*b*) 9$\frac{1}{4}$*d.*, (*c*) 1*s.* 9$\frac{1}{4}$*d.*, (*d*) 3*s.* 5$\frac{1}{4}$*d.*, (*e*) 6*s.* 9$\frac{1}{4}$*d.*, (*f*) 10*s.* 0$\frac{1}{4}$*d.*
2. (*a*) 1*s.* 1$\frac{1}{2}$*d.*, (*b*) 1*s.* 7$\frac{1}{2}$*d.*, (*c*) 9*s.* 1$\frac{1}{2}$*d.*, (*d*) 14*s.* 3$\frac{1}{2}$*d.*, (*e*) 16*s.* 7$\frac{1}{2}$*d.*, (*f*) 20*s.* 1$\frac{1}{2}$*d.*
3. (*a*) 1*s.* 3*d.*, (*b*) 4*s.* 9*d.*, (*c*) 7*s.* 3*d.*, (*d*) 12*s.* 11*d.*, (*e*) 17*s.* 3*d.*
4. (*a*) 4*s.* 8*d.*, (*b*) 6*s.* 8*d.*, (*c*) 7*s.* 8*d.*
5. (*a*) 5*s.* 4$\frac{1}{2}$*d.*, (*b*) 8*s.* 2*d.*, (*c*) 12*s.* 4$\frac{3}{4}$*d.*, (*d*) 18*s.* 10$\frac{1}{4}$*d.*

Exercises XIX. b. P. 59.

1. (*a*) £8. 1*s*. 10*d*., (*b*) £8. 19*s*. 8$\frac{3}{8}$*d*., (*c*) £1. 12*s*. 7*d*., (*d*) £19. 1*s*. 11$\frac{1}{2}$*d*.
2. (*a*) £1907. 12*s*. 9*d*., (*b*) £2427. 16*s*. 8*d*., (*c*) £346. 19*s*. 2*d*., (*d*) £10528. 1*s*. 9*d*.
3. (*a*) £12824. 18*s*. 0$\frac{3}{4}$*d*., (*b*) £9154. 7*s*. 10*d*., (*c*) £1519. 19*s*. 2$\frac{3}{4}$*d*., (*d*) £63490. 8*s*. 9*d*.
4. (*a*) £2425. 18*s*. 7$\frac{3}{4}$*d*., (*b*) £68547. 11*s*. 6*d*., (*c*) £22463. 16*s*. 4$\frac{1}{2}$*d*. (*d*) £471941. 2*s*. 8*d*.
5. (*a*) £99864. 19*s*. 10$\frac{1}{2}$*d*., (*b*) £310313. 1*s*. 10$\frac{1}{2}$*d*., (*c*) £66928. 1*s*. 10$\frac{1}{2}$*d*., (*d*) £191125. 12*s*. 10*d*.
6. £59. 17*s*. 11*d*. 7. 77. 8. £54. 15*s*. 7*d*. 9. £4. 2*s*. 0*d*. gain.
10. 6517. 11. 18*s*. 2*d*. 12. 24. 13. £166. 4*s*. 9*d*.
14. The first is better by £139. 10*s*. 11*d*. 15. 4*s*. 7*d*. in favour of payee.

Exercises XX. a. P. 62.

1. 3*s*. 9$\frac{1}{2}$*d*., 4*s*. 9$\frac{3}{4}$*d*., 5*s*. 8$\frac{1}{4}$*d*., 8*s*. 2*d*., 9*s*. 0$\frac{1}{2}$*d*., 32*s*. 8*d*., 36*s*. 9*d*. 47*s*. 3*d*., £3. 2*s*. 8$\frac{1}{2}$*d*.
2. 2*s*. 6*d*., 3*s*. 8*d*., 5*s*. 10*d*., 6*s*. 4*d*., 10*s*. 4*d*., 19*s*. 4*d*., 31*s*. 10*a*., 65*s*. 76*s*. 8*d*., 109*s*.
3. 2*s*. 3*d*., 3*s*. 6*d*., 4*s*. 9*d*., 5*s*. 6*d*., 8*s*. 3*d*., 9*s*. 9*d*., 13*s*. 3*d*., 27*s*. 6*d*. £11. 18*s*. 0*d*., £12. 4*s*. 9*d*.
4. 2*s*. 11$\frac{3}{4}$*d*., 9*s*. 2$\frac{1}{2}$*d*., 10*s*. 0$\frac{1}{4}$*d*., 11*s*. 4$\frac{1}{2}$*d*., 16*s*. 6$\frac{1}{4}$*d*., 22*s*. 2$\frac{1}{2}$*d*., 29*s*. 3*d*. 44*s*. 5*d*., 51*s*. 5$\frac{1}{2}$*d*., 114*s*. 10*d*., 129*s*. 8$\frac{3}{4}$*d*.
5. 27*s*. 6*d*., 31*s*. 9*d*., 51*s*. 7$\frac{1}{2}$*d*., 58*s*., 59*s*. 9*d*., 86*s*. 7$\frac{1}{2}$*d*., 111*s*. 9*d*. 123*s*., 145*s*. 6*d*., 154*s*. 6*d*., 237*s*.

Exercises XX. b. P. 63.

1. £440. 19*s*. 3*d*., £587. 19*s*. 0*d*., £734. 18*s*. 9*d*., £881. 18*s*. 6*d*.
2. £2506. 6*s*. 6$\frac{3}{4}$*d*., £3222. 8*s*. 5$\frac{1}{4}$*d*., £3938. 10*s*. 3$\frac{3}{4}$*d*., £1790. 4*s*. 8$\frac{1}{4}$*d*
3. £797. 7*s*. 8*d*., £647. 17*s*. 5$\frac{3}{4}$*d*., £946. 17*s*. 10$\frac{1}{4}$*d*., £697. 14*s*. 2$\frac{1}{2}$*d*.
4. £14327. 17*s*. 0*d*., £9551. 18*s*. 0*d*., £11939. 17*s*. 6*d*., £13531. 17*s*. 2*d*
5. £9281. 1*s*. 1$\frac{1}{4}$*d*., £8437. 6*s*. 5$\frac{1}{2}$*d*., £12655. 19*s*. 8$\frac{1}{4}$*d*., £16030. 18*s*. 3$\frac{1}{4}$*d*
6. £146295. 1*s*. 0*d*., £219442. 11*s*. 6*d*., £292590. 2*s*. 0*d*., £365737. 12*s*. 6*d*
7. £13951. 5*s*. 6*d*., £15501. 8*s*. 4*d*., £16276. 9*s*. 9*d*., £24802. 5*s*. 4*d*. £24220. 19*s*. 3$\frac{1}{4}$*d*.
8. £52500. 12*s*. 0*d*., £65625. 15*s*. 0*d*., £70000. 16*s*. 0*d*., £28711. 5*s*. 3$\frac{3}{4}$*d*.

9. £181128. 13*s.* 3¾*d.*, £466869. 11*s.* 11¼*d.*, £378995. 5*s.* 10½*d.*, £597186. 11*s.* 9¾*d.*
10. £585240. 19*s.* 6*d.*, £401244. 15*s.* 2¼*d.*, £538687. 14*s.* 3¾*d.*, £756675. 4*s.* 0*d.*

Exercises XX. c. P. 63.

1. £36. 19*s.* 0*d.*
2. £5. 12*s.* 6*d.*
3. £1148. 19*s.* 11*d.*
4. £284. 16*s.* 11*d.*
5. £112. 13*s.* 1½*d.*
6. £1147. 13*s.* 1½*d.*
7. £265. 3*s.* 3*d.*
8. £174. 3*s.* 0*d.*
9. £75. 5*s.* 7½*d.*
10. £473. 9*s.* 3*d.*
11. £100.
12. £9036. 11*s.* 3*d.*
13. £10570. 8*s.* 9*d.*
14. £1. 10*s.* 5*d.*
15. £28427. 10*s.* 8*d.*
16. £1141. 19*s.* 0*d.*
17. £17. 6*s.* 7*d.*
18. £22. 12*s.* 8*d.*
19. £306. 15*s.* 11*d.*
20. £37. 2*s.* 6*d.*

Exercises XXI. a. P. 67.

1. 1*s.* 2*d.*, 8*d.*, 7*d.*, 3½*d.*; 1*s.* 8*d.*, 1*s.* 4*d.*, 10*d.*, 8*d.*, 2*d.*
2. 10*d.*, 3*d.*; 1*s.* 9*d.*, 1*s.* 3*d.*, 5*d.*
3. 3*s.* 1½*d.*, 2*s.* 6*d.*, 2*s.* 1*d.*, 1*s.* 6¾*d.*, 10*d.*; 4*s.* 7*d.*, 3*s.* 8*d.*, 1*s.* 8*d.*, 10*d.*
4. 9*s.* 8*d.*, 6*s.* 0½*d.*, 3*s.* 0¼*d.*; 13*s.* 8¼*d.*, 9*s.* 1½*d.*, 6*s.* 1*d.*
5. £3. 6*s.* 10½*d.*, £2. 13*s.* 6*d.*, £2. 4*s.* 7*d.*, £1. 18*s.* 2½*d.* and ½*d.* over, £1. 13*s.* 5¼*d.*, £1. 9*s.* 8½*d.* and 1½*d.* over, £1. 2*s.* 3½*d.*
6. 8, 10, 28, 38, 70.
7. 4 and 4*d.* over, 7 and 4*d.* over, 9, 12 and 4*d.* over, 15 and 6*d.* over, 18.
8. £3. 18*s.* 11*d.*, £2. 16*s.* 4¼*d.* and 1¼*d.* over, £2. 3*s.* 10*d.* and 1*d.* over, £1. 15*s.* 10¼*d.* and 2¼*d.* over.
9. £36. 3*s.* 2½*d.* and 1*d.* over, £27. 2*s.* 5*d.*, £21. 13*s.* 11*d.* and 2*d.* over, £18. 1*s.* 7¼*d.* and 1*d.* over.

Exercises XXI. b. P. 67.

1. £43. 7*s.* 2¼*d.*, £38. 10*s.* 10*d.*, £33. 0*s.* 8½*d.* and 1½*d.* over, £28. 18*s.* 1½*d.*
2. £229. 15*s.* 3½*d.* and 4*d.* over, £163. 7*s.* 9*d.* and 11*d.* over, £141. 7*s.* 10½*d.* and 2*d.* over, £131. 5*s.* 10½*d.* and 8*d.* over.
3. £363. 3*s.* 10¼*d.* and 1*s.* 5¾*d.* over, £306. 8*s.* 10½*d.* and 1*s.* 8*d.* over, £280. 3*s.* 6½*d.* and 1*s.* 9½*d.* over, £262. 13*s.* 4*d.* and 4*d.* over.
4. £38. 7*s.* 0½*d.* and ¾*d.* over. 5. £83. 14*s.* 7½*d.* 6. £209. 15*s.* 8¾*d.*
7. £437. 18*s.* 2¼*d.* 8. £790. 0*s.* 9¾*d.* 9. £8063. 15*s.* 0½*d.*

10. £365. 13*s.* 6*d.*, undivided 5*s.* 6*d.*
11. £259. 14*s.* 9½*d.*, remainder 1*s.* 5½*d.* 12. 69 times. 13. 8 years.
14. Man, £3. 1*s.* 4*d.*; boy, £1. 10*s.* 8*d.* 15. 9*s.* 7½*d.* 16. 78.
17. 34 sums and 16*s.* 8*d.* remaining. 18. 108 children, 3*s.* 9*d.* remaining.
19. 235 coins. 20. 3054 rupees. 21. 58 of each.
22. (*a*) 4½*d.*, ¾*d.*; (*b*) 8*s.* 4*d.*, 1*s.* 11½*d.*, 3½*d.*; (*c*) 16*s.* 8*d.*, 3*s.* 10½*d.*, 6¾*d.*
(*d*) £4. 3*s.* 4*d.*, 19*s.* 2¾*d.*, 2*s.* 9*d.*; (*e*) £8. 6*s.* 8*d.*, £1. 18. 5½*d.*
5*s.* 5¾*d.*; (*f*) £41. 13*s.* 4*d.*, £9. 12*s.* 3¾*d.*, £1. 7*s.* 4¾*d.*
(*g*) £83. 6*s.* 8*d.*, £19. 4*s.* 7½*d.*, £2. 14*s.* 9½*d.*

Exercises XXII. P. 69.

1. £16. 11*s.* 6½*d.* 2. £1. 14*s.* 6*d.* 3. 270.
4. £149. 10*s.* 4*d.* 5. £29. 8*s.* 6*d.* 6. 840 lb.
7. £27. 0*s.* 9*d.* 8. £37. 7*s.* 3*d.* 9. £6. 5*s.* 0*d.*
10. £109. 4*s.* 0*d.* 11. 9+48 or 57. 12. £75. 11*s.* 3*d.*
13. £201. 4*s.* 6*d.* 14. 1156 yd. 15. £1. 18*s.* 9*d.*
16. 15048 dollars. 17. 1344 lb. 18. 2*s.* 6*d.* a yd.
19. 1st method by £1. 13*s.* 9*d.* 20. £1. 0*s.* 10*d.* gain.
21. £3. 14*s.* 9*d.* 22. Imports, £11. 13*s.* 1½*d.*; exports, £6. 4*s.* 8*d.*
23. £10. 16*s.* 3*d.* 24. £730. 16*s.* 0*d.* 25. £962.
26. £68. 6*s.* 5*d.*, £94. 13*s.* 7*d.*, £87.
27. £242. 7*s.* 11*d.* collected and £257. 12*s.* 1*d.* short.
28. £1. 2*s.* 6*d.* 29. 3½*d.* 30. £2. 6*s.* 10*d.*
31. £3. 7*s.* 11*d.* 32. £3. 3*s.* 4*d.* 33. £1. 2*s.* 0*d.*
34. £65. 7*s.* 6*d.* or 523 half-crowns. 35. 10.
36. 66 payments, £62. 7*s.* 6*d.* left over.
37. 11 payments, £43. 13*s.* 9*d.* left. 38. 3008 men.
39. 345, £711. 11*s.* 3*d.* 40. £3. 1*s.* 9*d.*
41. £6. 6*s.* 6*d.* each, £1. 10*s.* 0*d.* left.
42. £108. 4*s.* 2¾*d.* 43. £2. 7*s.* 3*d.* 44. 516.

Exercises XXIII. a. P. 76.

1. (*a*) 1 ft. 4 in., (*b*) 2 ft. 2 in., (*c*) 3 ft., (*d*) 4 ft. 2 in., (*e*) 8 ft. 4 in.,
(*f*) 16 ft. 8 in.
2. (*a*) 29, (*b*) 63, (*c*) 360, (*d*) 312, (*e*) 3600, (*f*) 364.
3. (*a*) 5280, (*b*) 660, (*c*) 1320, (*d*) 7920, (*e*) 660.

Exercises XXIII. c. P. 77.

1. 63360. 2. 5280. 3. 30.
4. 210 yd. 2 ft. 5. 7 m. 271 yd. 1 ft. 8 in. 6. 1500.
7. 96 yd. 2 ft. 4 in. 8. 8 m. 1593 yd. 1 ft. 9. 266 yd. 2 ft.
10. 5 m. 867 yd. 1 ft. 11. 261 ft. 12. 25416.
13. 22 ft. 6 in. 14. 41 yd. 1 ft. 0 in. 15. 11 m. 440 yd.

Exercises XXIV. P. 81.

1. 10 yds. 1 ft. 11 in. 2. 2 m. 1125 yd. 3. 190.
4. 295. 5. 105 yd. 1 ft. 8 in. 6. 678 yd. 8 in.
7. 9 yd. 0 ft. 7 in. 8. 252 yd. 2 ft. 9 in. 9. 107 yd. 1 ft.
10. 2 m. 1361 yd. 1 ft. 11. 3 m. 1608 yd.
12. (*a*) 366 ft. 0 in.; (*b*) 304 ft. 10 in.; (*c*) 198 ft.; (*d*) 80 ft. 1 in.;
(*e*) 6 ft. 0 in., 3 ft. 0 in., 1 ft. 0 in., 2 ft. 0 in., 0 ft. 6 in.,
17 ft. 6 in., 6 ft. 0 in., 1 ft. 0 in., 23 ft. 1 in., 19 ft. 9 in.,
73 ft. 0 in., 49 ft. 9 in., 0 ft. 10 in., 7 ft. 0 in., 40 ft. 0 in.;
(*f*) 35 ft. 0 in., 218 ft. 6 in., 10 ft. 6 in., 40 ft. 0 in., 2 ft. 0 in.,
1 ft. 3 in., 3 ft. 0 in., 10 ft. 11 in., 7 ft. 7 in., 1 ft. 6 in.,
17 ft. 0 in., 71 ft. 2 in., 12 ft. 6 in., 49 ft. 11 in., 8 ft. 0 in.,
11 ft. 6 in., 1 ft. 0 in., 37 ft. 0 in., 63 ft. 4 in.

Exercises XXVI. b. P. 86.

(A) 1. 60 mm. 2. 600 mm. 3. 6000 mm. 4. 6060 mm.
5. 6600 mm. 6. 6660 mm. 7. 7300. 8. 7003.
9. 303. 10. 52. 11. 18000. 12. 1010.
13. 1120. 14. 6020. 15. 6002. 16. 4200.
17. 4320. 18. 6006. 19. 7080. 20. 8020.
21. 8032. 22. 9091. 23. 2029. 24. 238.
25. 16803. 26. 5302. 27. 9027.

(B) 1. 7 cm. 6 mm. 2. 7 dm. 6 cm. 3. 7 dm. 6 mm.
4. 8 m. 2 dm. 6 cm. 5. 9 dm. 8 cm. 6. 5 m. 3 dm. 9 mm.
7. 23 m. 4 dm. 8. 9 m. 1 dm. 2 mm. 9. 9 dm. 1 cm. 2 mm.
10. 9 m. 1 cm. 2 mm. 11. 8 m. 7 dm. 1 cm. 6 mm.
12. 8 m. 7 dm. 6 cm. 13. 8 m. 7 dm.
14. 9 dm. 6 cm. 2 mm. 15. 83 m. 4 dm. 5 cm.

16. 8 m. 3 dm. 4 cm. 5 mm.
17. 8 m. 4 cm. 6 mm.
18. 7 m. 6 dm. 2 cm. 3 mm.
19. 9 m. 2 dm. 6 cm.
20. 1 m. 7 dm. 6 cm. 3 mm.

(c) 1. 4 dm. 2. 8 mm. 3. 3 m. 4. 1 decametre. 5. 2 dm.
6. 3 cm. 7. 2 dm. 8. 8 dm. 9. 7 cm. 10. 30 m.
11. 8 m. 12. 3 dm. 13. 70 m. 14. 6 dm. 15. 70 m.
16. 5 cm. 17. 1 m. 18. 10 m. 19. 1 dm. 20. 70 m.
21. 7 m. 22. 2 cm.

Exercises XXVII. P. 87.

	m.	dm.	cm.	mm.		m.	dm.	cm.	mm.		m.	dm.	cm.	mm.
1.	0	0	2	0	2.	0	2	0	0	3.	2	0	0	0
4.	1	2	0	0	5.	2	3	0	0	6.	20	5	0	0
7.	44	0	0	0	8.	44	0	0	0	9.	4	0	4	0
10.	23	2	0	0	11.	1	7	0	0	12.	34	5	0	0
13.	131	3	0	0	14.	0	1	0	0	15.	0	0	1	0
16.	0	0	0	1	17.	0	0	3	2	18.	0	2	3	3
19.	0	2	3	0	20.	0	2	0	3	21.	0	2	3	1
22.	10	6	1	2	23.	1	0	6	7	24.	2	4	1	3
25.	5	7	4	0	26.	8	8	8	4	27.	0	7	0	6

Exercises XXVIII. P. 89.

1. 3·28 metres. 2. (*a*) 4·16 dm., (*b*) 41·6 cm., (*c*) 416 mm.
3. (*a*) 70·25 dm., (*b*) 702·5 cm., (*c*) 7025 mm. 4. 14·069 m.
5. 0·296 m. 6. 13·236 m.
7. (*a*) 847·5 dm., (*b*) 8475 cm., (*c*) 84750 mm. 8. 71·8.

9. 7·1.	10. 0·7.	11. 31·7.	12. 3·02.	13. 6·2.
14. 3·202.	15. 4·003.	16. 37·03.	17. 20·02.	18. 300·4.
19. 230·2.	20. 3·6.	21. 412.	22. 1·8.	23. 4160.
24. 6020.	25. 60·2.	26. 61·24.	27. 1·2.	28. 412.
29. 3·024.	30. 1260.	31. 4 cm.	32. 2 mm.	33. 300.
34. 3000.	35. 4.	36. 50.	37. 300.	38. 0·1.
39. 0·001.	40. 2.	41. 100.	42. 100.	43. 500.
44. 1000.	45. 1000.	46. 20.	47. 200.	48. 250.
49. 4000.	50. 200.			

Exercises XXIX. P. 92.

	m.	dm.	cm.	mm.		m.	dm.	cm.	mm.		m.	dm.	cm.	mm.
1.	6	6	6	5	**2.**	15	7	7	9	**3.**	29	4	4	6
4.	23	2	6	5	**5.**	38	2	5	6	**6.**	26	1	4	8
7.	137	0	2	6	**8.**	51	8	2	1	**9.**	35	1	8	6
10.	40	1	4	7	**11.**	81	0	8	6	**12.**	84	3	0	7
13.	89	2	7	1	**14.**	20	1	5	5	**15.**	7	9	0	8
16.	0	7	2	0	**17.**	1	2	3	7	**18.**	4	3	7	0
19.	76	6	2	9	**20.**	96	3	1	3	**21.**	2	8	0	0
22.	5	7	6	5	**23.**	20	0	6	8	**24.**	3	7	7	9
25.	1	5	3	7	**26.**	2	4	0	1	**27.**	0	2	7	0
28.	4	5	8	0	**29.**	10	5	6	8	**30.**	3	7	7	2
31.	0	9	4	4	**32.**	0	9	0	9	**33.**	5	2	1	1
34.	6	5	8	9										

35. 41·161. **36.** 38·55 tons. **37.** 73·62 ft. **38.** 9·11 ft.

39. 28·17 gallons.

40. (i) 22·424, (ii) 506·309, (iii) 430·08, (iv) 191·851, (v) 367·537, (vi) 191·749, (vii) 522·879, (viii) 5·21, (ix) 70·182, (x) 47·261, (xi) 185·78, (xii) 99·91, (xiii) 15·28, (xiv) 100·84, (xv) 22·83, (xvi) 151·717, (xvii) 94·075, (xviii) 189·27, (xix) 108·723.

Exercises XXX. P. 96.

1. 65 m. 8 dm. 7 cm. 2 mm.

	m.	dm.	cm.	mm.		m.	dm.	cm.	mm.		m.	dm.	cm.	mm.
2. (i)	52	0	2	4	(ii)	110	9	2	5	(iii)	90	5	9	6
(iv)	86	3	8	2	(v)	1	5	0	4	(vi)	26	0	4	3
(vii)	96	2	6	4	(viii)	69	0	0	0	(ix)	73	0	0	0
(x)	47	1	0	0	(xi)	33	5	0	0	(xii)	121	5	3	6

3. 6 m. 2 dm. 0 cm. 6 mm.

	m.	dm.	cm.	mm.		m.	dm.	cm.	mm.		m.	dm.	cm.	mm.
4. (i)	0	5	2	7	(ii)	1	5	9	5	(iii)	0	3	4	9
(iv)	0	8	9	6	(v)	0	8	2	0	(vi)	2	3	7	7
(vii)	0	9	9	3	(viii)	1	1	8	9	(ix)	0	3	2	8
(x)	4	0	5	7	(xi)	0	7	5	0	(xii)	7	0	7	8
(xiii)	4	1	4	1	(xiv)	0	4	1	5	(xv)	0	7	4	8

5. 1·35 ft., 16·40 ft.

6. (i) (*a*) 26·883, (*b*) 44·805, (*c*) 62·727, (*d*) 80·649;
(ii) (*a*) 0·125, (*b*) 0·250, (*c*) 0·375, (*d*) 0·5;
(iii) (*a*) 98·112, (*b*) 126·144, (*c*) 112·128;
(iv) (*a*) 12·648, (*b*) 9·486, (*c*) 22·134, (*d*) 28·458.

7. (i) (*a*) 3·8, (*b*) 1·9, (*c*) 0·95; (ii) (*a*) 0·0207, (*b*) 0·0069;
(iii) (*a*) 4·543, (*b*) 1·947; (iv) (*a*) 124·875, (*b*) 208·125, (*c*) 69·375;
(v) (*a*) 0·5, (*b*) 0·25, (*c*) 0·125; (vi) (*a*) 0·75, (*b*) 0·375;
(vii) (*a*) 102·816, (*b*) 137·088, (*c*) 34·272;
(viii) (*a*) 0·0008, (*b*) 0·0006, (*c*) 0·0004, (*d*) 0·0003, (*e*) 0·0002;
(ix) (*a*) 0·20594, (*b*) 0·10297; (x) (*a*) 18·0012, (*b*) 9·0006, (*c*) 4·5003.
(xi) (*a*) 1·44, (*b*) 0·6, (*c*) 0·9, (*d*) 0·8.

8. (*a*) £0·7878, (*b*) 3·054 lb., (*c*) 28·4691 ft.

9. £1·776. **10.** 9·98 lb.

Exercises XXXI. a. P. 99.

1. (*a*) 4000, 40, 400; (*b*) 9000, 9, 0·9, 150, 18; (*c*) 100, 50, 5, 2, 3000, 300; (*d*) 40, 4, 4000, 56, 720; (*e*) 0·5, 5·0, 15·0, 0·0005; (*f*) 3·72, 0·0372, 0·372; (*g*) 0·48, 1·44, 14·4, 0·144.

2. 1·284 m. **3.** 5·824 m. **4.** 151·36 m., 227·04 m.

Exercises XXXI. b. P. 101.

1. 694·375 lb. **2.** 10 lb. **3.** 64·1875 lb.
4. 482·5 lb. **5.** £355·25. **6.** 242·52 ton.
7. 3223·68 m. **8.** 693 francs. **9.** £174·8565.
10. 21·33859 lb. **11.** 113·03859 grains.
12. (*a*) 88·375 francs, (*b*) 338·855 francs. **13.** 40·586 lb.
14. 0·4192 c. ft. **15.** 0·11616 c. ft. **16.** 6·72 c. ft.
17. (*a*) 16 ft. 4·85 in., (*b*) 9 ft. 10·8974 in., (*c*) 10 ft. 5·984 in., (*d*) 9 ft. 10·18874 in., (*e*) 118 ft. 1·43811 in., (*f*) 2 ft. 6·834584 in., (*g*) 3 ft. 0·149534 in., (*h*) 3 ft. 4·011731 in., (*i*) 7 ft. 9·7006 in., (*j*) 10·657459 in., (*k*) 1 ft. 11·169245 in., (*l*) 3·984244 in.

Exercises XXXI. c. P. 102.

1. (*a*) 5539·38, (*b*) 553·938.

2. (*a*) 6·788396, (*b*) 67883·96, (*c*) 67·88396, (*d*) 2518·8522, (*e*) 251·88522, (*f*) 283147·57, (*g*) 2831·4757.

3. (*a*) 8·19434, (*b*) 81·9434, (*c*) 819·434, (*d*) 55·165, (*e*) 551·65.
4. (*a*) 37807·5, (*b*) 378·075, (*c*) 3·78075, (*d*) 1397·3652, (*e*) 139·73652.
5. (*a*) 41499, (*b*) 414·99, (*c*) 4149·9, (*d*) 78300, (*e*) 7830, (*f*) 0·783.
6. (*a*) 8720, (*b*) 872, (*c*) 0·872, (*d*) 117720, (*e*) 1177·2, (*f*) 11772.
7. (*a*) 19109·275, (*b*) 1910·9275, (*c*) 191·09275, (*d*) 1·9109275, (*e*) 1618·668, (*f*) 16·18668.
8. (*a*) 5824·7424, (*b*) 592·704, (*c*) 0·614125, (*d*) 5·359375.
9. (*a*) 254·7216, (*b*) 6·0025, (*c*) 0·0027648.
10. (*a*) 104·04, (*b*) 1092·727, (*c*) 10100·25, (*d*) 250100·01.
11. (*a*) 803174·44, (*b*) 65·69202, (*c*) 893·62.
12. (*a*) 999998·56, (*b*) 9999·9375, (*c*) 6399·99.

Exercises XXXI. d. P. 103.

1. 1·8. **2.** 1·6. **3.** 500. **4.** 0·00006. **5.** 4·9. **6.** 0·064.
7. 0·000004. **8.** 640. **9.** 1.

Exercises XXXI. e. P. 104.

5. (*b*) (i) 0·25, (ii) 0·40, (iii) 0·51, (iv) 0·76, (v) 0·91.
(*c*) (i) 0·39, (ii) 0·71, (iii) 0·36, (iv) 0·59, (v) 0·83.
9. (*a*) 0·8, (*b*) 2·43, (*c*) 0·13, (*d*) 0·27, (*e*) 1·26.

Exercises XXXII. a. P. 109.

1. (*a*) 10, (*b*) 0·8, (*c*) 5, (*d*) 0·2. **2.** (*a*) 3·2, (*b*) 0·32, (*c*) 20, (*d*) 200.
3. (*a*) 3, (*b*) 40, (*c*) 0·4, (*d*) 80, (*e*) 0·08, (*f*) 0·6.
4. (*a*) 40, (*b*) 4, (*c*) 0·4, (*d*) 400, (*e*) 2000, (*f*) 0·3.
5. (*a*) 7·5, (*b*) 0·75, (*c*) 0·4, (*d*) 6, (*e*) 60, (*f*) 50, (*g*) 0·05.
6. (*a*) 6, (*b*) 0·6, (*c*) 0·8, (*d*) 4, (*e*) 400, (*f*) 20, (*g*) 2, (*h*) 0·2.
7. (*a*) 16, (*b*) 10, (*c*) 8, (*d*) 0·6, (*e*) 6, (*f*) 0·3.
8. (*a*) 3, (*b*) 0·1, (*c*) 8, (*d*) 10, (*e*) 9, (*f*) 30, (*g*) 0·2, (*h*) 2.
9. (*a*) 1·6, (*b*) 0·1, (*c*) 0·08, (*d*) 0·8, (*e*) 0·02, (*f*) 2, (*g*) 0·4.
10. (*a*) 0·8, (*b*) 12, (*c*) 60, (*d*) 3, (*e*) 0·2, (*f*) 0·02, (*g*) 16, (*h*) 160.
11. (*a*) 20, (*b*) 200, (*c*) 16. **12.** (*a*) 6·25, (*b*) 9, (*c*) 20.
13. (*a*) 8, (*b*) 4, (*c*) 16, (*d*) 32. **14.** (*a*) 0·5, (*b*) 15, (*c*) 25.

Exercises XXXII. b. P. 110.

1. 7·76. **2.** 26·923 c. in. **3.** 1·0936 and 0·9144.
4. 163·48. **5.** 43 and 1·506 pints. **6.** 9 rods and 5·67 in. over.
7. 2 m. and 5·26 in. over. **8.** 2·34 approx. **9.** 13. **10.** 8.
11. (*a*) 47, (*b*) 47, (*c*) 0·0047, (*d*) 0·00047, (*e*) 470.
12. (*a*) 9·3, (*b*) 9·3, (*c*) 0·093, (*d*) 0·000093, (*e*) 930000.
13. (*a*) 840, (*b*) 0·84, (*c*) 840, (*d*) 0·000084, (*e*) 0·84.
14. (*a*) 623, (*b*) 0·000623, (*c*) 0·00623, (*d*) 623, (*e*) 0·0623.
15. (*a*) 735·6, (*b*) 7356, (*c*) 73560000, (*d*) 0·0000007356, (*e*) 0·00007356.
16. (*a*) 95, (*b*) 0·095, (*c*) 0·095, (*d*) 9·5, (*e*) 0·00095.
17. (*a*) 795·06, (*b*) 7950·6, (*c*) 0·000079506, (*d*) 0·79506.
18. (*a*) 0·040075, (*b*) 0·040075, (*c*) 0·40075, (*d*) 4007·5, (*e*) 400·75.
19. (*a*) 80·963, (*b*) 0·080963, (*c*) 8·0963, (*d*) 8·0963, (*e*) 80963.
20. (*a*) 4·8596, (*b*) 0·048596, (*c*) 4·8596, (*d*) 0·048596, (*e*) 48596.
21. (*a*) 0·056, (*b*) 5600, (*c*) 0·056, (*d*) 560.
22. (*a*) 708·5, (*b*) 0·07085, (*c*) 0·007085, (*d*) 7085000.
23. (*a*) 753·8, (*b*) 0·07538, (*c*) 75·38, (*d*) 7538, (*e*) 75380.
24. (*a*) 3854·06, (*b*) 0·385406, (*c*) 38·5406, (*d*) 0·0385406, (*e*) 3854·06.
25. (*a*) 0·0973, (*b*) 973, (*c*) 973, (*d*) 97·3, (*e*) 9·73.

Exercises XXXIII. P. 114.

1. 0·28775. **2.** 0·479. **3.** 0·951. **4.** 0·533.
5. 0·412875. **6.** 0·8715. **7.** 0·40075. **8.** 0·847.
9. 26 fur. 0 ch. 80 lks. **10.** 2 fur. 6 ch. 27·2 lks.
11. 8 fur. 4 ch. 96 lks. **12.** 0 fur. 7 ch. 12 lks.
13. 17 fur. 3 ch. 4 lks. **14.** 34 fur. 8 ch. 88 lks.
15. 57 fur. 0 ch. 0 lks. **16.** 3 fur. 0 ch. 0 lks.
17. 0 fur. 0 ch. 16 lks. **18.** 54 fur. 4 ch. 80 lks.
19. 5 days 3 hrs. **20.** 12. **21.** 58·7 mls. per hr.
22. 160·38 yds. **23.** 0·8 miles.
24. (i) 5 ft. 4·4 in., (ii) 5 ft. 0·8 in., (iii) 5 ft. 4 in., (iv) 5 ft. 0 in., (v) 5 ft. 3·2 in., (vi) 4 ft. 9·2 in., (vii) 5 ft. 1·2 in.
25. (i) 671112000, (ii) 8949600, (iii) 66960, (iv) 1040·4, (v) 801.
26. 8 min. 20 sec. **27.** 738 decametres.

Exercises XXXIV. P. 116.

1. 4 sq. in. **2.** 8 sq. cm. **3.** 6 sq. in. **4.** 1804 sq. mm.
5. 12 sq. cm. **6.** 15 sq. cm. **7.** 31 sq. cm. **8.** 120 sq. cm.
9. 27·72 sq. cm. **10.** $a \times b$.

Exercises XXXV. a. P. 120.

1. 11 sq. yd. 4 sq. ft. 80 sq. in. **2.** 27 sq. yd. **3.** 31·4914 sq. cm.
4. 71·2 sq. in. **5.** 65 sq. yd. 3 sq. ft.
6. 39 sq. yd. 7 sq. ft. 136 sq. in. **7.** 2 sq. ft. 104 sq. in.
8. 300 sq. ft. **9.** 359 sq. ft. 48 sq. in. **10.** 100 sq. ft.
11. 1·0124 acres nearly.

Exercises XXXV. b. P. 121.

(i) 2, (ii) 12, (iii) 1, (iv) 25, (v) 36, (vi) 3, (vii) 50, (viii) 24, (ix) 40, (x) 10·5.

1. 350. **2.** 700. **3.** 280. **4.** 4900. **5.** 560. **6.** 350.
7. 350. **8.** 280. **9.** 420. **10.** 1400. **11.** 8400. **12.** 210.

Exercises XXXV. c. P. 122.

1. 1st case, 78 ft.; 2nd case, 90 ft., 30 sq. ft. wasted.
2. 128 ft. **3.** 30 flagstones. **4.** 195 sq. ft. **5.** 400 slates.
6. 936 slates. **7.** 720. **8.** 48 sq. ft. **9.** 572 sq. ft.
11. 8 sq. in. **12.** 744 sq. ft., £18. 12*s.* 0*d.*
13. 540 sq. ft., £13. 10*s.* 0*d.* **14.** 5 sq. ft. **15.** 6 sq. ft., 4*s.* 6*d.*
16. 12 sq. in. **17.** 440 sq. ft. **18.** 220 ft. **19.** £1. 19*s.* 0*d.*

Exercises XXXV. d. P. 125.

1. 1 sq. ft. 18 sq. in. **2.** 88 sq. in. **3.** 52 sq. ft.
4. £42. 12*s.* 6*d.* **5.** 160 yd.
6. (*a*) 1550 sq. in., (*b*) 10·76, (*c*) 1·196 sq. yd.
7. (i) 20 m. 1 dm. 2 cm., (ii) 4048·144 sq. m.
8. 37293 sq. ft. **9.** 976.
10. Carpet, 239·25 sq. ft.; margin, 83·75 sq. ft. **11.** 5·25 m.
12. 3150 cm. **13.** 168 sq. ft., 108 sq. in. **14.** 27 sq. ft. 72 sq. in.
15. (*a*) 414 ft., (*b*) 342 ft., (*c*) 304 ft. **16.** 10 ft.

Exercises XXXVI. P. 128.

1. 155×10^2 sq. m. 2. 620 ac. 3. 1385 sq. m. 4. 227 sq. m.
5. 156 sq. m. 6. 490 sq. m. 7. 275 sq. m. 8. 165 sq. m.

Exercises XXXVII. a. P. 131.

1. (*a*) 23 fr. 20 c., (*b*) 193 fr. 60 c., (*c*) 83 fr. 70 c., (*d*) 9 fr. 36 c., (*e*) 1945 fr. 30 c., (*f*) 876 fr. 20 c., (*g*) 897 fr. 60 c., (*h*) 1 fr. 76·23 c.
2. (*a*) 6 lires 30 c., (*b*) 7 lires 83 c., (*c*) 9 dol. 30 c., (*d*) 3 dol. 75·6 c., (*e*) 8 dol. 21 c., (*f*) 356 drachmas 80 leptas, (*g*) 932 M. 13 pf., (*h*) 13 M. 2 pf.
3. (*a*) (i) 1 dol. 50 c., (ii) 2 dol. 60 c., (iii) 103 dol.;
(*b*) (i) 150 fr., (ii) 120 fr., (iii) 100 fr.; (*c*) 11 dr. 20 leptas;
(*d*) (i) 1824 lires, (ii) 320 lires; (*e*) (i) 515 M., (ii) 101 M. 50 pf.

Exercises XXXVII. b. P. 131.

1. 76 c. 2. 4 fr. 70 c. 3. 131 fr. 10 c.
4. 89 lire 90 centesimos. 5. 307 dol. 20 c. 6. 2353 M. 86 pf.
7. 75 M. 76 pf. (approx). 8. 1200 dol. 15 c. 9. 48384 fr. 0 c.
10. 14 dol. 18 c. 11. 326 fr. 12. 101412 lires 48 c.
13. 3301 dol. 65 c. 14. 7 fr. 60 c. 15. 1 kr. 5 M. 90 pf.
16. 76 kr. 6 M. 71·6 pf. 17. 6 drachmas 80 leptas.

Exercises XXXVIII. a. P. 133.

1. 24 cub. in. 2. (*a*) 12, (*b*) 6, (*c*) 40, (*d*) 432, (*e*) 144.
3. 48, 24, 12, 80, 864, 288. 4. 84, 42, 280, 2924, 1008.
5. (*a*) 192 c. in., (*b*) 72 c. in., (*c*) 576 c. in., (*d*) 360 c. ft., (*e*) 375 c. in., (*f*) 6300 c.c., (*g*) $x \times y \times z$ c.c., (*h*) $a \times b \times c$ c.c.
6. 1728 c. in., 27 c. ft. 7. (*a*) 1000 c.c., (*b*) 1000000 c.c.

Exercises XXXVIII. b. P. 134.

1. 121·5 c. in., 5760. 2. 2835 c. ft. 3. 17600 c.c.
4. 2 c. ft. 1086 c. in. 5. 1 ft. 4 in. 6. 0·025 cm.
7. 207·125 c. in. 8. 14 ft. 9. 13 ft. 8 in.; volume, 1476 c. in.
10. 75. 11. Content, 1800 c. cm.; substance, 1080 c. cm.
12. 3·36 sq. yd. 13. 57200. 14. 180000 c. ft., 11250000 lb.
15. 0·002 ft. 16. 70. 17. 45360 tons. 18. 1 c. ft. 1512 c. in.

Exercises XXXIX. b. P. 140.

1. 2079 bottles. **2.** 7 gal. 3 qt. 1 pt. **3.** 1 c. ft. 5 c. in.
4. 3·33 ft. **5.** 5·12 in. **6.** 72600 gal.
7. 59·319 c. in. **8.** 5·05568 c. in. **9.** 7 pt., 2·16 c. in. over.
10. 26 f℥ 4 f℥ 13 m. **11.** 1 pt. 14 f℥ 4 f℥ 38·4 m.
12. 28 f℥ 3 f℥ 45 m. **13.** A, 0·421 litre; B, 0·239 litre; C, 0·356 litre.
14. 0·864 in.

Exercises XL. b. P. 145.

1. 67 ton 6 cwt. 2 qr. **2.** 7 ton 12 cwt. 3 qr. **3.** 3 qr. 9 lb. 10 oz.
4. (*a*) 482 lb., (*b*) 1988 lb., (*c*) 2173 lb., (*d*) 31433 lb., (*e*) 479 lb., (*f*) 1777944 lb., (*g*) 43601 lb.; sum, 829 ton 10 cwt. 0 qr. 20 lb.
5. (*a*) 84 lb. 1 oz., (*b*) 63 cwt., (*c*) 8 ton 10 cwt. 3 lb., (*d*) 3 qr. 15 lb. 9 oz., (*e*) 27 cwt. 2 qr. 20 lb. 1 oz. 12 dr., (*f*) 86 ton 13 cwt.
6. 1 ton. **7.** 105007 c. ft. **8.** 99750 c. ft.
9. 102144 c. ft. **10.** 382 cwt. 2 qr. **11.** 787·39 ft.
12. 57 ton 8 cwt. 3 qr. 11 lb. 14 oz. **13.** 0·42 cm.
14. 26 weights; 1·92 lb. over. **15.** 40 litres; 0·18 gal. left. **16.** 4204 ft.
17. (*a*) 6720 kilo., (*b*) 19 cwt. 2 qr. 16 lb., (*c*) 245 kilo., (*d*) 3 cwt. 1 qr. 17 lb. 11·2 oz., (*e*) 3 lb. 6·208 oz.
18. 1 ton 17 cwt. 3 qr. 18 lb. 14 oz.

	ton.	cwt.	qr.	lb.	oz.		ton.	cwt.	qr.	lb.	oz.
19. (*a*)	8	14	0	23	4	(*b*)	13	18	2	26	0
(*c*)	27	17	1	24	0	(*d*)	48	15	2	7	0
(*e*)	87	2	0	8	8	(*f*)	365	16	2	24	8
(*g*)	566	3	1	27	4						

20. 1 ton 11 cwt. 0 qr. 3 lb. 8 oz. **21.** 5 lb. 1 oz.

	ton.	cwt.	qr.	lb.	oz.	dr.
22. (*a*)	88	5	1	22	4	8
(*b*)	64	3	3	23	13	7+3
(*c*)	20	3	2	3	7	14+6
(*d*)	7	18	2	21	8	2+14
(*e*)	3	8	2	6	13	8+112
(*f*)	1	18	0	19	3	15+306

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
23. (*a*)	3	9	11	11	(*b*)	5	3	16	1	(*c*)	34	7	12	12
(*d*)	10	0	6	6	(*e*)	17	9	5	15	(*f*)	13	0	15	10

24. (*a*) 14·08 drs., (*b*) 15 oz. 0·64 dr., (*c*) 16 lb. 1 oz. 14·72 dr., (*d*) 49 lb. 0 oz. 12·544 dr.

Exercises XLI. P. 147.

1. 3 min. 56 sec., 23 hr. 55 min. 40 sec. 2. 365 days 5 hr. 48 min. 47 sec.
3. 0·2545, 0·2564, 0·2459, 0·2438. 4. 0 hr. 45 min. approx.
5. 0·7784 days. 6. 0·089 days. 7. 29 days 12 hr. 44 min.
9. (*a*) 1, (*b*) 1·102, (*c*) 1·23, (*d*) 1·38, (*e*) 1·62, (*f*) 2·06, (*g*) 3·36.

Exercises XLII. P. 149.

1. 122·836123 and 77·163877. 2. 9·98. 3. £0. 17*s*. 6*d*.
4. 2050. 5. 8 gal. 3 qt. 1 pt. 6. 5·75. 7. 10 lb. 1 oz.
8. 2220. 9. 59·8. 10. 32. 11. 6*d*.
12. £53. 13*s*. 0*d*. 13. 19·8219661336 and 19·822.
14. 23780000. 15. £10. 16. 3·8 lb.
17. 2061, 1 qr. left. 18. (*a*) £2. 11*s*. 8*d*., (*b*) 0·3614.
19. 45·168. 20. (*a*) £7224. 11*s*. 0*d*.; 40·75.
21. 40 sq. yd. 4 sq. ft. 24 sq. in. 22. 10065, 0·375. 23. 270.
24. (*a*) 0·67, (*b*) 18·70, (*c*) 5·78, (*d*) 19·2, (*e*) 67·225, (*f*) 0·31, (*g*) 14·085, (*h*) 4·115, (*i*) 16·2, (*j*) 67·62.
25. 6·25. 26. 126·72. 27. £121. 0*s*. 4*d*. 28. 5·4198.
29. (*a*) 59·04, 64·8, 40·3, (*b*) 637, 20·9, 22·93, (*c*) 57910, 2085, 1295, (*d*) 300, 9·84, 6·71, (*e*) 30 min., (*f*) 1·41.
30. (*a*) 202, (*b*) 30, 0·3. 31. 30·15 inches of mercury.
32. 115 sq. ft. 72 sq. in. 33. 1134. 34. 846, 0·0046875.
35. £2340. 12*s*. 4½*d*. 36. 54·3.
37. (*a*) 3·58 in., (*b*) 3·17 in. 38. Hull 59°·9, Oxford 61°·3.
39. (*a*) 1·75, (*b*) 3·68, (*c*) 3·94, (*d*) 2·94, (*e*) 2·84, (*f*) 1·68.
40. 22 ft. per sec. 41. 48·6261 sq. in., 313·7055 sq. cm., 6·45.
42. 7 mm. 43. 985·6. 44. 13 ft. 45. £1. 17*s*. 4*d*.
46. 16 hrs. 40 min. 47. 0·279464.
48. 275 sq. ft. 72 sq. in.; £6. 17*s*. 9*d*.
49. 9 yds. 1 ft. 8·1672 in. 50. 50 ton 17 cwt. 3 qr. 12 lb.
51. (*a*) 290, (*b*) 0·25196, (*c*) £0. 17*s*. 3·35*d*. 52. £0. 4*s*. 8*d*.
53. 12 hrs. 5 min., 11 hrs. 56 min.

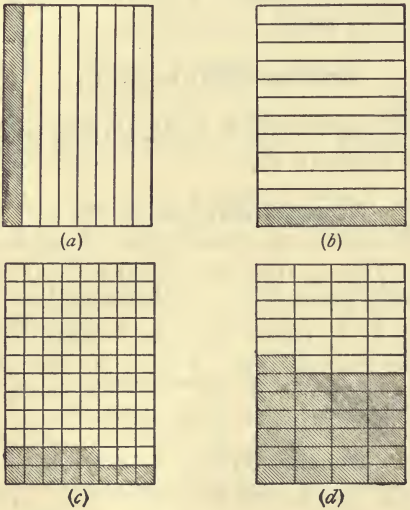
Exercises XLIII. a. P. 158.

1. 4*d*. 2. 8*d*. 3. 9*d*. 4. 12*s*. 6*d*. 5. 1*s*. 9*d*.
6. £1. 12*s*. 0*d*. 7. 8 in. 8. 1 ft. 3 in. 9. 2*s*. 1½*d*.
10. 6 oz. 11. 104 lb. 12. 1 cwt. 68 lb. 13. £2. 4*s*. 0*d*.
14. 1 stone. 15. 78 lb. 16. 90 in. 17. 18*s*. 18. 100 yds.

Exercises XLIII. b. P. 158.

1. 2 cwt. 16 lb. **2.** £6. 16*s.* 6*d.* **3.** £14. 7*s.* 1*d.*
4. 2 ton 10 cwt. 3 qr. 21 lb. **5.** 2 lb. 10 oz. 10 dr.
6. 24 mls. 1 ch. **7.** 63 yd. 2 ft. **8.** 6 m. 5 dm. 3 cm. 2 mm.
9. 2144 gms. **10.** 21 ac. 2 ro. 16 p. **11.** 121. **12.** 11*s.* 4*d.*
13. 48 yd. 2 in. **14.** 100 ton 13 cwt. 87 lb.

Exercises XLIII. c. P. 159.



Exercises XLIV. P. 161.

1. (*a*) $\frac{71}{18}$, (*b*) $\frac{52}{9}$, (*c*) $\frac{291}{16}$, (*d*) $\frac{726}{25}$, (*e*) $\frac{2483}{40}$, (*f*) $\frac{2189}{12}$, (*g*) $\frac{2113}{108}$, (*h*) $\frac{3433}{96}$, (*i*) $\frac{20179}{240}$, (*j*) $\frac{5835}{112}$.

2. (*a*) $14\frac{12}{25}$, (*b*) $27\frac{43}{71}$, (*c*) $16\frac{16}{17}$, (*d*) $23\frac{29}{32}$, (*e*) $6\frac{104}{295}$, (*f*) $62\frac{69}{112}$, (*g*) $5\frac{703}{1760}$, (*h*) $18\frac{575}{841}$, (*i*) $1\frac{101}{144}$, (*j*) $20\frac{15}{36}$.

3. 11*s.* 11*d.* **4.** £2. 5*s.* 0*d.* **5.** 4 ton 8 cwt.
6. 39 yd. **7.** 11 cwt. 2 qr. 12 lb. **8.** £26. 9*s.* $7\frac{1}{4}$*d.*

Exercises XLV. a. P. 163.

1. (*a*) $\frac{4}{20}$, (*b*) $\frac{8}{40}$, (*c*) $\frac{16}{80}$, (*d*) $\frac{48}{240}$, (*e*) $\frac{96}{480}$.

2. (*a*) $\frac{2}{3}$, (*b*) $\frac{3}{4}$, (*c*) $\frac{1}{4}$, (*d*) $\frac{1}{3}$, (*e*) $\frac{1}{2}$, (*f*) $\frac{3}{20}$, (*g*) $\frac{3}{4}$.

3. $\left(\frac{4}{12}\ \frac{15}{45}\ \frac{8}{24}\ \frac{7}{21}\right)$, $\left(\frac{15}{16}\ \frac{75}{80}\ \frac{30}{32}\right)$, $\left(\frac{8}{16}\ \frac{60}{120}\ \frac{7}{14}\right)$, $\left(\frac{22}{24}\right)$.

5. (*a*) $\frac{144}{216}$, $\frac{30}{45}$, $\frac{24}{36}$, $\frac{48}{72}$, $\frac{96}{144}$, $\frac{96}{144}$;
(*b*) $\frac{162}{216}$, $\frac{30}{40}$, $\frac{24}{32}$, $\frac{54}{72}$, $\frac{108}{144}$, $\frac{96}{128}$;
(*c*) $\frac{108}{216}$, $\frac{30}{60}$, $\frac{24}{48}$, $\frac{36}{72}$, $\frac{72}{144}$, $\frac{96}{192}$.

6. (*a*) 21 lb., (*b*) 5 lb., (*c*) 21 lb. **7.** (*a*) 8 in., (*b*) 9 in.

8. £24. **9.** $\frac{2}{11}$ and 48.

Exercises XLV. b. P. 164.

1. $\frac{6}{7}$, $\frac{11}{15}$, $\frac{12}{17}$.

2. (*a*) $\frac{13}{75}$, (*b*) $\frac{15}{27}$, (*c*) $\frac{21}{25}$, (*d*) $\frac{40}{73}$, (*e*) $\frac{27}{38}$, (*f*) $\frac{150}{173}$, (*g*) $\frac{207}{415}$, (*h*) $\frac{49}{50}$, (*i*) $\frac{12}{35}$, (*j*) $\frac{30}{38}$, (*k*) $\frac{4}{17}$.

Exercises XLVI. a. P. 167.

1. 3 cwt. 16 lb., 2 cwt. 91 lb., 5 cwt. 107 lb. **2.** £8. 0*s.* 8*d.*, £$8\frac{1}{30}$.

3. (*a*) $6\frac{23}{24}$, (*b*) $7\frac{25}{36}$, (*c*) $11\frac{17}{48}$, (*d*) $19\frac{7}{9}$, (*e*) $10\frac{7}{72}$, (*f*) $14\frac{83}{120}$, (*g*) $57\frac{141}{192}$, (*h*) $106\frac{59}{240}$, (*i*) $23\frac{16}{51}$, (*j*) $122\frac{31}{60}$.

4. $\frac{8}{15}$, $\frac{7}{18}$, $\frac{7}{36}$, $\frac{1}{4}$. **5.** $\frac{5}{72}$, $\frac{1}{56}$, $\frac{17}{120}$, $\frac{79}{216}$. **6.** $\frac{61}{264}$, $\frac{11}{125}$, $\frac{23}{240}$, $\frac{283}{567}$.

7. $5\frac{175}{192}$, $3\frac{11}{320}$, $22\frac{1}{52}$, $696\frac{77}{104}$. **8.** $\frac{13}{24}$. **9.** $\frac{83}{144}$. **10.** $1\frac{13}{70}$.

11. £6. 1*s.* 1*d.* **12.** £2. 5*s.* $11\frac{1}{2}d.$ **13.** 13 tons 19 cwt. 55 lb.

14. 2 mls. 7 fur. 195 yds. **15.** 30 yds. 1 in.

Exercises XLVII. a. P. 171.

1. (*a*) $\frac{3}{5}$, (*b*) $\frac{3}{50}$, (*c*) $\frac{3}{500}$, (*d*) $\frac{1}{8}$, (*e*) $\frac{3}{4}$, (*f*) $\frac{8}{25}$, (*g*) $\frac{3}{8}$, (*h*) $\frac{11}{20}$, (*i*) $\frac{27}{400}$, (*j*) $\frac{83}{125}$, (*k*) $\frac{7}{8}$, (*l*) $\frac{1}{640}$, (*m*) $7\frac{16}{25}$, (*n*) $590\frac{7}{8}$, (*o*) $\frac{253}{4000}$, (*p*) $\frac{3143}{4000}$, (*q*) $\frac{21}{400}$, (*r*) $\frac{12257}{1250}$, (*s*) $41\frac{3}{16}$, (*t*) $\frac{3}{32}$, (*u*) $\frac{317}{400}$, (*v*) $\frac{31}{8000}$, (*w*) $18\frac{11}{16}$, (*x*) $62\frac{41}{4000}$,

2. (*a*) $\frac{2}{9}$, $\frac{3}{14}$, $\frac{7}{33}$, $\frac{27}{131}$, (*b*) $\frac{6}{8}$, $\frac{17}{24}$, $\frac{25}{36}$, $\frac{8}{12}$, (*c*) $\frac{326}{625}$, $\frac{13}{25}$, $\frac{194}{375}$, (*d*) $\frac{3}{4}$, $\frac{47}{64}$, $\frac{5}{8}$, $\frac{9}{16}$, $\frac{11}{32}$, (*e*) $\frac{4}{15}$, $\frac{7}{27}$, $\frac{2}{9}$, $\frac{29}{135}$.

Exercises XLVII. b. P. 172.

2. 3·66666, 3·63, 3·625, 3·58333. **3.** $5\frac{17}{40}$. **4.** $1\frac{29}{300}$. **5.** $1\frac{48}{100}$.

Exercises XLVIII. a. P. 174.

2. (*a*) $6\frac{1}{4}$, (*b*) $5\frac{1}{4}$, (*c*) $7\frac{1}{2}$, (*d*) $5\frac{1}{2}$, (*e*) $6\frac{1}{8}$, (*f*) 6, (*g*) $3\frac{1}{8}$, (*h*) $5\frac{1}{4}$, (*i*) $4\frac{1}{12}$, (*j*) $5\frac{3}{4}$.

3. (*a*) $\frac{3}{8}$, (*b*) $\frac{5}{8}$. **4.** (*a*) $1\frac{1}{8}$, (*b*) $\frac{3}{8}$. **5.** $1\frac{7}{16}$. **6.** $\frac{4}{32}$.

7. (*a*) $\frac{3}{35}$, (*b*) $\frac{6}{49}$, (*c*) $\frac{4}{50}$, (*d*) $\frac{5}{32}$.

Exercises XLVIII. b. P. 175.

1. $2\frac{7}{24}$. **2.** (*a*) $4\frac{11}{20}$, (*b*) $1\frac{3}{5}$, (*c*) 21.

3. (*a*) $\frac{3}{20}$, (*b*) $\frac{3}{28}$, (*c*) $\frac{19}{240}$, (*d*) $\frac{4}{13}$, (*e*) $\frac{13}{128}$.

Exercises XLIX. a. P. 176.

1. (*a*) $\frac{35}{64}$, (*b*) $\frac{3}{16}$ (*c*) $\frac{18}{64}$, (*d*) $\frac{39}{64}$, (*e*) $\frac{14}{64}$, (*f*) $\frac{4}{64}$, (*g*) $\frac{6}{64}$, (*h*) $\frac{18}{64}$.

2. (*a*) $\frac{63}{100}$, (*b*) $\frac{6}{100}$; (*a*) $\frac{9}{100}$, (*b*) $\frac{49}{100}$. **3.** 28 sq. cm., $\frac{28}{100}$.

Exercises XLIX. b. P. 177.

1. (*a*) $\frac{4}{21}$, (*b*) $\frac{9}{35}$, (*c*) $\frac{17}{88}$, (*d*) $\frac{27}{35}$.

2. (*a*) $\frac{19}{10}$, (*b*) $11\frac{1}{4}$, (*c*) $18\frac{1}{2}$, (*d*) $\frac{65}{84}$.

3. (*a*) $\frac{49}{55}$, (*b*) $\frac{7}{10}$, (*c*) $14\frac{1}{6}$. **4.** (*a*) $3\frac{59}{160}$, (*b*) $\frac{60}{2873}$.

5. (*a*) 240, (*b*) $3\frac{9}{19}$. **6.** $\frac{27}{595}$.

Exercises XLIX. c. P. 177.

2. (*a*) 10, (*b*) 7, (*c*) 6, (*d*) 9, (*e*) 4, (*f*) 4, (*g*) 6, (*h*) 6, (*i*) 10, (*j*) 8.

3. $\frac{12}{7}$. **4.** $\frac{32}{45}$. **5.** (*a*) $\frac{104}{48}$, $\frac{104}{80}$, (*b*) $\frac{66}{70}$, $\frac{66}{14}$. **6.** (*a*) $\frac{15}{13}$, (*b*) $\frac{40}{26}$.

Exercises XLIX. d. P. 178.

1. (*a*) 48 lb., (*b*) 70 lb., (*c*) $\frac{70}{48}$. **2.** $\frac{5}{6}$. **3.** $\frac{20}{27}$. **4.** $2\frac{4}{5}$. **5.** $\frac{25}{16}$.

6. $2\frac{2}{3}$. **7.** 128. **8.** 9. **9.** $\frac{1}{28}$. **10.** $22\frac{1}{2}$. **11.** 20. **12.** 7.

Exercises L. P. 180.

1. (*a*) $\frac{5}{8}$, (*b*) $3\frac{3}{4}$, (*c*) $1\frac{1}{2}$, (*d*) $\frac{9}{80}$, (*e*) $\frac{1052}{}$.

2. (*a*) $\frac{5}{3}$, (*b*) $\frac{4}{9}$, (*c*) $\frac{384}{279}$ or $\frac{128}{93}$, (*d*) $\frac{45}{82}$, (*e*) $\frac{2}{27}$.

3. (*a*) $\frac{2}{2}$, (*b*) $\frac{85}{8}$, (*c*) $2\frac{1}{2}$, (*d*) $\frac{2258}{}$, (*e*) $\frac{17}{16}$.

4. (*a*) $\frac{3}{4}$, (*b*) $\frac{49}{192}$, (*c*) $\frac{1129}{}$, (*d*) $\frac{17923}{}$, (*e*) $\frac{1}{5}$.

Exercises LI. P. 180.

1. $\frac{7}{10}$. 2. $\frac{3}{10}$. 3. $\frac{21}{31}$. 4. $\frac{14}{19}$. 5. $\frac{7}{16}$.
6. $\frac{17}{18}$. 7. $\frac{1}{704}$. 8. $\frac{5}{72}$. 9. $\frac{1}{2}$, $\frac{3}{14}$, $\frac{2}{7}$. 10. 907 tons 4 cwt.
11. $\frac{131}{476}$, $2\frac{39}{476}$. 12. $\frac{5}{9}$. 13. $1\frac{23}{44}$. 14. $\frac{47}{90}$.
15. $\frac{5}{7}$. 16. $1\frac{1}{12}$. 17. (*b*)(*c*)(*a*)(*d*). 18. $1\frac{1}{24}$.
19. (*a*) $5\frac{5}{9}$, (*b*) $1\frac{2}{5}$, (*c*) $2\frac{22}{65}$. 20. (*a*) $3\frac{19}{25}$, (*b*) $1\frac{73}{96}$.
21. (*a*) $1\frac{506}{7389}$, (*b*) $11\frac{2}{3}$. 22. (*a*) 3, (*b*) $13\frac{137}{144}$.
23. $3\frac{13}{240}$, $2\frac{11}{968}$, 2, $2\frac{29}{260}$, $2\frac{21}{32}$, $3\frac{47}{192}$, $3\frac{143}{192}$, $3\frac{181}{192}$, $3\frac{23}{32}$, $3\frac{266557}{818840}$, $3\frac{179}{240}$.
24. (*a*) $\frac{35}{102}$, (*b*) $\frac{115}{527}$. 25. (*a*) $\frac{2}{148859}$, (*b*) $\frac{1}{24}$.
26. (*a*) $\frac{25}{13}$, (*b*) $\frac{347}{2933}$, (*c*) $\frac{5183}{7223}$.

Exercises LII. a. P. 184.

1. (*a*) $5^2\times3^2$, (*b*) $2^2\times3\times11$, (*c*) $11\times3^2\times2^3$, (*d*) $5^3\times11$, (*e*) $3^2\times2^3\times7$, (*f*) 2^{10}, (*g*) 2^8, (*h*) 3×2^7, (*i*) 5×2^6.
2. (*a*) $\frac{4}{15}$, (*b*) $\frac{7}{24}$, (*c*) $\frac{8}{11}$, (*d*) $\frac{111}{212}$, (*e*) $\frac{20}{11}$, (*f*) $\frac{4}{9}$, (*g*) $\frac{725}{55}$, (*h*) $\frac{275}{55}$, (*i*) $\frac{11}{111}$, (*j*) $\frac{1}{101}$, (*k*) $\frac{111}{1111}$, (*l*) $\frac{5}{8}$, (*m*) $\frac{2}{7}$, (*n*) $\frac{7}{13}$, (*o*) $\frac{9}{25}$, (*p*) $\frac{13}{51}$.

Exercises LII. b. P. 185.

1. (*a*) 36, (*b*) 2700, (*c*) 37, (*d*) 144, (*e*) 25, (*f*) 9, (*g*) 96. 2. 4 : 5 : 6.
3. 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
4. 5 and 4536.

Exercises LII. c. P. 187.

1. (*a*) 1·3 ft., (*b*) 1·7, (*c*) 1·4. 2. 1·5 ft. 3. 6 mm., 19 and 14.
4. (*a*) 3, (*b*) 4, (*c*) 5. 5. 713.

Exercises LII. d. P. 189.

1. 6 in., 37. 2. 15*s*. 3. (*a*) 131, (*b*) 131, (*c*) 233, (*d*) 109, (*e*) 25.
4. (*a*) $\frac{11}{15}$, (*b*) $\frac{7}{8}$, (*c*) $\frac{59}{78}$, (*d*) $\frac{169}{154}$, (*e*) $\frac{105}{164}$, (*f*) $\frac{12}{37}$, (*g*) $\frac{16}{33}$, (*h*) $\frac{3}{19}$, (*i*) $\frac{15}{23}$, (*j*) $\frac{4}{5}$, (*k*) $\frac{12}{35}$, (*l*) $\frac{48}{143}$, (*m*) $\frac{37}{73}$, (*n*) $\frac{43}{127}$, (*o*) $\frac{29}{47}$, (*p*) $\frac{79}{151}$.
5. 6 c.c. 6. 429 wts. of 12. 7. A cube of 1·5 in. edge.
8. 8944. 9. 7·6 ft. 10. 326 and 49 loads.
11. 94 in. 12. 32. 13. 24 lb.
14. 96 soldiers, 49 rows. 15. 6*s*. 4*d*. 16. 63 balls, 90 groups.

Exercises LIII. a. P. 192.

1. $3^2 \times 2^2$. 2. $3^5 \times 5^4$. 3. $2^4 \times 5^3 \times 3^2$. 4. $11^2 \times 2^4 \times 5^3 \times 3^3$.
5. $7^4 \times 3^7 \times 5^4 \times 2^3$. 6. $13^2 \times 2^5$. 7. $2^{10} \times 3^4$.
8. $5^3 \times 2^6$. 9. $2^5 \times 7^1 \times 3^2$. 10. $3^4 \times 5 \times 2^4$.

Exercises LIII. b. P. 193.

1. $3^4 \times 5^3 \times 2^4 \times 7$. 2. $7^2 \times 2^6 \times 3^3 \times 5 \times 11$, 4656960.
3. (*a*) 390, (*b*) 19200, (*c*) 444, (*d*) 504.
4. (*a*) $\frac{30}{60}, \frac{20}{60}, \frac{15}{60}, \frac{24}{60}$; (*b*) $\frac{275}{600}, \frac{288}{600}, \frac{90}{600}, \frac{280}{600}$; (*c*) $\frac{385}{1365}, \frac{255}{1365}, \frac{390}{1365}, \frac{819}{1365}$; (*d*) $\frac{420}{1428}, \frac{651}{1428}, \frac{1309}{1428}, \frac{48}{1428}$; (*e*) $\frac{864}{1872}, \frac{832}{1872}, \frac{702}{1872}, \frac{143}{1872}$.
5. (*a*) $\frac{209}{780}, \frac{125}{468}, \frac{52}{195}$; (*b*) $\frac{10}{24}, \frac{32}{77}, \frac{23}{56}, \frac{17}{42}$; (*c*) $\frac{59}{504}, \frac{85}{728}, \frac{109}{936}$.
6. 10*s*. 7. 12 ft. 8. 108 c.c. 9. 4 sq. ft., 16 and 9.
10. 12 sq. ft., 64, 27, 12. 11. $\frac{11}{228}$. 12. $\frac{89}{432}$. 13. $\frac{125}{288}$.
14. $\frac{83}{140}$. 15. $\frac{89}{270}$. 16. $\frac{101}{420}$. 17. $\frac{19}{432}$. 18. $\frac{15}{2023}$.
19. 204. 20. 19. 21. At every $\frac{1}{4}$ of an inch.
22. 3026. 23. 603. 24. 751. 25. 600.
26. 348 lb. 27. £1. 28. £24. 1*s*. 0*d*. 29. 1836.
30. 1120. 31. 93 yds. 1 ft. 32. 19*s*. and 24 fr. 33. 54.
34. 246. 35. 54 and 72. 36. 224. 37. 0·45 inches.
38. (*a*) 7, (*b*) 1, (*c*) 31. 39. 69696. 40. (*a*) 5184, (*b*) 93636.
41. (*a*) 421875, (*b*) 1728, (*c*) 42875, (*d*) 287496.
42. 21472, 502. 43. 225. 44. 13, 34, ..., increasing by 21.

Exercises LIV. P. 202.

1. 25 kilo. 2. 770 lb. 3. 150. 4. 9·9.
5. 60 sec. 6. 66. 7. 95·68 min. 8. 14.
9. 50. 10. 52500 cub. ft. 11. 6*s*. 3*d*. 12. 60,000 lb.
13. £1. 4*s*. $4\frac{1}{2}$*d*. 14. 216. 15. 24 yds. 2 ft. 8 in. 16. £9500.
17. 100·1 sq. ft. 18. 14080 lb. 19. 200. 20. 10*s*. 10*d*.
21. 42. 22. 0·0804384 lb. 23. 309300. 24. 4·09 lb.
25. 350 ft. 26. £1800. 27. £18. 18*s*. 0*d*. 28. £4. 16*s*. 0*d*.
29. 6*s*. 9*d*. 30. 1200 tons. 31. 174 tons. 32. $\frac{1}{26}$ sq. in.
33. 140 ft. 34. 9 cwt. 35. 7 lb. 15 oz. 36. 63 in.
37. 5500 gal. 38. 94·5 lb. 39. 8240 lb.

Exercises LVI. P. 209.

1. 3·729 nearly. **2.** 17 ft. 3·24 in. **3.** 806·4. **4.** 633·024. **5.** 1050.
6. 22 ft. per sec. **7.** 35·325 ft. per sec., 540. **8.** 30. **9.** 69·8.
10. 0·2355 miles. **11.** 0·1168 cu. ft. **12.** 16800 cu. ft.
13. 1 ton 13 cwt. 2·4 qr. **14.** 12·21792 tons, *i.e.* 12 tons 4 cwt. 1·4336 qr.
15. 13300 cu. ft. gas, 2100 lb. coke, 98 lb. tar.
16. 2·1525 kilo. **17.** 2·46 gms. **18.** 0·68 lb.
19. Passengers £172254, goods £179550. **20.** £100, £1375, £825.
21. Sports £30, football £45, cricket £120. **22.** £75, £100, £175.
23. 19·2864 in., 20·0592 in., 9·6544 in.
24. 147150 lb. 8 oz. true to nearest oz.
25. £799. 4*s.* 10¾*d.*, £985. 13*s.* 0¼*d.*, £640. 2*s.* 1*d.* **26.** 221, 184, 77.
27. (*a*) £1040. 12*s.* 6*d.*, (*b*) £1271. 17*s.* 6*d.*, (*c*) £1965. 12*s.* 6*d.*, (*d*) £3121. 17*s.* 6*d.*
28. £1. 5*s.* 5¼*d.*, £1. 9*s.* 6¾*d.* **29.** 10·4 tin, 89·6 copper.
30. 2 tons 16·025 cwt. **31.** 20. **32.** 91·67 gold, 8·33 copper.
33. 11·54 tin, 11·54 zinc, 76·92 copper. **34.** 85·47 tin, 14·53 antimony.
35. 39·62 copper, 20·13 sulphur, 40·25 oxygen.
36. 18·18 lime, 54·55 sand, 27·27 water.
37. 74·67 carbonate of lime, 6·00 clay, 19·33 sand : 15 tons 2·4 cwt.
38. 78·84 clay, 0·55 coal dust, 20·60 nitro-glycerine, 6·633 kilo.
39. 57·5 sodium, 40 oxygen, 2·5 hydrogen. **40.** 1 ton 16·4 cwt.
41. £21. 16*s.* 4¼*d.* **42.** £41. 13*s.* 4*d.* **43.** $\frac{1}{30}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$.
44. 25, 12½, 2½, 2, 75, 16⅔. **45.** (*b*) 16·33, (*c*) 2·202.
46. (*a*) 0·63, (*b*) 102, (*c*) 0·49, (*d*) 0·30. **47.** 1·862 tons.
48. 6⅔%, £2. 8*s.* 11*d.* nearly. **49.** 4*s.* 7*d.* **50.** 10*s.*
51. 1*s.* 6*d.* **52.** 5⅝. **53.** 3. **54.** 27·2 loss. **55.** £21. 5*s.* 0*d.*
56. £19. 5*s.* 4*d.* **57.** 2·5. **58.** $4\frac{16}{21}$. **59.** £192. 10*s.* 0*d.*
60. 45 to 35. **61.** 2⅛. **62.** 0·0153.
63. 1 sq. ml. 207 ac. 6 sq. ch. **64.** £9900. **65.** 2·34375.

Exercises LVII. b. P. 218.

1. 600 yds. **2.** 55 yds. **3.** 28·5 ft. **4.** 7·29375 mls.
5. 1·305 mls. **6.** 19·23 yds. **7.** 66·332 yds.
8. 6·524 metres. **9.** 83·64 cm. **10.** 10·53 ft.

Exercises LVIII. P. 222.

1. (*a*) 6 ft. 2 in., (*b*) 14 ft. 2 in., (*c*) 18 ft. 8 in., (*d*) 31 ft. 3 in., (*e*) 14 ft. 2 in., (*f*) 15 ft. 3 in., (*g*) 21 ft. 2 in.
3. (*a*) 45 miles, (*b*) 53 miles, (*c*) 2 miles.
4. (*a*) 107 miles, (*b*) 37 miles, (*c*) 94 miles, (*d*) 103 miles, (*e*) 97 miles, (*f*) 117 miles, (*g*) 63 miles.
5. (*a*) 22 miles, (*b*) 26 miles.

Exercises LIX. P. 224.

1. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59; 3^{15}.
2. £503. **3.** £1774. 13*s.* 1*d.* **4.** £5. 4*s.* 6*d.*; 0·25.
5. A £2. 12*s.* 6*d.*, B £4. 16*s.* 3*d.*, C £4. 10*s.* 0*d.*
6. 0·20432; $\frac{1}{320000}$. **7.** $\frac{2320}{5800}$. **8.** 1262250. **9.** $5\frac{1}{4}$ lb.
10. 252 lb. sulphur, 378 lb. charcoal, 1890 lb. nitre.
11. $\frac{7}{192}$. **12.** (*a*) $20\frac{5}{12}$, (*b*) 1 ton 15 cwt. 3 qr., (*c*) 3200.
13. 32. **14.** 11. **15.** 4 tons 0 cwt. 3 qr. **16.** £6. 6*s.* 0*d.*
17. $2^4 \times 3^2 \times 11^2$; 6 chains or 0·075 mile. **18.** £7. 4*s.* $2\frac{1}{2}$*d.*
19. 11. **20.** £390; $4\frac{1}{3}$. **21.** £1. 18*s.* $5\frac{1}{4}$*d.*
22. (*a*) $1\frac{1}{14}$, (*b*) 0·2. **23.** (*a*) $\frac{17}{30}$, (*b*) $1\frac{5}{22}$. **24.** $1\frac{2}{7}$ days.
25. £75. **26.** 45. **27.** 2025. **28.** £425. **29.** 56718·75.
30. £64. **31.** 8. **32.** $3\frac{1}{3}$. **33.** 11, 13. **34.** 8.
35. A wins by $\frac{1}{7}$ mile. **36.** £212. 10*s.* 0*d.* **37.** £0. 12*s.* $0\frac{1}{2}$*d.*
38. £0. 16*s.* $1\frac{1}{2}$*d.*; $\frac{1}{4}$. **39.** A £6. 5*s.* 0*d.*; B £6. 15*s.* 0*d.*; C £7. 10*s.* 0*d.*
40. $18\frac{2}{9}$. **41.** $390\frac{13}{28}$. **42.** £5. 16*s.* $6\frac{10}{27}$*d.*
43. 165. **44.** £5. **45.** 25. **46.** 3, 7, 11. **47.** 0·00288; 80.
48. 4 cwt. copper, 10 cwt. zinc. **49.** £2000. **50.** £78. 15*s.* 0*d.*
51. 7 hrs. **52.** £0. 9*s.* $4\frac{1}{2}$*d.*; £0. 5*s.* $7\frac{1}{2}$*d.*; £0. 1*s.* $10\frac{1}{2}$*d.*; $2\frac{1}{2}$*d.*, $1\frac{1}{2}$*d.*, 1*d.*
53. $3\frac{1}{2}$. **54.** £385. **55.** A £300, B £98, C £112, D £120.
56. 792. **57.** 0·0014999. **58.** 7. **59.** 1671 oz.
61. A £540, B £675, C £1200. **62.** 3%. **63.** 200 : 243. **64.** 45.
65. £526. **66.** $\frac{304}{357}$. **67.** $4\frac{345}{456}\frac{523}{119}$. **68.** $\frac{211}{336}, \frac{125}{336}$. **69.** $82\frac{7}{24}$.
70. £83000. **71.** £33. 6*s.* 8*d.*
72. £7173. 6*s.* 8*d.*; £8070; £8966. 13*s.* 4*d.*; £8608. **73.** £60. 15*s.* 0*d.*
74. £151. 1*s.* 8*d.* **75.** £2418. 17*s.* $3\frac{1}{2}$*d.* **76.** 140, 135, 126.
77. $\frac{1}{12}$. **78.** 0·635. **79.** £15. **80.** 25. **81.** £2. 9*s.* $3\frac{1}{2}$*d.*
82. 0·20625, £4. 5*s.* 9·6*d.* **83.** (*a*) $\frac{17}{46}$, (*b*) $\frac{7}{12}$, (*c*) $25\frac{2}{9}$. **84.** 520.

85. (*a*) 189 miles, (*b*) 28 miles per hour, (*c*) 10.30 a.m., 5.15 p.m.
86. £328. 4*s*. 4½*d*. **87.** $\frac{9}{17}$. **88.** £3. 10*s*. 0*d*.
89. man £8. 18*s*. 1½*d*.; woman £7. 2*s*. 6*d*.; boy £5. 6*s*. 10½*d*.
90. £0. 1*s*. 4½*d*. **91.** 3. **92.** £603. 5*s*. 0*d*.
93. A £17309. 5*s*. 0*d*., B £8654. 12*s*. 6*d*., C £6923. 14*s*. 0*d*., D £5769. 15*s*. 0*d*.
94. £2160. **95.** 480, 15 of 32, 10 of 48, 12 of 40.
96. £10. 17*s*. 11$\frac{5}{8}$*d*. **97.** £10. 10*s*. 7*d*. **98.** A £90, B £81, C £108.
99. $\frac{47}{576}$. **100.** $5\times3^2\times11$, $3\times11\times13$, $5\times3\times13$, 6435.
101. £306. 16*s*. 0*d*. profit. **102.** £76. 17*s*. 1*d*. **103.** £1. 6*s*. 8*d*.; $\frac{2}{9}$.
104. 45 weeks. **105.** £14. **106.** 904. **107.** $72\frac{19}{24}\frac{01}{64}$.
108. £1240, £806. **109.** 74088. **110.** 25·25. **111.** $10\frac{5}{7}$.
112. £479. 11*s*. 10*d*. **113.** 108909. **114.** ½.
115. 50000, 6000, 6600, 3740. **116.** $10\frac{47}{110}$.
117. 113·0016 grs. **118.** A pays B 2*s*. 7*d*., C pays B 3*s*. 2*d*.
119. A £13. 10*s*. 0*d*., B £14. 8*s*. 0*d*., C £14. 14*s*. 0*d*.
120. £35 per horse, £14 per cow. **121.** A £1. 4*s*. 0*d*., B £0. 18*s*. 0*d*.
122. £1234. 4*s*. 0*d*., £617. 2*s*. 0*d*., £205. 14*s*. 0*d*.
123. A £202. 10*s*. 0*d*., B £225, C £112. 10*s*. 0*d*., D £75.
124. £56250. **125.** $34\frac{4}{9}$. **126.** £0. 10*s*. 11½*d*.
127. £0. 4*s*. 0$\frac{3}{4}$*d*.; 0·05078125. **128.** £117. 14*s*. 11*d*.
129. £12. **130.** £683. 13*s*. 0½*d*. **131.** 3·25. **132.** 330.

Exercises LX. a. P. 237.

1. (*a*) 3*s*. 6*d*., (*b*) 3*s*. 10*d*., (*c*) 3*s*. 7*d*., (*d*) 4*s*. 7½*d*.
2. (*a*) 7*s*. 5*d*., (*b*) 4*s*. 2½*d*., (*c*) 6*s*. 10*d*., (*d*) 8*s*. 2½*d*.
3. (*a*) 4*s*. 9*d*., (*b*) 3*s*. 11½*d*., (*c*) 5*s*. 11½*d*., (*d*) 11*s*. 4*d*.
4. (*a*) 12*s*. 6*d*., (*b*) 14*s*. 7½*d*., (*c*) 18*s*. 5*d*. **5.** £1. 1*s*. 1½*d*.

Exercises LX. b. P. 237.

1. £4. 7*s*. 6½*d*. **2.** £107. 7*s*. 2*d*. **3.** 10*s*. 2*d*. **4.** 5*s*. 1½*d*.
5. 3*s*. 6½*d*. **6.** 1578 lb. **7.** 380 cwt. **8.** 100 qr.
9. 69 qr. **10.** 26600. **11.** 12100 tons. **12.** 12200 tons.
13. £5. 1*s*. 11*d*. **14.** £9. 19*s*. 8*d*. **15.** £2. 10*s*. 7*d*. **16.** £4. 15*s*. 4*d*.
17. £3. 12*s*. 11*d*. **18.** £8. 8*s*. 5*d*. **19.** 7. **20.** 1126000. **21.** 24.
22. 513. **23.** 636. **24.** 521000. **25.** 2179.
26. 1½*d*. in the £. **27.** $8\frac{2}{11}$ miles. **28.** 3 tons 2 qr.

Exercises LXI. a. P. 242.

1. £26·060. 2. £26·073. 3. £35. 2*s*. 5*d*.
4. £35. 2*s*. 2*d*. 5. £98·922. 6. £15·583.
7. £83. 13*s*. 11*d*. 8. £34. 2*s*. $2\frac{3}{4}d$. 9. £17. 17*s*. $1\frac{3}{4}d$.

Exercises LXI. b. P. 242.

1. £2·1. 2. £2·15. 3. £5·25. 4. £5·4.
5. £3·85. 6. £4·75. 7. £102·65. 8. £116·95.
9. £0·001. 10. £0·003. 11. £0·005. 12. £0·009.
13. £0·024. 14. £2·034. 15. £12·041. 16. £11·534.
17. £12·431. 18. £12·481. 19. £12·091. 20. £101·169.
21. £13·106. 22. £13·365. 23. £14·305. 24. £114·255.

Exercises LXI. c. P. 242.

1. £2·675. 2. £3·419. 3. £101. 2*s*. $5\frac{1}{2}d$. 4. £6·851.
5. £3·751. 6. £7·804. 7. £3·705. 8. £3. 5*s*. 9*d*.
9. £6. 2*s*. $0\frac{1}{2}d$. 10. £12. 6*s*. $0\frac{1}{4}d$. 11. £8. 5*s*. $10\frac{1}{4}d$. 12. £16·929.
13. £3·817. 14. £1. 14*s*. $6\frac{1}{4}d$. 15. £2. 6*s*. $4\frac{1}{4}d$. 16. £100·100.
17. £100·092. 18. £23·896. 19. £15·336. 20. £3. 3*s*. 4*d*.
21. £2. 2*s*. $8\frac{1}{2}d$. 22. £2·177. 23. £7·059. 24. £13. 0*s*. $1\frac{1}{2}d$.
25. £8. 2*s*. $0\frac{3}{4}d$. 26. £9·131.

Exercises LXII. P. 244.

1. (*a*) (i) £186. 13*s*. 4*d*., (ii) £1866. 13*s*. 4*d*., (iii) £1493. 6*s*. 8*d*., (iv) £896;
   (*b*) (i) £1. 11*s*. $2\frac{3}{4}d$., (ii) £0. 4*s*. $5\frac{1}{2}d$., (iii) 10*s*. 5*d*.;
   (*c*) (i) £751928. 2*s*. 6*d*., (ii) £361503. 18*s*. $1\frac{1}{2}d$.
2. £150.
3. (*a*) £438. 10*s*. 10*d*., (*b*) £911. 17*s*. 6*d*., (*c*) £155. 4*s*. 2*d*.,
   (*d*) £1582. 19*s*. $4\frac{1}{2}d$.
4. (*a*) £0. 4*s*. 2*d*., (*b*) £2. 6*s*. 8*d*., (*c*) £21, (*d*) £8.

Exercises LXIII. a. P. 246.

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{15}$, $\frac{1}{20}$, $\frac{1}{24}$, $\frac{1}{16}$.
2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$.
3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$.
4. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{16}$.
5. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{14}$, $\frac{1}{28}$.

Exercises LXIII. b. P. 247.

1. (*a*) $\frac{1}{6}$, (*b*) $\frac{1}{4}$, (*c*) $\frac{1}{10}$, (*d*) $\frac{1}{15}$, (*e*) $\frac{1}{16}$, (*f*) $\frac{1}{3}$, (*g*) $\frac{1}{32}$, (*h*) $\frac{1}{48}$.
2. (*a*) $\frac{1}{4}$, (*b*) $\frac{1}{5}$, (*c*) $\frac{1}{2}$, (*d*) $\frac{1}{8}$, (*e*) $\frac{1}{16}$, (*f*) $\frac{1}{24}$, (*g*) $\frac{1}{3}$, (*h*) $\frac{1}{80}$.
3. (*a*) $\frac{1}{3}$, (*b*) $\frac{1}{2}$, (*c*) $\frac{1}{6}$, (*d*) $\frac{1}{16}$, (*e*) $\frac{1}{24}$, (*f*) $\frac{1}{32}$.
4. (*a*) $\frac{1}{3}$, (*b*) $\frac{1}{5}$, (*c*) $\frac{1}{6}$, (*d*) $\frac{1}{10}$, (*e*) $\frac{1}{12}$.
5. (*a*) 9*s*., (*b*) 13*s*. 6*d*., (*c*) 18*s*.
6. (*a*) 9*s*. 4*d*., (*b*) 13*s*. 4*a*., (*c*) 12*s*.
7. (*a*) £2, (*b*) £4. 13*s*. 4*d*., (*c*) £5. 6*s*. 8*d*.
8. (*a*) 2*s*. 6*d*., (*b*) 2*s*. 1*d*., (*c*) 1*s*. 8*d*.
9. (*a*) £1, (*b*) £2, (*c*) £2*s*. 1*s*. 4*d*., (*d*) £1. 18*s*. 8*d*.
10. (*a*) £1, (*b*) £3, (*c*) £4, (*d*) £4. 3*s*. 4*d*., (*e*) £3. 16*s*. 8*d*.
11. (*a*) 12*s*., (*b*) 9*s*., (*c*) £5. 11*s*. 0*d*., (*d*) 16*s*., (*e*) 16*s*. 4*d*.
12. (*a*) £1, (*b*) £2, (*c*) £2. 1*s*. 8*d*., (*d*) £1. 18*s*. 4*d*., (*e*) £4. 0*s*. 10*d*., (*f*) £5. 0*s*. 10*d*.
13. (*a*) £1, (*b*) £2, (*c*) £2. 0*s*. 7½*d*., (*d*) £3, (*e*) £3. 2*s*. 6*d*., (*f*) £5. 18*s*. 9*d*.
14. (*a*) £1, (*b*) £2, (*c*) £2. 3*s*. 4*d*., (*d*) £4, (*e*) £54. 3*s*. 4*d*., (*f*) £200.
15. (*a*) £1, (*b*) £9, (*c*) £2, (*d*) £16. 17*s*. 6*d*., (*e*) £54, (*f*) £109. 2*s*. 6*d*.

Exercises LXIII. c. P. 248.

1. (*a*) £16. 8*s*. 10*d*., (*b*) £24. 13*s*. 3*d*., (*c*) £32. 17*s*. 8*d*., (*d*) £49. 6*s*. 6*d*., (*e*) £65. 15*s*. 4*d*., (*f*) £82. 4*s*. 2*d*.
2. (*a*) £14. 16*s*. 10½*d*., (*b*) £27. 4*s*. 3¼*d*., (*c*) £32. 3*s*. 2¾*d*., (*d*) £47. 0*s*. 1¼*d*., (*e*) £54. 8*s*. 6½*d*., (*f*) £71. 14*s*. 10¾*d*., (*g*) £86. 11*s*. 9¼*d*., (*h*) £66. 15*s*. 11¼*d*., (*i*) £89. 1*s*. 3*d*., (*j*) £103. 18*s*. 1½*d*.
3. (*a*) £65. 14*s*. 0*d*., (*b*) £99. 15*s*. 4*d*., (*c*) £136. 5*s*. 4*d*., (*d*) £160. 12*s*. 0*d*., (*e*) £199. 10*s*. 8*d*., (*f*) £214. 2*s*. 8*d*., (*g*) £255. 10*s*. 0*d*., (*h*) £282. 5*s*. 4*d*., (*i*) £311. 9*s*. 4*d*.
4. (*a*) £457. 2*s*. 6½*d*., (*b*) £2545. 3*s*. 9¾*d*., (*c*) £2936. 9*s*. 2¼*d*., (*d*) £3428. 9*s*. 0¾*d*., (*e*) £3486. 11*s*. 3*d*., (*f*) £3409. 1*s*. 8*d*., (*g*) £3564. 0*s*. 10*d*., (*h*) £3602. 15*s*.
5. (*a*) £11325. 10*s*. 8*d*., (*b*) £18938. 9*s*. 9*d*., (*c*) £23337. 4*s*. 10½*d*., (*d*) £101763. 13*s*. 5½*d*.
6. (*a*) £17157. 13*s*. 1½*d*., (*b*) £25460. 17*s*. 2¼*d*., (*c*) £31444. 4*s*. 4½*d*., (*d*) £39605. 0*s*. 3¾*d*.
7. (*a*) £107017. 9*s*. 7½*d*., (*b*) £160415. 0*s*. 3¾*d*., (*c*) £113615. 14*s*. 4½*d*., (*d*) £268470. 8*s*. 5¼*d*.

8. (*a*) £69659. 12*s*. 6*d*., (*b*) £80558. 15*s*. 0*d*., (*c*) £61666. 18*s*. 8*d*., (*d*) £41069. 3*s*. 4*d*., (*e*) £151403. 1*s*. 3*d*.
9. (*a*) £390813. 2*s*. 8½*d*., (*b*) £151702. 19*s*. 5¼*d*., (*c*) £292019. 9*s*. 7½*d*., (*d*) £409601. 18*s*. 2*d*.
10. (*a*) £21518. 16*s*. 3*d*., (*b*) £35864. 13*s*. 9*d*., (*c*) £54908. 2*s*. 8*d*., (*d*) £45006. 13*s*. 4*d*., (*e*) £19409. 2*s*. 6*d*.

Exercises LXIII. d. P. 249.

1. (*a*) £50. 13*s*. 7½*d*., (*b*) £68. 11*s*. 4½*d*., (*c*) £101. 7*s*. 3*d*., (*d*) £152. 0*s*. 10½*d*., (*e*) £281. 14*s*. 6¾*d*., (*f*) £324. 19*s*. 1½*d*.
2. (*a*) £136. 5*s*. 1*d*., (*b*) £343. 19*s*. 4*d*., (*c*) £599. 3*s*. 1*d*., (*d*) £2271. 13*s*. 1¼*d*.
3. (*a*) £1292. 11*s*. 7*d*., (*b*) £583. 12*s*. 9*d*., (*c*) £2238. 18*s*. 8*d*., (*d*) £3927. 4*s*. 0*d*.
4. (*a*) £14. 7*s*. 0*d*., (*b*) £144. 19*s*. 6*d*., (*c*) £571. 7*s*. 2½*d*., (*d*) £873. 14*s*. 10*d*.
5. (*a*) £141. 18*s*. 4*d*., (*b*) £2890. 5*s*. 6*d*., (*c*) £4008. 15*s*. 8*d*., (*d*) £6151. 18*s*. 1*d*.
6. (*a*) £455. 9*s*. 2*d*., (*b*) £1300. 12*s*. 11*d*., (*c*) £3358. 15*s*. 9*d*., (*d*) £5771. 2*s*. 6*d*.
7. (*a*) £97. 14*s*. 4*d*., (*b*) £122. 5*s*. 11*d*., (*c*) £596. 0*s*. 1*d*., (*d*) £981.
8. (*a*) £1147. 18*s*. 4*d*., (*b*) £5939. 10*s*. 8*d*., (*c*) £6877. 15*s*. 7*d*., (*d*) £12519. 13*s*. 9*d*.
9. (*a*) £1512. 13*s*. 0*d*., (*b*) £8291. 18*s*. 8*d*., (*c*) £15756. 7*s*. 1*d*., (*d*) £7324. 10*s*. 6*d*.
10. £142. 8*s*. 10*d*., (*b*) £214. 7*s*. 4*d*., (*c*) £1710. 4*s*. 0*d*., (*d*) £3964. 15*s*. 11*d*.

Exercises LXIII. e. P. 249.

1. (*a*) 2*s*. 6*d*., (*b*) 1*s*. 8*d*., (*c*) 9*s*. 6*d*.
2. (*a*) 14*s*. 3*d*., (*b*) 40*s*. 6*d*., (*c*) £17. 10*s*. 6*d*.
3. (*a*) 3*d*., (*b*) 4*d*., (*c*) 2*s*. **4.** (*a*) 11*d*., (*b*) 4¼*d*., (*c*) 1¼*d*.
5. (*a*) £1. 17*s*. 6*d*., (*b*) £5. 17*s*. 6*d*., (*c*) £36.
6. (*a*) 2½*d*., (*b*) 4*s*. 10½*d*., (*c*) 2*d*.
7. (*a*) £11. 12*s*. 9*d*., (*b*) 6¾*d*., (*c*) 7*s*. 6*d*.
8. (*a*) 5*d*., (*b*) 10½*d*., (*c*) 22*s*. 6*d*. **9.** (*a*) 2¼*d*., (*b*) 5*d*., (*c*) 8*s*. 1½*d*.
10. (*a*) 1¼*d*., (*b*) £1. 2*s*. 6*d*.

Exercises LXIII. f. P. 250.

1. (*a*) £26. 16*s*. 6¼*d*., (*b*) £66. 0*s*. 8*d*., (*c*) £123. 16*s*. 3*d*.
2. (*a*) £240. 9*s*. 0*d*., (*b*) £330. 12*s*. 4½*d*., (*c*) £490. 18*s*. 4½*d*.
3. (*a*) £93. 14*s*. 11¾*d*., (*b*) £354. 18*s*. 2*d*., (*c*) £1499. 19*s*. 8*d*.
4. (*a*) £496. 18*s*. 8¼*d*., (*b*) £851. 17*s*. 9*d*., (*c*) £3738. 16*s*. 10*d*.
5. (*a*) £1. 12*s*. 8½*d*., (*b*) £2. 18*s*. 10½*d*., (*c*) £4. 5*s*. 0½*d*.
6. (*a*) £231. 19*s*. 1½*d*., (*b*) £651. 11*s*. 3*d*., (*c*) £1652. 15*s*. 11¼*d*.
7. (*a*) £475. 11*s*. 10½*d*., (*b*) £2055. 12*s*. 5¼*d*.
8. (*a*) £131. 8*s*. 11¾*d*., (*b*) £120. 10*s*. 3⅜*d*.
9. (*a*) £402. 3*s*. 8*d*., (*b*) £848. 14*s*. 6*d*.
10. (*a*) £3. 19*s*. 7*d*., (*b*) £6. 14*s*. 3¾*d*.
11. (*a*) £218. 10*s*. 3*d*., (*b*) £327. 15*s*. 5*d*.
12. (*a*) 11*s*. 6¾*d*., (*b*) £1. 3*s*. 5¼*d*.
13. (*a*) £302. 2*s*. 4½*d*., (*b*) £402. 16*s*. 6*d*.
14. (*a*) £6. 2*s*. 8½*d*., (*b*) £2. 7*s*. 3½*d*.
15. (*a*) £860. 17*s*. 1*d*., (*b*) £227. 8*s*. 6½*d*.
16. (*a*) £4. 17*s*. 5*d*., (*b*) £19. 6*s*. 9*d*.
17. (*a*) £2. 4*s*. 8*d*., (*b*) £8. 0*s*. 2*d*.
18. (*a*) £24. 5*s*. 4½*d*., (*b*) 10*s*. 0½*d*.
19. (*a*) £144. 13*s*. 0*d*., (*b*) £11. 10*s*. 0*d*.

Exercises LXIV. P. 251.

1. £109. 4*s*. 4½*d*.
2. £586. 3*s*. 2*d*.
3. 16 tons 1 cwt. 3 qr.
4. £132.
5. £211. 2*s*. 8¼*d*.
6. £10850. 10*s*. 9*d*.
7. £33. 2*s*. $7\frac{9}{16}$*d*.
8. £162. 1*s*. 10½*d*.
9. £21. 9*s*. 4*d*.
10. 796,603·5 tons.
11. £7. 14*s*. 6*d*.
12. 281 mi. 978⅜ yds.
13. 5 cwt. 25·9375 lb.
14. £3. 2*s*. 4⅛*d*.
15. £1524. 11*s*. 2½*d*. approx.
16. £270.
17. £7. 17*s*. 2$\frac{2}{5}$*d*.
18. 57 ton 4 cwt. 2 qr. $7\frac{7}{20}$ lb.
19. £16. 7*s*. 0*d*.
20. (*a*) £31. 5*s*. 4*d*., (*b*) £1017. 14*s*. 8*d*., (*c*) £11284. 14*s*. 0*d*., (*d*) £557. 1*s*. 8*d*., (*e*) £10609. 11*s*. 0*d*., (*f*) £2891. 8*s*. 0*d*., (*g*) £8916. 1*s*. 0*d*. (*h*) £6804. 18*s*. 8*d*., (*i*) £1286. 11*s*. 4*d*., (*j*) £65083. 4*s*. 0*d*. (*k*) £1234. 0*s*. 0*d*.
21. (*a*) £2138. 15*s*. 8*d*., (*b*) £2153. 5*s*. 0*d*., (*c*) £20944.
22. (*a*) £15, £435, (*b*) £520, £500, (*c*) £700, £668. 10*s*. 0*d*., (*d*) £75, £1425, (*e*) £800, £40, (*f*) £350, £9. 10*s*. 0*d*.

23. (*a*) £13. 15*s*. 10*d*., (*b*) £741. 0*s*. 10*d*., (*c*) £109. 2*s*. 2*d*.,
(*d*) £12334. 13*s*. 2*d*., (*e*) £48583. 4*s*. 10*d*., (*f*) £10. 12*s*. 10*d*.,
(*g*) £56. 17*s*. 7*d*., (*h*) £228. 19*s*. 6*d*., (*i*) £45060. 10*s*. 6*d*.,
(*j*) £11633. 15*s*. 4*d*. (*k*) £12. 12*s*. 5*d*., (*l*) £76. 17*s*. 10*d*.,
(*m*) £220. 5*s*. 2*d*., (*n*) £255180. 16*s*. 0*d*.

24. (*a*) £400, (*b*) £6800, (*c*) £5600, (*d*) £78000,
(*e*) £5000, (*f*) £8125, (*g*) £17000, (*h*) £560000.

Exercises LXV. a. P. 256.

2. 0·5 sq. in. approx. **3.** 0·95 sq. cm. **4.** 1·0 sq. in. approx.

Exercises LXV. b. P. 256.

1. 1·24 sq. in. **2.** 648·48 cub. in., 625·33 cub. in.
3. 0·55 litre. **4.** 0·0183 ac.
5. (*a*) 1644·3025 and 1636·2025 sq. in. (*b*) 126·5625 and 124·3225 sq. in.,
(*c*) 355·3225 and 351·5625 sq. in., (*d*) 583·2225 and 578·4025 sq. in.,
(*e*) 642·6225 and 637·5625 sq. in., (*f*) 3606·0025 and 3594·0025 sq. in.
6. 1128·7, 1047.
7. (*a*) 11463 and 10938·6, (*b*) 6944·6 and 6596·8,
(*c*) 15000·0 and 14415·8, (*d*) 47388·2 and 46099·9,
(*e*) 2299·5 and 2245·5, (*f*) 32801·0 and 32459·0.
8. 0·22 and 0·23.
9. (*a*) 5·88 and 6·00, (*b*) 4·96 and 5·04, (*c*) 6·58 and 6·68,
(*d*) 1·87 and 1·88, (*e*) 2·44 and 2·54, (*f*) 2·49 and 2·57.
10. 95778 cm. and 94757 cm. **11.** 18·4 and 18·1.

Exercises LXVI. a. P. 264.

1. 97000. **2.** 21×10^4. **3.** 336×10^3. **4.** 27×10^2.
5. $1{\cdot}6 \times 10^5$. **6.** $4{\cdot}6 \times 10^6$. **7.** $4{\cdot}2 \times 10^3$. **8.** $2{\cdot}3 \times 10^6$.
9. $4{\cdot}4 \div 10^3$. **10.** $1{\cdot}8 \times 10^6$. **11.** 30×10^6. **12.** $2{\cdot}2 \times 10^2$.
13. 96×10^1. **14.** $4{\cdot}0 \times 10^6$. **15.** $1{\cdot}0 \times 10^3$. **16.** $3{\cdot}0 \times 10^7$.
17. $9{\cdot}4 \times 10^4$. **18.** $3{\cdot}6 \times 10^3$. **19.** $1{\cdot}3 \times 10^6$. **20.** $1{\cdot}8 \div 10^1$.
21. $4{\cdot}5 \times 10^1$. **22.** $1{\cdot}1 \times 10^5$. **23.** $1{\cdot}8 \times 10^1$. **24.** $6{\cdot}0 \times 10^3$.
25. $2{\cdot}9 \times 10^3$.

Exercises LXVI. b. P. 265.

1. $4{\cdot}724 \times 10^4$ gr.
2. (*a*) 131×10^3, 72×10^3, (*b*) $2{\cdot}3 \times 10^3$, $1{\cdot}1 \times 10^3$, (*c*) $3{\cdot}50 \times 10^3$, $1{\cdot}91 \times 10^3$, (*d*) $1{\cdot}69 \times 10^4$, $9{\cdot}5 \times 10^3$, (*e*) $3{\cdot}3 \times 10^3$, $2{\cdot}0 \times 10^3$, (*f*) $4{\cdot}7 \times 10^3$, $3{\cdot}0 \times 10^3$, (*g*) $2{\cdot}0 \times 10^3$, $1{\cdot}1 \times 10^3$, (*h*) $1{\cdot}11 \times 10^3$, $7{\cdot}6 \times 10^2$, (*i*) $6{\cdot}6 \times 10^3$, $4{\cdot}6 \times 10^3$, (*j*) $3{\cdot}0 \times 10^3$, $1{\cdot}8 \times 10^3$, (*k*) $9{\cdot}3 \times 10^3$, $4{\cdot}9 \times 10^3$, (*l*) $2{\cdot}7 \times 10^3$, $1{\cdot}9 \times 10^3$, (*m*) $5{\cdot}3 \times 10^3$. $2{\cdot}4 \times 10^3$.
3. 88 cub. metres.
4. 24·40 ft.
5. 57560, reliance on 1st three figures.
6. $3{\cdot}50 \times 10^4$ tons.
7.

Elevation in feet	0	328	656	984	1312	1640
Bar height in inches	29·92	29·57	29·21	28·85	28·50	28·19

8. 144·8.
9. 2.
10. 60000.
11. 390655.
12. 327235·8.
13. 61879·2.
14. 29629.
15. 17·5364.
16. 835216·8.
17. 107·98.
18. 981598.
19. 1·0612.
20. 9·01.
21. 18018.
22. 9994·5.
23. 4229.
24. 28763×10^3.
25. 3619×10^2.
26. 179442×10^2.
27. 1827×10^3.
28. 65771×10^2.
29. 403×10^4.
30. 8831×10^4.

Exercises LXVII. P. 271.

1. 6·458.
2. 3869.
3. 13 lb.
4. (*a*) 60·8233, 27′ 0·19″.
5. 1 : 2566, 1 : 1841, 1 : 1609.
6. (*a*) 4287, (*b*) 5202, (*c*) 626, (*d*) 554, (*e*) 1161.
7. 561·24 ft.
8. (*a*) 999·41, (*b*) 31·265, (*c*) 25768, (*d*) 0·81688, (*e*) $208{\cdot}01 \times 10^3$, (*f*) 106·54.
9. 7·680 in.
10. 1·609 kilom.

Exercises LXVIII. a. P. 273.

1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400.
2. 0, 1, 4, 5, 6, 9.

Exercises LXVIII. b. P. 273.

1. 42.
2. 210.
3. 75.
4. 231.
5. 84.
6. 36.
7. 48.
8. 1540.
9. 54.
10. 63.
11. 216.
12. 66.
13. 256.
14. 1155.
15. 192.

Exercises LXVIII. c. P. 273.

1. (*a*) 2, (*b*) 3, (*c*) 6, (*d*) 10, (*e*) 9, (*f*) 14.
2. (*a*) 2, (*b*) 4, (*c*) 12, (*d*) 8, (*e*) 8, (*f*) 10.
3. (*a*) 3, (*b*) 7, (*c*) 14, (*d*) 14, (*e*) 13.
4. (*a*) 1, (*b*) 2, (*c*) 2, (*d*) 2, (*e*) 1, (*f*) 2, (*g*) 3, (*h*) 3.
5. (*a*) 1, (*b*) 2, (*c*) 2, (*d*) 3.
6. (*a*) 1 and 1, (*b*) 1 and 2, (*c*) 1 and 2, (*d*) 2 and 1, (*e*) 2 and 2.

Exercises LXVIII. d. P. 277.

1. (*a*) 13, (*b*) 17, (*c*) 23, (*d*) 29, (*e*) 47, (*f*) 53.
2. (*a*) 58, (*b*) 64, (*c*) 74, (*d*) 85, (*e*) 89, (*f*) 93, (*g*) 97, (*h*) 99.
3. (*a*) 504 yds., (*b*) 1012 yds., (*c*) 1228 metres, (*d*) 1752 ft., (*e*) 2396 yds.
4. (*a*) 672, (*b*) 709, (*c*) 856, (*d*) 945, (*e*) 2041, (*f*) 3872, (*g*) 4018, (*h*) 5306, (*i*) 5084, (*j*) 70963.
5. (*a*) 23·04, (*b*) 70·082, (*c*) 0·924, (*d*) 491·62, (*e*) 6·57, (*f*) 30·0905, (*g*) 0·0529, (*h*) 0·02538, (*i*) 0·008904, (*j*) 64·007.
6. 29, 23 over. **7.** 278 yds. **8.** 68·08 sq. cm., 219·04 sq. cm.
9. 10 ft. 7 in. nearly. **10.** 182 ft., 730 : 728. **11.** 19 people, 19*d*.

Exercises LXVIII. e. P. 278.

1. $\frac{24}{25}$. **2.** $\frac{32}{31}$. **3.** $\frac{198}{291}$. **4.** $\frac{93}{445}$. **5.** $\frac{219}{1198}$.
6. $\frac{761}{824}$. **7.** 1·59. **8.** 0·77. **9.** 0·86.

Exercises LXIX. a. P. 280.

1. £46. 16*s*. 9*d*. **2.** £27. 14*s*. 1½*d*. **3.** £32. 4*s*. 1½*d*.

Exercises LXIX. b. P. 282.

1. £26. 10*s*. 6½*d*. **2.** £38. 10*s*. 4*d*. **3.** £40. 19*s*. 7*d*.

Exercises LXIX. c. P. 282.

1. £1. 5*s*. 0*d*. **2.** ½*d*. **3.** £0. 1*s*. 8*a*. **4.** £0. 6*s*. 3*d*.
5. £0. 6*s*. 8*a*. **6.** £0. 3*s*. 4*a*. **7.** £40. **8.** £1. **9.** £168.
10. £42. **11.** £36. **12.** £120. **13.** 1. **14.** 9.
15. 6. **16.** 12. **17.** 4. **18.** 9.

Exercises LXIX. d. P. 285.

1. £131. 5*s*. 0*d*. **2.** £34. 12*s*. 5*d*. **3.** £356. 18*s*. 9*d*.
4. £19. 5*s*. 6*d*. **5.** £86. 14*s*. 1*d*. **6.** £153. 13*s*. 9*d*.
7. £16. 15*s*. 6*d*. **8.** £836. 3*s*. 3*d*. **9.** £3. 6*s*. 7*d*.
10. £54. 17*s*. 10*d*. **11.** £19. 12*s*. 4*d*. **12.** £44.
13. £571. 6*s*. 2*d*. **14.** (*a*) £1. 9*s*. 7*d*., (*b*) £5. 6*s*. 7½*d*.
15. £13. 12*s*. 10*d*. **16.** £170. 0*s*. 5*d*. **17.** £1012. 9*s*. 7*d*.
18. £7. **19.** 2 yrs. **20.** 3 yrs. **21.** 5 yrs.
22. 5 yrs. **23.** 5 yrs. **24.** 2½ yrs. **25.** 4 yrs.
26. (*a*) £2. 7*s*. 3*d*., (*b*) £4. 10*s*. 11*d*., (*c*) £61. 12*s*. 11*d*., (*d*) £108. 14*s*. 11*d*.
27. 4 yrs. **28.** 15 yrs. **29.** 30 yrs. **30.** 0·2 yrs.
31. 2½. **32.** 2. **33.** 3⅓. **34.** 2⅝. **35.** 3. **36.** 2½.
37. 2½. **38.** 2¼. **39.** 4½. **40.** 60. **41.** 2⅜. **42.** 4.
43. £159. 3*s*. 4*d*. **44.** £394. 4*s*. 0*d*. **45.** £439. 8*s*. 6*d*.
46. £575. **47.** £2700. **48.** £240. **49.** £312.
50. £1053. 6*s*. 8*d*. **51.** £318. 15*s*. 0*d*. **52.** £150.
53. (*a*) 30, (*b*) 60, (*c*) 26⅔, (*d*) 26⅔, (*e*) 15·3, (*f*) 30. **54.** (*a*) 25, (*b*) 15.

Revision Papers. P. 289.

A **1.** 68 tons 8 cwt. **2.** £1. 8*s*. 10½*d*. **3.** 3704.
4. 11 yds. 0 ft. 4 in. **5.** 1¾. **6.** 6¼. **7.** 7·55. **8.** £1·624.
9. 4·191 sq. ft. **10.** 68. **11.** $\frac{15}{112}$. **12.** 0·0875. **13.** 2.
14. 5. **15.** £0. 4*s*. 10*d*. **16.** 587 ft. 6 in.

B **1.** $\frac{31}{48}$. **2.** 42 sq. yds. **3.** 3388. **4.** 0·0852, 0·0075.
5. 0·0353073, 0·00161. **6.** £28. 1*s*. 0*d*. **7.** 16380.
8. 455. **9.** 0·1̇3̇. **10.** 760 lb.

C **1.** 1167 ft. **3.** 5 tons 13 cwt. 2 qr. 9 lb. 13 oz., 5916264 in.
4. 565, 1½ lb. **5.** £0. 4*s*. 2*d*. gain. **6.** £5 gain.
7. 10%. **8.** 180. **9.** £10. 5*s*. 0*d*.

D **1.** 9 hrs. **2.** £87. 10*s*. 0*d*. **3.** 20 min. 50 sec. **4.** 85%.
5. 700 sq. ft. **6.** £441. **7.** £14. 10*s*. 7½*d*. **8.** 3⅛%.
9. $5\frac{29}{33}$. **10.** £1. 2*s*. 6*d*. **11.** 3⅓%.

E **1.** $\frac{3}{5}$. **2.** 1562·5. **3.** £531. 13*s*. 4*d*.
4. 31 lb. 4 oz. 16 dwt. 16 gr. **5.** 0·941̇6̇. **6.** £1092. 15*s*. 8*d*.
7. 1155. **8.** £7. 15*s*. 0*d*. **9.** £25. 13*s*. 4*d*.

F **1.** £670. 8*s.* 1*d.* to nearest 1*d.* **2.** 7500, 0·0051. **3.** 437.
4. $\frac{5}{18}$. **5.** $\frac{4}{15}$. **6.** 14·553. **7.** £688. 10*s.* 8*d.*
9. 70. **10.** 20. **11.** £1. 5*s.* 0*d.* **12.** 7 : 5. **13.** $33\frac{1}{128}$.
14. £600, £594, £609, £621, £596, £646.

G **1.** 0·697. **2.** 1295 litres, 23 litres. **3.** 8·279...%, 690900 approx.
4. Manitoba 25, Ontario 17. **5.** £19. 11*s.* 10*d.*, £109. 11*s.* 10*d.*
6. 1·85 sq. in., 73 lb. approx.

H **1.** £680, £510, £340. **3.** 70% town, 30% country. **4.** 37.
5. 21 min. **6.** 12800. **7.** 1256·636, 4·53 gal.
8. 43·93 yds. **9.** 1650 sq. ft. **10.** 360.

PART II

Exercises LXXII. P. 300.

1. 3·74 times. **2.** 0·1325 mm. **3.** 1617·1 yd.
4. 8·75 ft. **5.** 0·5 mm. **6.** 0·125 in.
7. 5·84 turns approx. **8.** 0·14 in., 0·8736 in. **9.** 593 cm. approx.
10. 0·05 in. **11.** Thread 0·04 in., handle 1·5 in. nearly.
12. 2·632 mm. **13.** 1·0094 m. **14.** $\frac{29}{36^3}$, $\frac{24}{36^2}$.
18. $\frac{250}{481}$, $\frac{250}{479}$, $\frac{480}{481}$, $\frac{480}{479}$ secs. **19.** 288 secs.

Exercises LXXIV. a. P. 311.

1. 8·64. **2.** 9·43. **3.** 27·9. **4.** 29·0. **5.** 7·43.
6. 8·32. **7.** 18·3. **8.** 9·60. **9.** 1·386. **10.** 2·83.
11. 1·96. **12.** 3·92. **13.** 3·05. **14.** 8·17. **15.** 1·03.
16. 0·99. **17.** 0·051. **18.** 0·94. **19.** 0·4986. **20.** 3·006.

Exercises LXXIV. b. P. 312.

1. (*a*) 31·1, (*b*) 20·2, (*c*) 78·8, (*d*) 104·6, (*e*) 158.
2. (*a*) 4·03, (*b*) 8·03, (*c*) 517, (*d*) 203, (*e*) 40·4.
3. (*a*) £78·3, (*b*) £56·4, (*c*) £115·2, (*d*) £34·3.
4. 7·5 too small (−). **5.** 7·5 too high (+). **6.** 0·0001+.
7. 0·0000003+. **8.** 4·04−. **9.** 54+. **10.** 0·052−.
11. 43−. **12.** 0·0368+. **13.** 0·126+. **14.** 0·63−.
15. 0·6−. **16.** 29−. **17.** 100−. **18.** 0·02−.

Exercises LXXIV. c. P. 317.

1. (*a*) 41·9, (*b*) 6, (*c*) 6·4, (*d*) 8·4. **2.** 490.
3. (*a*) 87·93, (*b*) 144·6, (*c*) 291·6. **4.** 97 miles nearly.
5. 49 yd. **6.** 743 ft. nearly. **7.** 38·4 ft. **8.** 12 ft.
9. 188 yd. by 23·5 yd. **10.** 7·3 in. **11.** 2 ft. 4 in. **12.** 5 ft. 11 in.
13. 35. **14.** (*a*) 17 in., (*b*) 61 ft., (*c*) 26·6 ft., (*d*) 26·51 ft.
15. 12 ch. 21 lk. **16.** 9·80 cm. **17.** 192 ft.

Exercises LXXIV. e. P. 319.

1. (*a*) Final position 13·5 miles to E. and 14·1 to N., 19·5 miles between first and last position.
2. (*a*) 7·33 miles to E., 1·68 to N., distance 7·52 mi.; (*b*) 2·2 mi. W., 3·08 mi. N., distance 3·78 mi.; (*c*) 11·6 mi. E., 4·3 mi. N., distance 12·37 mi.
3. (*a*) 0·17365, (*b*) 0·93969, (*c*) 0·50000, (*d*) 0·64279.

Exercises LXXIV. f. P. 321.

1. 100·005. 2. 1200. 3. 300·01.
4. (*a*) 2nd, (*b*) 2nd, (*c*) 3rd, (*d*) 5th, (*e*) 3rd, (*f*) 3rd, (*g*) 3rd.

Exercises LXXIV. g. P. 322.

1. 8·12. 2. 25·01. 3. 40·5. 4. 49·22. 5. 32·25.
6. 55·3. 7. 15·36. 8. 10·5. 9. 44·36. 10. 42·65.

Exercises LXXV. P. 324.

1. 7·69 tons. 2. 0·267 in. approx. 3. £360. 8*s.* 4*d.*
4. £3. 3*s.* 9*d.* 5. 509·48 gr. 6. 253·2 min.
7. 175 lb. 8. 10·625. 9. 1250 lb.
10. 35 in. 11. 544·7 lb. 12. 31·62 ton. 13. 156·8 lb.

Exercises LXXVI. a. P. 329.

3. (*a*) $\frac{1}{3}$, (*b*) $\frac{1}{5}$, (*c*) $\frac{1}{6}$, (*d*) $\frac{1}{8}$, (*e*) $\frac{1}{20}$.
4. (*a*) $63\frac{1}{2}°$, (*b*) $71\frac{1}{2}°$, (*c*) 76°, (*d*) $26\frac{1}{2}°$, (*e*) $18\frac{1}{2}°$, (*f*) 14°.
5. (*a*) 6°, (*b*) $11\frac{1}{2}°$, (*c*) $17\frac{1}{2}°$, (*d*) $23\frac{1}{2}°$, (*e*) 30°, (*f*) 37°, (*g*) $44\frac{1}{2}°$, (*h*) 53°.

Exercises LXXVI. b. P. 330.

1. (*a*) 1·05316; (*b*) 0·56004; (*c*) 0·33535; (*d*) 0·45383; (*e*) 0·26624; (*f*) 0·59838; (*g*) 0·21004; (*h*) 1·45220.
2. (*a*) 13° 26′ 23″; (*b*) 47° 10′; (*c*) 56° 34′ 1″; (*d*) 178° 42′.
3. (*a*) 34°; (*b*) 0·504; (*c*) 78°; (*d*) 290·9; (*e*) 168; (*f*) 155.

Exercises LXXVII. P. 331.

6. (*a*) 26, (*b*) 16, (*c*) 38, (*d*) 20. 7. 1·7, 7, 6, 14, 62.
8. (*b*) 302×10^6, (*c*) 110×10^6.

Exercises LXXVIII. b. P. 336.

1. 8 sq. cm. **2.** $8\frac{1}{2}$ cub. ft. **3.** 11 ft.
4. 7 ft. **5.** 132 sq. ft. **6.** 616 cub. ft.
7. 53900 c.c. **8.** 2 in. **9.** $\frac{11}{6}$ cub. ft.
10. $\frac{5500}{7}$ oz. **11.** 9900 lb. **12.** 3 cm.

Exercises LXXVIII. c. P. 337.

1. 754 c.c. approx. **2.** 277 cub. ft. approx.
3. $152{\cdot}4 \times 10$ cub. ft. approx. **4.** 1706 cub. in.
5. 1 metre, 8·61 cm. **6.** 0·04274... cub. ft.
7. 1838 lb. **8.** 20·4 lb. **9.** 21·36 sq. ft.
10. (*a*) (i) 3583×10, (ii) $16{\cdot}43 \times 10^4$; (*b*) (i) 1018×10^2, (ii) 1264×10; (*c*) (i) 0·1257, (ii) 2·177; (*d*) (i) 2479×10, (ii) 1933.
11. 0·022 cm. **12.** 0·056 mm. **13.** 1·025, 1·14 approx. **14.** 0·25 cm.

Exercises LXXIX. a. P. 338.

1. 1, 8, 27, 64 c.c. **3.** B 2, 2, 2, 8; C 3, 3, 3, 27; D 4, 4, 4, 64.
4. B 2^3, C 3^3, D 4^3. **5.** (*a*) £61076·64, (*b*) 7634580, (*c*) 954·3225.
6. 24 ton 6 cwt. **7.** $x(0{\cdot}343)$. **8.** 1440^3. **9.** $243 \times \frac{91}{216}$.

Exercises LXXIX. b. P. 340.

4. B 57×10^6, C 36×10^6, D 19×10^6, E 13×10^6.

Exercises LXXX. a. P. 344.

1. 2 ft. **2.** 6·4 ft. **3.** 2 days. **4.** 9 days.
5. 42 cub. in. **6.** 54 hrs. **7.** 7200 **8.** 80%.
9. 20%. **10.** 160 ft. **11.** $3\frac{1}{9}$ lb. **12.** 50%.
13. 4 years. **14.** $5\frac{1}{3}$%. **15.** 30. **16.** 30.
17. $16\frac{2}{3}$. **18.** £22. 8*s*. **19.** $42\frac{2}{3}$ hr. **20.** 1·28 hr.

Exercises LXXX. b. P. 347.

1. (*a*) 745, (*b*) 27·1.
2. (*a*), (*e*), (*l*), (*m*); (*b*), (*f*); (*g*); (*c*), (*d*), (*h*), (*i*), (*k*).
5. 5·2 amp., 36 amp. hrs. **7.** (*a*), (*e*), (*f*); (*b*), (*f*), (*c*).
8. (*b*) 833, (*c*) 926, (*d*) 560, (*e*) 195, (*f*) 667. **9.** 54 oz., $7\frac{3}{4}$ in.

Exercises LXXXI. P. 351.

1. £60. **2.** 8·68 days. **3.** £59. 18*s.* 7*d.*
4. 27 days more. **5.** 3 hr. approx. **6.** 3·49 hr.
7. 2·31 cm. **8.** 64. **9.** 100.
10. 70 days in all. **11.** 97·22 tons. **12.** (*a*) 5 miles approx., (*b*) 24 ft.
13. $93\frac{3}{11}$ days. **14.** (*a*) 2 tons 19 cwt. 1 qr. 11 lb., (*b*) 40, (*c*) £40.
15. 7. **16.** 62. **17.** (*a*) 5·90 cm., (*b*) 0·0377 cm.
18. (*a*) 1·181 kilo. per sq. cm., (*b*) 12·35 c.c. **19.** 9*d.*
20. £21. 14*s.* 11*d.* **21.** £272. 5*s.* **22.** 201·9 cub. ft.
23. (*a*) 3·2 ft. approx., (*b*) 752 lb. per sq. in. **24.** 36 days.
25. 20 days. **26.** 80. **27.** (*a*) 155 ft. per sec., (*b*) 150 ft.
28. 19·91 ft. per sec. **29.** 14·2 min. **30.** 71·7.
31. 573 kilo. **32.** Countesses 1·04 times as thick as 'doubles.'
33. 262·5 days. **34.** 265·68 sq. yd. **35.** 1·29 mm.
37. 0·7556°. **38.** A £675, B £315, C £126.
39. A £163. 8*s.*, B £28. 14*s.*, C £39. 14*s.*, D £38. 4*s.*
40. £4. 12*s.*, £6. **41.** 18*s.*, 24*s.* **42.** £60.
43. 97 sec. **44.** $1{\cdot}405 \times 10^4$ ft. **45.** (*a*) 3·11 cm., (*b*) 301 grams.
46. (*a*) 2182 lb., (*b*) $1333\frac{1}{3}$ lb. **47.** 4 in. **48.** 59·6 miles per hour.
49. (*a*) 9·8 sec., (*b*) 7·18 ft.

Exercises LXXXII. P. 363.

1. 1000 sq. lks. **2.** 1·9 sq. ch. **3.** 5000 sq. lks.
4. 4000 sq. lks. **5.** 7250 sq. lks. **6.** 2150 sq. lks.
7. 8950 sq. lks. **8.** 5·35 sq. ch.

Miscellaneous Problems LXXXIII. P. 365.

1. $16\frac{4}{11}$ min. past 12.
2. (*a*) $27\frac{3}{11}$ min. past 5, (*b*) $16\frac{4}{11}$ min. past 3, (*c*) $5\frac{5}{11}$ min. to 11.
3. (*a*) $43\frac{7}{11}$ min. past 2, (*b*) $5\frac{5}{11}$ min. to 5, (*c*) $16\frac{4}{11}$ min. past 9.
4. $12\frac{3}{11}$ sec. **5.** $42\frac{6}{35}$ sec. **6.** $8\frac{5}{55}\frac{2}{5}$ sec.
7. (*a*) $\frac{1}{15}$, (*b*) $\frac{1}{6}$, (*c*) $\frac{1}{30}$, (*d*) 30 min. **8.** $4\frac{5}{18}$ min. **9.** 70·18 sec.
10. 23·5 yd. **11.** $42\frac{6}{7}$ min. past 6; $5\frac{1}{7}$ miles from A.
14. 334 days approx. **15.** 15 cwt., 12·8 nearly.
16. 11 lb. of the cheap to each 8 lb. of the dear. **17.** 26 : 19.

18. $38\frac{2}{11}$ min. past 10, $24\frac{6}{11}$ miles, trains 35 miles apart, approach at 55 miles per hour. **19.** 2 min., 880 yd. **20.** $5\frac{3}{5}$ mile. **22.** £15. **23.** $3\frac{9}{11}$ hr. **24.** 32 min., 42 min. **25.** 1800 yd., $9\frac{2}{3}\frac{0}{5}\frac{7}{2}$ and $9\frac{3}{8}$ m. per hr. **26.** 2 miles, 24 min. after. **27.** 60 days, 12 hr. 16 min. **28.** 120 days. **29.** $16\frac{5}{1}\frac{6}{9}\frac{}{9}$ days. **30.** 5 days. **31.** 8 days. **32.** 5 days. **33.** 30 boys. **34.** $15\frac{1}{2}\frac{5}{3}$ past 24, 1 hr. $2\frac{1}{2}\frac{4}{3}$ min. **35.** £0. 9*s.* 8*d.* **36.** £3. 4*s.* **37.** 11 : 80. **38.** 3 hr. **39.** 5 miles. **40.** B, $4\frac{2}{4}\frac{3}{8}$ ft. **41.** 36 miles. **42.** $3\frac{1}{3}$ gal. and $8\frac{2}{3}$ gal. **43.** 8 : 9.

Revision Exercises LXXXIV. P. 370.

Miscellaneous.

A **1.** 324. **2.** £30. 9*s.* 0*d.* **3.** $2^2\times3^2\times13$, $11\times3\times17$, $7\times13\times17$, $11\times13\times17$; 7056. **4.** 4. **5.** 30·03, 4·4 kilo. **6.** £5. 17*s.* 1*d.* **7.** 36·251, 1521. **8.** $10\frac{1}{4}d.$ **9.** 633 tons.

B **1.** 2*s.* **2.** 2·646. **3.** £126 **4.** 2 ft. **6.** 1*s.* 8*d.* **7.** 3%. **8.** 5662. **9.** 6 cm. 4 mm. **10.** 48·6261 sq. in., 313·7055 sq. cm., 6·451, true to units, units and 1st dec. figures respectively.

C **1.** 2928. **2.** £1255. 6*s.* 8*d.* **3.** £1. 0*s.* 10*d.* **4.** 7%. **5.** 45. **6.** $6\sqrt{96}$, $4\sqrt{96}$. **7.** 247·1. **8.** 114. **9.** (*a*) 24886, (*b*) 4·021. **10.** 7749.

D **1.** 858357, remainder 48. **2.** 20, £4. 9*s.* 2*d.* **3.** (i) $2\frac{1}{16}$; (ii) 7·038. **4.** (i) 2 ton 17 cwt. 2 qr.; (ii) 11 miles 3 fur. 176 yd. **5.** 218 fr. 51 cent. **6.** $8\frac{4}{7}d.$ **7.** £328. 14*s.* 9*d.* **8.** 252·960. **9.** £0. 19*s.* 0*d.* approx.

E **1.** (*a*) 0·7275; (*b*) £26. 19*s.* $1\frac{1}{4}d.$ **2.** (*a*) 295·502; (*b*) 13097·53. **3.** 20. **4.** 04. **5.** 149, 1317160. **6.** $4\frac{1}{2}$%. **7.** 1st cl. : 2nd cl. :: 9 : 10. **8.** 6*s.* 8*d.*, 6*s.* 10*d.*, 6*s.* 5*d.* **9.** £309, £146, £175, £149, £203.

F 1. £4072. 7*s*. 8*d*. 2. 14400. 3. $16\frac{1}{2}$. 4. 160.
5. 951, 3487. 6. 2·2. 7. 3·280899.
8. $\frac{1}{791}$. 9. 3·4. 10. £75. 16*s*. 4*d*. nearly.

G 1. $409\frac{1}{2}$ sq. ft., 46 sheets. 2. $5937\frac{1}{2}$ cub. ft., $37109\frac{3}{8}$ gal.
3. 310284. 4. 8. 5. 1·55838. 6. 200%.
7. 1897, 1904. 8. £0. 5*s*. 4*d*. 9. 6 : 1, 32.
10. 442·604, no decimal figure.

H 1. 540, 3780. 2. $\frac{47}{55}$. 3. 0·00000008562.
4. (i) 83·56; (ii) 14, 124. 5. 55·86591. 6. 135%.
7. 237·17. 8. 385. 9. 989 cub. in. 10. £2. 9*s*. 0*d*.

I 1. £8. 6*s*. 8*d*. 2. 3 and 11. 3. (i) $\frac{13}{80}$; (ii) $\frac{143}{1160}$.
4. 1100 yd., 10100 yd. 5. 1, 1·0625, 0·002, 0.
6. 491 sq. m., $\frac{1}{4}$ sq. m. approx. 7. 42, 6, 25, 26.
8. £43911. 12*s*. 3*d*. 9. £358. 19*s*. 6*d*. 10. 1·3215 ac.

J 1. £547. 17*s*. 3*d*. 2. 327. 3. 112 yd. 28 in. 4. 1.
5. 32. 6. 0·0395, 0·006812. 7. 0·513. 8. £7373. 17*s*. $7\frac{1}{4}$*d*.
9. $40\frac{1}{2}$ bushels. 10. £224. 18*s*. 6*d*. 11. 47, 54.

K 1. $5\frac{29}{33}$. 2. $\frac{19110}{9360}$, $\frac{19110}{31850}$, $\frac{19110}{2457}$, $\frac{19110}{153770}$, $\frac{19110}{20580}$.
3. 13 hrs. 32 min. 4. 27, 13. 5. 1 day 4 hr.
6. 1776 sq. yd., 1191 cub. yd. 7. $2\frac{6}{7}$. 8. 0·028, $7{\cdot}91 \times 10^{-3}$.
9. 156. 10. 2940 miles.

L 1. 9 cwt. 1 qr. 26 lbs. 2. £200. 3. 3*s*. 4*d*. 4. £7630.
5. 29225 1960. 6. 6 mo. 7. 39·37 in.
8. $\frac{1}{2534{\cdot}4}$, 62·5 sq. in. 9. 529×10^4. 10. 52·9°.

M 1. 3·937. 2. £18. 17*s*. 8*d*. 3. 876576, 5218. 4. $10\frac{1}{8}$ hr.
5. 192 cub. yd., 208 sq. yd.; $v=\frac{V}{8}$, $a=\frac{A}{4}$, $d=\frac{D}{2}$. 6. 420.
7. (*a*) Yes; (*b*) No; (*c*) No. 8. £1. 8*s*. $1\frac{1}{2}$*d*., £0. 2*s*. $9\frac{3}{4}$*d*.
9. 1658 mm., 3317 mm.

N 1. 990 or 3366. 2. 6411, 6835. 3. £603. 6*s*. 8*d*.
4. 12·8044. 5. (i) 58 sq. ft. 45 sq. in.; (ii) 19 sq. ft. 99 sq. in.
6. 0·007782. 7. 44 ft. 1 in. 8. 20 ft.
9. 26·1%. 10. 9·13 oz. approx.

Exercises LXXXV. a. P. 389.

1. (*a*) 3^5, (*b*) 4^6, (*c*) 10^{11}, (*d*) 4^{13}, (*e*) 100^5, (*f*) 25^{63}, (*g*) $(\frac{1}{6})^{21}$, (*h*) x^{15}, (*i*) a^{x+y}, (*j*) $\left(\frac{1}{b}\right)^{2x+2y}$.

2. $\frac{7}{10}$. 3. $\frac{6}{10}$. 4. $\frac{7}{10}$. 5. $\frac{8}{6}$. 6. $\frac{5}{12}$. 7. $\frac{1}{5}$. 8. $\frac{4}{8}$.

Exercises LXXXV. b. P. 390.

1. $\frac{10^{13}}{10^3}$. 2. $\frac{10^{\frac{5}{6}}}{10^{\frac{7}{8}}}$. 3. $11^{\frac{3}{8}-\frac{5}{10}}$. 4. $\frac{N^{2x}}{N^b}$.

5. $\frac{n^x}{n^y}$. 6. 3^{x-y}. 7. $\frac{100^{12}}{100^{\frac{1}{8}}}$. 8. $\frac{N^{10}}{N^a}$.

9. a^{x-b}. 10. $a^{\frac{p}{q}-\frac{c}{d}}$. 11. a^{x-x}. 12. b^{2-2}.

13. $c^{100-100}$. 14. $(0{\cdot}01)^{\frac{3}{4}-\frac{3}{4}}$. 15. $\frac{x^y}{x^y}$.

Exercises LXXXV. c. P. 390.

1. 2^{-2}. 2. (*a*) 10^{-3}, (*b*) 10^2, (*c*) 10^{-2}, (*d*) 10^{-4}.
3. (*a*) 5^{-4}, (*b*) 5^3, (*c*) 5^{-3}, (*e*) 5^{-2}, (*f*) 5^{-6}.
4. (*a*) 2^{-2}, (*b*) 2^{-3}, (*c*) 2^{-10}, (*d*) 2^{-8}. 5. (*a*) 3^{-1}, (*b*) 3^{-3}, (*c*) 3^{-6}.
6. 2 and 3. 7. -1 and -2. 8. -3 and -4. 9. 2 and 3.
10. 5 and 6. 11. -5 and -6. 12. 5 and 6. 13. 1 and 2.
14. 0 and -1. 15. 0 and -1. 16. 1 and 2. 17. 1 and 2.
18. -2 and -3. 19. 0 and 1. 20. -2 and -3.
21. 1 and 2. 22. -5 and -6. 23. -6 and -7.

Exercises LXXXV. d. P. 391.

1. (*a*) 2, (*b*) -1, (*c*) -4, (*d*) 8, (*e*) 6, (*f*) -5, (*g*) 10, (*h*) -2.
2. (*a*) 1, (*b*) 3, (*c*) -1, (*d*) 5, (*e*) -4, (*f*) -3, (*g*) 3.
3. (*a*) $\frac{1}{6}$, (*b*) 0·5, (*c*) $-(0{\cdot}5)$, (*d*) -1.
4. (*a*) 1, (*b*) -1, (*c*) -2, (*d*) 3, (*e*) 4, (*f*) -5.
5. (*a*) -1, (*b*) 2, (*c*) 3, (*d*) -3, (*e*) -2.
6. (*a*) 0·5, (*b*) 1·5, (*c*) -2, (*d*) $-2{\cdot}5$.
7. (*a*) 0 25, (*b*) 0·75, (*c*) 0·5, (*d*) $-(0{\cdot}5)$, (*e*) $-1{\cdot}25$.
8. (*a*) -1, (*b*) -2, (*c*) 3, (*d*) 2, (*e*) 1, (*f*) 5, (*g*) -5.
9. (*a*) $0{\cdot}1=10^{-1}$, (*b*) $10^2=100$, (*c*) $10^5=100000$,
(*d*) $3^4=81$, (*e*) $4^{-1{\cdot}5}=\frac{1}{8}=0{\cdot}125$, (*f*) $2^{10}=1024$,
(*g*) $10^3=0{\cdot}001$, (*h*) $5^{-3}=\frac{1}{125}=0{\cdot}008$, (*i*) $x^2=x^2$,
(*j*) $8=4^{1{\cdot}5}$, (*k*) $8=16^{\frac{3}{4}}$, (*l*) $16=8^{\frac{4}{3}}$.

Exercises LXXXV. e. P. 392.

1. (*a*) 1 and 2, (*b*) 2 and 3, (*c*) 0 and 1, (*d*) 4 and 5, (*e*) 5 and 6, (*f*) 0 and 1, (*g*) 1 and 2.
2. (*a*) -2 and -3, (*b*) -3 and -2, (*c*) -2 and -1, (*d*) -5 and -4.
3. (*a*) -1 and 0, (*b*) -2 and -1, (*c*) 3 and 4, (*d*) 2 and 3.
4. (*a*) 3 and 4, (*b*) 0 and -1.

Exercises LXXXV. f. P. 394.

1. (*c*) 0·23 and 0·307, (*d*) 0·27 and 0·303, (*e*) 0 and 0·5, (*f*) 0·466 and 0·533, (*g*) 0·43 and 0·478, (*h*) 0·47 and 0·53, (*i*) 0·71 and 0·855, (*j*) 0·846 and 0·92.
2. (*a*) 0·60206, (*b*) 0·69897, (*c*) 0·77815, (*d*) 0·90309. **5.** 0·95424.

Exercises LXXXVI. a. P. 399.

1. (*a*) $6{\cdot}248 \times 10^5$ gr., (*b*) $1{\cdot}428 \times 10^5$ gr., (*c*) $3{\cdot}879 \times 10^5$ gr., (*d*) 3·066 × 10 lb., (*e*) 0·4623 gr., (*f*) 337·9 gr., (*g*) $1{\cdot}56 \times 10^4$ lb., (*h*) $1{\cdot}533 \times 10^{10}$ lb.
2. (*a*) 1309, (*b*) 1093·6, (*c*) 725·9, (*d*) 2703, (*e*) 733, (*f*) $2{\cdot}319 \times 10^{-5}$, (*g*) $2{\cdot}79 \times 10^4$, (*h*) 61·02.
3. (*a*) 43·5, (*c*) 4·58, (*h*) 9·95, (*k*) 10·5. **4.** (*a*) 35·88 c. ft., (*b*) 37·5 c. ft.
5. (*a*) 1269 gal., 0·056 in., (*b*) 3807 gal., 0·168 in., (*c*) 5076 gal., 0·22 in.
6. (*a*) 0·007770, (*b*) 0·001551, (*c*) 0·001168, (*d*) 0·0005714, (*e*) 0·0004007, (*f*) 0·00003776. **7.** 14924 lb.
8. (*a*) 0·0004525, 0·07539, (*b*) 0·094, 0·2953, (*c*) 0·088, 0·2764.
9. (*a*) 15·712, (*b*) 0·089607, (*c*) 53·51.
10. (*a*) $1{\cdot}2875 \times 10^6$, (*b*) 1·8345, (*c*) 0·38773, (*d*) 0·3650.

Exercises LXXXVI. b. P. 403.

1. 96·43. **2.** 0·00071 in. **3.** 399×10^2.
4. (*a*) 170·2, (*b*) 3464, (*c*) 626 × 10 c. ft.
5. (*a*) $G = \dfrac{\sqrt{(3{\cdot}447D)^5 H}}{L}$, (*b*) 9. **6.** (*a*) 167·6, (*b*) 9·61.
7. (*a*) 3887, (*b*) 18·5. **8.** (*a*) 32·9 ft., (*b*) 1759 lb.
9. $4{\cdot}57 \times 10^2$. **10.** (*a*) 1·96, (*b*) 12·3, (*c*) 49·2. **11.** (*a*) 250, (*b*) 96·3 in.
12. (*a*) 3213×10^2, (*b*) 0·0047. **13.** (*a*) 20, (*b*) 5, (*c*) 30.

Exercises LXXXVII. a. P. 418.

1. $£1000(1{\cdot}04)^3$. **2.** $£500(1{\cdot}02)^3$. **3.** $£500(1{\cdot}03)^2$.
4. $£1200(1{\cdot}025)^4$. **5.** $£800(1{\cdot}03)^5$. **6.** $£800(1{\cdot}05)^3$.
7. $£900(1{\cdot}04)^6$. **8.** $£750(1{\cdot}04)^6(1{\cdot}02)$. **9.** $£100(1{\cdot}06)^4(1{\cdot}03)$.
10. $£400(1{\cdot}04)^{10}$. **11.** $£400(1{\cdot}0325)^{10}$.
12. $£500(1{\cdot}05)^2(1{\cdot}025)$. **13.** $£750(1{\cdot}06)^3(1{\cdot}03)$.
14. $£1000(1{\cdot}04)^4(1{\cdot}02)$. **15.** $£1000(1{\cdot}045)^4(1{\cdot}0225)$.

Exercises LXXXVII. b. P. 419.

1. £500$\{(1{\cdot}02)^6 - 1\}$. **2.** £500$\{(1{\cdot}02)^7 - 1\}$. **3.** £500$\{(1{\cdot}01)^{13} - 1\}$.
4. £1000$\{(1{\cdot}005)^{12} - 1\}$. **5.** £200$\{(1{\cdot}0075)^3 - 1\}$.
6. £1000$\{(1{\cdot}01)^{10} - 1\}$. **7.** £1200$\{(1{\cdot}005)^{30} - 1\}$. **8.** $\{(1{\cdot}01)^3 - 1\}$.

Exercises LXXXVII. c. P. 420.

1. (*a*) £153. 15*s.* 0*d.*, (*b*) £13. 13*s.*, (*c*) £19. 8*s.* 3*d.*,
(*d*) £38. 19*s.* to the nearest shilling, (*e*) £72. 5*s.* 3*d.*,
(*f*) £126. 2*s.* 0½*d.*, (*g*) £422. 18*s.* to the nearest shilling,
(*h*) £83. 17*s.* approx., (*i*) £11904 approx.

2. (*a*) 4·04, (*b*) 4·07, (*c*) 6·168 approx., (*d*) 6·09.

3. Interest table :

	(*a*)	(*b*)	(*c*)	(*d*)
yrs.	1 p.c.	1¼ p.c.	2 p.c.	2½ p.c.
1	1·0100000	1·0125000	1·0200000	1·0250000
2	1·0201000	1·0251562	1·0404000	1·0506250
3	1·0303010	1·0379707	1·0612080	1·0768906
4	1·0406040	1·0509443	1·0824322	1·1038129
5	1·0510105	1·0640811	1·1040808	1·1314082
6	1·0615202	1·0773821	1·1261624	1·1596934
7	1·0721354	1·0908494	1·1486949	1·1886857
8	1·0828567	1·1044850	1·1716788	1·2184058
9	1·0936853	1·1182911	1·1951124	1·2488659
10	1·1046222	1·1322697	1·2190146	1·2800875

	(*e*)	(*f*)	(*g*)
yrs.	3 p.c.	4 p.c.	5 p.c.
1	1·0300000	1·0400000	1·0500000
2	1·0609000	1·0816000	1·1025000
3	1·0927270	1·1248640	1·1576250
4	1·1255088	1·1698586	1·2155062
5	1·1592741	1·2166529	1·2762816
6	1·1940563	1·2653190	1·3400957
7	1·2298780	1·3159318	1·4071005
8	1·2667743	1·3685691	1·4774555
9	1·3047775	1·4233119	1·5513283
10	1·3439108	1·4802444	1·6288947

4. (*a*) £28. 17*s*. 0*d*., (*b*) £34. 1*s*. 6½*d*., (*c*) £24. 10*s*. 7*d*., (*d*) £89. 9*s*. 8½*d*., (*e*) £514. 14*s*. approx., (*f*) £410. 12*s*. approx., (*g*) £34. 5*s*. 9*d*.

Exercises LXXXVII. d. P. 421.

1. 5 yrs. **2.** 3%. **3.** 4%. **4.** £1200.
5. £1219 approx. **6.** 2½ yrs. **7.** (*a*) 1·005%, (*b*) 1·06%, (*c*) 1·06%.
8. 5·11, 5·12, 5·125.

Exercises LXXXVII. e. P. 425.

1. (*a*) $\frac{4}{9}$, (*b*) $\frac{41}{333}$, (*c*) $\frac{234}{990}$ or $2\frac{13}{55}$. **2.** (*a*) £100, (*b*) £348. 8*s*.
4. (*a*) £853. 0*s*. 5*d*., (*b*) £2640. 15*s*. 6*d*., (*c*) £507. 19*s*. (*d*) £1507. 6*s*. 8*d*.
5. 1·966, 60 ft. **6.** 70%. **7.** 0·547 lb. **8.** 0·28 approx.
9. (*a*) £3·0909, (*b*) £4·1836, (*c*) 5·3091. **11.** £1154. 10*s*. 3*d*.
12. £1712. 8*s*. 0*d*. **13.** £7. 13*s*. 2*d*. **14.** £133. 18*s*. 8*d*.
15. Interest paid in each half-year instalment, £20, £18·174, £16·311, £14·411, £12·472, £10·495, £8·478, £6·421, £4·323, £2·183. Principal paid £91·327, £93·153, £95·016, £96·917, £98·855, £100·832, £102·849, £104·906, £107·004, £109·144.

Exercises LXXXVII. g. P. 430.

1. (*c*) 26 approx. **2.** (*e*) 30 yrs., (*f*) £23, (*g*) 46 yrs. approx.
3. 172. **4.** (*e*) £10. 5*s*., (*f*) 21 yrs. nearly.
5. (*a*) £9, (*b*) 4·8% approx. **6.** (*a*) 18 yrs. approx., (*b*) 14 yrs. approx.
7. 8¼ yrs. approx. **8.** 44½ yrs. approx.
9. (*a*) 6·4 yrs., (*b*) 17 yrs. approx.

Exercises LXXXVIII. P. 434.

1. (*a*) 3·5, (*b*) 60, (*c*) 1710. **2.** (*a*) 56, (*b*) 60, (*c*) 5280.
3. (*a*) 9, (*b*) 16, (*c*) 336. **4.** (*a*) 17·5, (*b*) 24, (*c*) 660.
5. (*a*) 6·5, (*b*) 24, (*c*) 924. **6.** (*a*) 5·5, (*b*) 12, (*c*) 126.
7. (*a*) 34·5, (*b*) 60, (*c*) 2730. **8.** (*a*) 7, (*b*) 60, (*c*) 1080.
9. (*a*) 0, (*b*) $\sqrt{b^2-\frac{a^2}{4}}$, (*c*) $\frac{a}{2}\sqrt{b^2-\frac{a^2}{4}}$. **10.** (*a*) 0, (*b*) $\frac{a\sqrt{3}}{2}$, (*c*) $\frac{a^2\sqrt{3}}{4}$.
11. 103 in. **12.** 100 in.

14. (*a*) 21751, (*b*) 2293, (*c*) 3588×10^2, (*d*) 6068·8, (*e*) 152850, (*f*) 791, (*g*) 3068×10^2, (*h*) 44212·8, (*i*) 735368, (*j*) 3243×10^2, (*k*) 1764192, (*l*) 96510·7.

15. (*a*) 1 : 3, (*b*) 1 : 8, (*c*) 4 : 5, (*d*) 9 : 7, (*e*) 16 : 9, (*f*) $x^2 : (y^2 - x^2)$, (*g*) $1 : (x^2 - 1)$.

16. (*a*) $\frac{7}{40}$, (*b*) $\frac{14}{55}$, (*c*) $\frac{355}{132}$, (*d*) $\frac{403}{1320}$. 17. 11·3 in.

Exercises LXXXIX. P. 437.

1. £2. 9*s*. 9*d*.
2. £4. 19*s*. 9*d*.
3. £0. 4*s*. 1½*d*.
4. £49. 19*s*. 4*d*.
5. £72. 0*s*. 4*d*.
6. £7. 9*s*. 7*d*.
7. £1. 6*s*. 9*d*.
8. £13. 8*s*. 11½*d*.
9. £2. 0*s*. 1*d*.
10. £27. 0*s*. 2½*d*.
11. £1. 11*s*. 6½*d*.
12. £1. 0*s*. 1¼*d*.
13. £4. 13*s*. 2½*d*.
14. £26. 10*s*. 11*d*.
15. £2. 6*s*. 11¾*a*.
16. £4. 5*s*. 4½*d*.
17. £1. 11*s*. 4½*d*.
18. £9. 18*s*. 0¾*d*.
19. £88. 17*s*. 8*d*.
20. £1. 3*s*. 0¾*d*.
21. Jan. 23, 1906.
22. £876
23. £1250.
24. July 1, 1905.
25. Sept. 10, 1905.
26. 3%.
27. 3%.
28. July 1, 1905.
29. Jan. 30.
30. Jan. 8.
31. 2½%.

Exercises XC. P. 440.

1. (*a*) 1 in., (*b*) $\sqrt{15}$.
2. (*a*) 3·6 in., (*b*) 4·8 in.
3. (*a*) (i) 14·94, (ii) 1·25; (*b*) (i) 17·4, (ii) 2·25; (*c*) (i) 8·912, (ii) 1·25; (*d*) (i) 32·25, (ii) 16; (*e*) (i) 11·4, (ii) 8·128; (*f*) (i) 12, (ii) 5; (*g*) (i) 60, (ii) 11; (*h*) (i) 15, (ii) 8; (*i*) (i) 20, (ii) 15; (*j*) (i) $\sqrt{l^2 - \frac{s^2}{4\pi^2}}$, (ii) $\frac{s}{2\pi}$.
4. 188·5 sq. ft.
5. 10 sq. in (approx.).
6. 55·0 sq. in.
7. (*a*) 254·4 sq. in., (*b*) 235·6 sq. ft., (*c*) 175·9 sq. ft., (*d*) 189·4 sq. ft., (*e*) 115·6 sq. ft.
8. 348·72 sq. in.
9. $(21 \cdot 6\pi)$ or 67·858 sq in.
10. 321π.
11. 285 sq. in. (approx.).
12. (*a*) 75π or 235·62 sq. cm., (*b*) 104π or 326·73 sq. cm., (*c*) 1891π or 5940·7 sq. in., (*d*) 17π or 53·4 sq. in., (*e*) 26840π or 843×10^2 sq. in.

Exercises XCI. a. P. 442.

1. 30·39 sq. in.
2. 475·3 sq. in.
3. 12·70 sq. cm.
4. 1932×10^2 sq. cm.
5. 2865 sq. cm.
6. 1810 sq. in.
7. 3·801 sq. ft.
8. 7917 sq. cm.
9. 5542 sq. cm.
10. 42·7 cm.
11. 46·3 ft.
12. 5·1 in.
13. 10·3 cm.
14. 30·2 ft.
15. 20·1 cm.
16. 15·2 in.
17. 38·2 yd.

Exercises XCI. b. P. 443.

1. 201×10^6 sq. mls.
2. 44·178 sq. in.
3. 40·55 cm.
4. 50·265 sq. in., 0·160 lb.
5. (*a*) $33{\cdot}51 \times 10^6$, (*b*) $13{\cdot}9 \times 10^6$, (*c*) $27{\cdot}93 \times 10^6$, (*d*) $50{\cdot}8 \times 10^6$, (*e*) $39{\cdot}4 \times 10^6$, (*f*) $64{\cdot}88 \times 10^6$, (*g*) $5{\cdot}586 \times 10^5$.
6. (*a*) $8{\cdot}33 \times 10^6$, (*b*) $52{\cdot}1 \times 10^6$, (*c*) $80{\cdot}2 \times 10^6$, (*d*) 30×10^6, (*e*) 100×10^6, (*f*) $2{\cdot}75 \times 10^5$, (*g*) $1{\cdot}27 \times 10^5$, (*h*) $14{\cdot}46 \times 10^6$.
7. 40π or 125·7 sq. in.
8. 64π or 201·06 sq. ft.
9. 160·758 sq. cm.
10. 163·363 sq. in.
11. 179·071 sq. in.
12. 1266·7 mm.
13. 2264 lb.

Exercises XCII. P. 453.

1. (*a*) 44·12 cub. ft., (*b*) 136·19 cub. ft., (*c*) 9·2 in., (*d*) 7·5 cm.
2. 7·57 gal.
3. 4 gal. (approx.).
4. 422 gal.
6. 3 mm. nearly.
7. (*a*) surf 9161 sq. cm., vol. 825×10^2 c.c.; (*b*) surf 16·62 sq. ft., vol. 6·37 cub. ft.; (*c*) 128·7 sq. in., dia. 6·4 in.; (*d*) surf 283·529 sq. in., dia. 9·5 in.; (*e*) vol. 203·69 c.c., dia. 7·3 cm.
8. $14\frac{2}{3}$ cub. in.
9. 150·8 cub. in.
10. 53·4 cub. in.
11. 268 cub. in.
12. 396 cub. ft.
13. 180 grms.
14. 79·8 cub. in.
15. 9·45 lb.
16. 4974 cub. ft.

Exercises XCIII. P. 458.

1. (*a*) 1460 c.c., (*b*) 735 c.c., (*c*) 3750 c.c.
2. 2600 cub. in.
3. 217 cub. yds.

Revision Exercises XCIV. P. 459.

Miscellaneous.

A
1. 1325610.
2. 4·644 in.
3. £4. 19*s.* 0*a.*
4. $1\frac{10}{11}$ ac., £3465.
5. £5533. 6*s.* 8*d.*
6. $1\frac{4}{11}$ days.
7. £105.
8. 1 m. 4 dm. 9 cm. 8 mm.
9. 135 years nearly.
10. 0·26275 ac.

B
1. $9\frac{1}{11}$.
2. £52083 approx.
3. 13·4.
4. 2 T. 18 cwt.
5. £47. 14*s.* $2\frac{1}{4}$*d.*, 27*s.* $11\frac{1}{4}$*d.*
6. 495 cub. ft.
7. 204 yd., $\frac{3}{28}$ min.
8. £3. 2*s.*
9. $\sqrt{57}$, 6 in. roughly.
10. 2 yrs. 9 mo. approx.

C **1.** 23, 3·89 in. **2.** Difference = 0·0037 in. **3.** 5*s*.
4. 3·14. **5.** £11. **6.** £1. 6*s*. 8*d*. **7.** £866 approx.
8. 102·6 hrs. approx. **9.** 29·85 ac. **10.** 89304.

D **1.** 17810, 9·995, 0·40156625, 2120·2698. **2.** 32%.
3. £1. 10*s*. 10½*d*. **4.** £120. **5.** 2 yd. **6.** 5.
7. £37. 4*s*. 0*d*. **8.** **9.** £1066. 13*s*. 4*d*.
10. 1·2 ac. approx.

E **1.** 3·141597, 1·00000127. **2.** £2248. 6*s*. 3*d*. **3.** 8 secs.
4. £3, £2. 8*s*. 0*d*., £0. 12*s*. 0*d*. **5.** 50 miles.
7. 53 approx. **8.** 16 a shilling. **9.** £130. 11*s*. 6*d*.
10. 0·247, 0·987, 2·221, 3·948, 6·169, 8·883, 12·091, 15·792. No. Yes.

F **1.** 7 cwt. 2 qr. 18 lb. **2.** 502386, 505087, 507788. **3.** 1561·6.
4. 108. **5.** $\pi\sqrt{\frac{l}{g}}$.
6. \$6168 × 10^6, \$594 × 10^7, \$4717 × 10^6, 62 × 10^6 app. **7.** 11¼ mile.
8. 26·3 approx. **9.** 1½ in. **10.** 4 ac. 2 ro. 23 po. to nearest pole.

G **2.** 640 mm. **3.** £161. 9*s*. 6*d*. **4.** £453. 10*s*. 3¾*d*.
5. £4. 12*s*. 9¾*d*. **6.** 27 miles. **7.** 161·133 approx.
8. 1·59%. **9.** £0. 6*s*. 9*d*. **10.** £194. 4*s*. approx.

H **1.** 3956. **2.** £631. 11*s*. 7*d*. **3.** 21⅜%.
4. 1·53 m., 0·76 m. **5.** $\frac{2}{3}\frac{7}{4}$ liquid, $\frac{7}{3\,4}$ water.
6. 0·0576723, 2·538 ft. approx. **7.** 4½%. **8.** 24.
9. 1 : 0·993, 1 : 7·777. **10.** 36 ac. 0 ro. 23 po. approx.

I **1.** $\frac{1}{2}\frac{5}{9}\frac{2}{9}$, 4·00001. **2.** 101·6 nearly. **3.** 100%. **4.** 192 in.
5. 119, 119 ft., 119 sq. ft. **6.** 6 cub. ft. 192 cub. in.; 4 yd.
7. 0·815. **8.** £57. 6*s*. 4½*a*. **9.** **10.** £280·07

J **1.** (i) 1·0004 nearly; (ii) 0·7022 nearly. **2.** 5·68 m., 1·602 sq. m.
3. 56. **4.** 18·5625. **5.** 24240 3rd, 9696 1st. **6.** £126. 2*s*. 0*d*.
7. £715. **8.** $30\frac{1}{8}$. **9.** £3·495 × 10^4.
10. 4 ac. 0 ro. 25 sq. p. approx.

K **1.** £18524225. 10*s*. 0*d*. **2.** 4%, 9. **3.** (*a*) $117\frac{2}{2}\frac{1}{2}$ yd.; (*b*) 3¾ min.
4. 15 min. 16 sec. **5.** 194·4 ton.
6. 14592 cub. in., 8064 cub. in., 6528 cub. in.
7. (*a*) £1503160; (*b*) £2000000. **8.** £1. 13*s*. 4*d*.
9. £146. 13*s*. 4*d*. **10.** (*a*) 4 yr. 5½ mo. approx.; (*b*) £35⅛ approx.

L 1. 376, 379. 2. 20 casks, 21 mm. 3. 62·5.
4. $2{\cdot}408$, $a^3\sqrt{2}$. 5. 13. 6. 67·08 ft., 12·1 ft.
7. £1. 19*s*. $8\frac{1}{4}d$. 8. £625. 9. 2 : 5.
10. 465 yd. and 846 yd. approx.

M 1. $1234\frac{3}{7}$. 2. 1 min. $52\frac{1}{2}$ sec. 3. £819. 6*s*. 3*d*.
4. 25. 5. 2825761, 2560000.
6. £3550, £3124, £2982, amounts would be £3536, £3128, £3992.
7. £1000. 8. £20. 5*s*. 4*d*., 2·09.
9. 18019, 25806, 9·055... ac. 10. 8, $40\frac{1}{2}$ just outside C.

N 1. 1 ro. 19 sq. po. $15\frac{1}{4}$ sq. yd. 2. $11^2 \times 7^2 \times 13^2$.
3. 54, 0·00096 gr. 4. 1210. 5. $8\frac{1}{3}$ min. 6. 12 days.
7. 22 hrs. 8. 27. 9. £11. 0*s*. 8*d*. 10. 1·16 ac. approx.

Exercises XCV. P. 480.

1. 69 to nearest day. 2. 71 days to nearest day.
3. 83 days. 4. 22; Aug. 23.
5. (*a*) 80, Sept. 20; (*b*) Aug. 15.
6. (*a*) £650, Nov. 25, 1906.; (*b*) Nov. 29, £1585; (*c*) £400, Dec. 3; (*d*) £1500, Mar. 7 in the following year.

Exercises XCVI. P. 485.

1. (*a*) 7·98805. (*b*) 7·93787. 2. (*a*) 5·65518. (*b*) 9·44984.
3. (*a*) £3. 17*s*. $10\frac{1}{2}d$. (*b*) £4. 4*s*. $11\frac{1}{2}d$. 4. £3. 17*s*. $5\frac{1}{2}d$.
5. 935. 6. 132. 7. 0·941 in each case.
8. 0·678 approx. 9. 0·663 approx.
10. (*a*) £0. 9*s*. $9\frac{1}{2}d$. (*b*) 7*s*. 11·4*d*. (*c*) £2. 1*s*. 1*d*.
11. (*a*) 0·90. (*b*) 0·98. (*c*) 0·917. (*d*) 0·900.
(*e*) 0·874. (*f*) 0·900. (*g*) 0·873. (*h*) 0·916.
(*i*) 0·919. (*j*) 0·917. (*k*) 0·900. 12. 25.22.
13. (*a*) £1 = 6·305 roubles. (*b*) £1 = 9·762 yen. (*c*) £1 = 18·1595 kr.
14. (*a*) 0·11 fr. (*b*) 3927 fr. 15. 20·31 and 20·53.

Exercises XCVII. P. 489.

1. (i) (*a*) £15,251. 17*s*. $3\frac{1}{2}d$. (*b*) £15,233. 18*s*. $5\frac{1}{4}d$.
(*c*) £15,349. 9*s*. $1\frac{3}{4}d$. in 3 mo., or £15,234. 6*s*. $8\frac{1}{2}d$. at present.
(*d*) £15,337. 4*s*. 11*d*. in 3 mo., or £15,222. 4*s*. 4*d*. at present.

(ii) (*a*) £1731. 4*s*. 6*d*., (*b*) £1729. 3*s*. 0*d*., (*c*) £1742. 5*s*. 3*d*., (*d*) £1740. 17*s*. 6½*d*.

(iii) (*a*) £2305. 12*s*. 1*d*., (*b*) £2302. 17*s*. 10*d*., (*c*) £2320. 7*s*. 1*d*. (*d*) £2318. 2*s*. 3½*d*.

2. (*a*) £22,009. 4*s*. 2½*d*. (*b*) £22,127. 3*s*. 10¼*d*.
(*c*) £22,135. 7*s*. 4*d*. in 3 mo. (*d*) £22,168. 0*s*. 6*d*. in 3 mo.

3. 2½%. 4. 25·40.

Exercises XCVIII. a. P. 494.

1. (*a*) £1453. 2*s*. 6*d*., (*b*) £2124. 12*s*. 6*d*., (*c*) £1150. 10*s*. 0*d*.
2. (*a*) £2408. 15*s*. 0*d*., (*b*) £4120. 10*s*. 0*d*., (*c*) £4850.
3. (*a*) £440; (*b*) (i) £690, (ii) £186. 5*s*., (iii) £20.

Exercises XCVIII. b. P. 498.

1. £1131. 12*s*. 0*d*. 2. £831. 11*s*. 6*d*. 3. £1038. 15*s*. 0*d*.
4. £22975. 5. £1084. 2*s*. 6*d*. 6. £542. 18*s*. 6*d*.
7. £3701. 5*s*. 0*d*. 8. £4440. 9. £459. 7*s*. 6*d*.
10. £9837. 10*s*. 0*d*. 11. £1254. 8*s*. 0*d*. 12. £5812. 10*s*. 0*d*.
13. (*a*) £3·046, (*b*) 3·503, (*c*) 3·429, (*d*) 3·483, (*e*) 3·333, (*f*) 3·670, (*g*) 3·431, (*h*) 3·315, (*i*) 3·352, (*j*) 4·124, (*k*) 3·9025, (*l*) 3·465, (*m*) 3·449, (*n*) 3·297, (*o*) 3·390.
14. (*a*) £45. 8*s*. 0*d*. gain, (*b*) £109. 13*s*. 0*d*. loss, (*c*) £123. 10*s*. 0*d*. loss, (*d*) £126. 17*s*. 0*d*. gain, (*e*) £4. 2*s*. 0*d*. loss, (*f*) £37. 2*s*. 0*d*. loss, (*g*) £10. 10*s*. 0*d*. gain, (*h*) £29. 15*s*. 0*d*. loss, (*i*) £2. 2*s*. 0*d*. loss, (*j*) £424. 18*s*. 0*d*. loss.
15. (*a*) £94, (*b*) £135, (*c*) £124. 4*s*. 0*d*., (*d*) £102, (*e*) £108¼, (*f*) £151½.
16. (*a*) 5⅜, (*b*) 5¼, (*c*) 3⅓, (*d*) 2 9/16, (*e*) 5½, (*f*) 6⅛, (*g*) 2.
17. (*a*) 4, 5½, (*b*) £3. 4*s*. 6*d*., 2½, 8, (*c*) £6. 16*s*. 4*d*., 5½, 7½, (*d*) 3, 6, (*e*) 3, 5½, 7, (*f*) 2¾, 4, 6. 18. 300000.
19. £22000, 10%. 20. £1650. 21. £1487. 10*s*. 0*d*.
22. £510. 23. £811411, £34485.
24. £357. 2*s*. 10*d*., £1642. 17*s*. 2*d*., £1785. 4*s*. 3*d*.
25. (*a*) £326925, (*b*) £0. 18*s*. 10*a*. 26. (*a*) 1139500, (*b*) 1·449*d*.
27. 5%. 28. 31157 approx. 29. £843333⅓. 30. £1197023.
31. £11538. 32. £9727. 33. £753951. 34. £5·62.

Exercises XCIX. P. 510.

1. (*a*) 5 hr. 25 min. (*b*) 4 hr. 30 min. (*c*) 5 hr. 13 min.

2. (*a*) 5 hr. 37 min. (*b*) 5 hr. 53 min. (*c*) 5 hr. 38 min.

3. (*a*) 4 hr. 33 min. (*b*) 250 and 255 min. (*c*) 4 hr. 15 min.
4 hr. 18 min.

4. (*a*) 4 hr. 26 min. (*b*) 3 hr. 52 min., 3 hr. 58 min., 4 hr. 10 min., 4 hr. 30 min. (*c*) 4 hr. 22 min.

6. Mode 4 hr. 15 min.—4 hr. 30 min.; 5 hr. 45 min.—6 hr.; 4 hr.—4 hr. 15 min.; 3 hr. 50 min.—4 hr. 5 min.

7. (*a*) 23·67. (*b*) 23·65. (*c*) 23·655.

8.

	(*a*)	(*b*)	(*c*)
	£ s. d.	£ s. d.	£ s. d.
Arith. Av.	1 . 19 . 5·4	2 . 8 . 10½	3 . 4 . 8
Median	1 . 19 . 8—11	2 . 8 . 10—11	3 . 4 . 8—10
Mode	1 . 19 . 0—3	2 . 8 . 4	£ s. d. £ s. d. 3 . 4 . 3—3 . 4 . 5 3 . 4 . 6—3 . 4 . 8 3 . 5 . 6—3 . 5 . 8

	(*d*)	(*e*)
	£ s. d.	£ s. d.
Arith. Av.	4 . 11 . 3	6 . 19 . 11
Median	4 . 11 . 1—3	6 . 18 . 11 }
Mode	4 . 11 . 0 } to 4 . 11 . 6 }	6 . 19 . 5 } 7 . 2 . 0 } to 7 . 3 . 0 }

9. (*a*) 6 st. 1¼ lb. and 6 st. 2 lb. (*b*) 6 st. to 6¼ st. (*c*) 6 st. 1½ lb.

10. (*a*) (i) 45·63. (ii) 46·6—47·1. (*b*) (i) 35·78. (ii) 36·6—36·7.
(*c*) (i) 23·65. (ii) 22·8. (*d*) (i) 28·72. (ii) 29·5.
(*e*) (i) 42·83. (ii) 42·9—42·2. (*f*) (i) 12·01. (ii) 11·8—12·2.
(*g*) (i) 29·4. (ii) 29·6—29·8. (*h*) (i) 3·18. (ii) 3·2.
(*i*) (i) 7·41. (ii) 7·2. (*j*) (i) 26·42. (ii) 25·4—26.

11. (*a*) (i) 100—109. (ii) 105·7. (*b*) (i) 120—129·9. (ii) 129·4.
(*c*) (i) 130—139·9. (ii) 118·2. (*d*) (i) 95—99·9. (ii) 99·1.
(*e*) (i) 90—94·9. (ii) 97·7. (*f*) (i) 96. (ii) 92.
(*g*) (i) 95·4. (ii) 95·4. (*h*) (i) 120. (ii) 120.
(*i*) (i) 80. (ii) 90.

12. (*a*) $4\frac{13}{20}$, $4\frac{11}{14}$, $3\frac{5}{8}$, $3\frac{3}{4}$, $2\frac{7}{8}$, $3\frac{171}{400}$, $3\frac{38}{75}$, $2\frac{779}{900}$, $2\frac{33}{40}$.
(*b*) 3·64 true to 2nd dec. place. (*c*) 3—3·5. (*d*) $3\frac{1}{3}$.

13. (i) (*a*) $93\frac{1}{2}$, $92\frac{1}{4}$, $91\frac{9}{20}$, $92\frac{11}{20}$, $95\frac{29}{80}$, $100\frac{1}{4}$.
(*b*) 94·2. (*c*) 92·5—93. (*d*) 93.
(ii) (*a*) 1884-1888, £1,847,240; 1889-1893, £9,015,740; 1894-1898, £4,371,420; 1899-1903, £8,353,120; (*b*) £6,141,900 to nearest £100; (*c*) mode in groups of £1,000,000 between £2,000,000 and £3,000,000; (*d*) £5,780,600.

14. (*a*) $59\frac{33}{40}$, $59\frac{57}{80}$, $59\frac{1}{2}$, $60\frac{5}{80}$, $61\frac{11}{20}$, $61\frac{27}{80}$, $60\frac{59}{80}$, $59\frac{63}{80}$, $53\frac{13}{20}$, $52\frac{13}{20}$, $44\frac{67}{80}$, $38\frac{17}{40}$, $28\frac{41}{80}$.
(*b*) 53·9281. (*c*) 60·4. (*d*) $60\frac{1}{2}$. (*e*) $59\frac{1}{2}$.

15. Value from arithmetic mean 30·12—30·24.

16. (*a*) 22—23. (*b*) 25·6—25·8. (*c*) 26·0.

17. 10, 0·633 cm.

18. (*a*) 32·04—32·026. (*b*) $(3700 \pm 2) \times 10^{-8}$ cm.
(*c*) 492·57—490·23.

19. (*a*) 49*s.* 11*d.*, 88·1%. (*b*) 49*s.* 11*d.*, 61·6%. (*c*) 27*s.* 3*d.*, 48·0%.

Exercises C. P. 524.

1. (*a*) 24. (*b*) 120. (*c*) 720. **2.** (*a*) 24. (*b*) 210. (*c*) 120.
3. 14950. **4.** (*a*) $\frac{3}{36}$. (*b*) $\frac{30}{36}$. (*c*) $\frac{3}{36}$.
5. 1365. **6.** 12150. **7.** $\frac{1}{5525}$.

Exercises CI. P. 528.

1. (*a*) £7. 13*s.* 2*d.* (*b*) £7. 5*s.* 10*d.*
2. (*a*) £7. 19*s.* 3*a.* (*b*) £7. 11*s.* 11*d.*
3. (*a*) £9. 8*s.* 1*d.* (*b*) £8. 17*s.* 1*d.*
4. (*a*) £5. 8*s.* 7*d.* (*b*) £5. 2*s.* 9*d.*
5. (*a*) £14. 17*s.* 1*d.* (*b*) £13. 9*s.*
6. £1. 3*s.* 8*d.* **7.** £1. 9*s.* 8*d.* **8.** £2. 5*s.* 7*d.* **9.** £2. 10*s.* 2*d.*
10. £3. 9*s.* 3*d.* **11.** £1. 5*s.* 6*d.* **12.** £3. 18*s.* 5*d.*
13. (*a*) 18·93. (*b*) 18·31. (*c*) 17·71. (*d*) 17·12.
(*e*) 20·68. (*f*) 20·01. (*g*) 19·38. (*h*) 18·66.
14. (*a*) £631. 10*s.* 7*d.* (*b*) £1066. 3*s.* 9*d.* (*c*) £1293. 18*s.* 4*d.*
(*d*) £5313. 3*s.* (*e*) £958. 0*s.* 6*d.*

Revision Exercises CII. P. 529.

Miscellaneous.

A **1.** 16836, $2^3 \times 3^2 \times 5 \times 11 \times 37 = 146520$. **2.** (i) 0·253037; (ii) £0. 14*s*. 11*d*.
3. 39·32. **4.** £47. 2*s*. 10*d*. approx. **5.** 33 min. past 1.
6. 10 hr. 48 min. **7.** 16·7. **8.** 0·9.
9. £1552. 10*s*. 0*d*., £26347. 10*s*. 0*d*. **10.** 2·89 miles from **C**.

B **1.** $2^3 \times 3^3 \times 7^3 \times 11^3$. **2.** 453·1. **3.** 12 miles 936 yd.
4. 212½ ft. **5.** £241. 5*s*. 6*d*., £308. 6*s*. 0*d*., £290. 8*s*. 6*d*. approx.
6. £17. 10*s*. 0*d*. **7.** 1·02. **8.** £5. 2*s*. 7¼*d*.
9. £115. 9*s*. 9*d*. nearly. **10.** $a = -41{\cdot}6$, $b = 11{\cdot}24$.

C **1.** 496. **2.** 112 sq. yd., £1. 16*s*. 0*d*. **3.** 3·3227.
4. £350. **5.** 11336 to nearest sov. **6.** 4331.
7. 17. **8.** $\pi(36 + 14\sqrt{13})$.
9. £1. 10*s*. 8¾*d*. per oz., 8. **10.** 44, 44.

D **1.** 19. **2.** 417000. **3.** 62 lb. 6·8 oz.
4. £17. 10*s*. 0*d*., £70, £157. 10*s*. 0*d*. **5.** £3. 3*s*. 8*d*.
7. 5·56 sq. mile. **8.** 15.
9. £14400, 83⅓. **10.** 9.

E **1.** (i) $56\frac{29}{70}$; (ii) 924.
2. $2 \times 13 \times 41$, $3 \times 5 \times 11^2$, $2^3 \times 3 \times 79$, $2^3 \times 3 \times 5 \times 11^2 \times 79 \times 13 \times 41$.
3. 20526 cub. ft. 171 cub. in.
4. £600 to good, less £13. 7*s*. 6*d*. bank charges.
5. £77. 11*s*. 0*d*. **6.** 2·09787 kilo.
7. 2 hr. 47 min. to nearest min. **8.** $4\sqrt{3}$ or 6·928 in.
9. $£\left(100 + \frac{x}{10}\right)$, £350, 4250. **10.** 2·646 in.

F **1.** 0·508857. **3.** 703, 1234455346. **4.** 81097 sq. mile.
5. 48·64. **6.** No, 1*s*. 3½*d*. **7.** 5·7596 cub. ft., 323·95 lb.
8. £1912. 9*s*. 9½*d*. **9.** £23. 14*s*. 0*d*.
10. (1) 8 h. 9½ m. a.m.; (2) 3 hr. 46 m.; (3) Dec. 21.

G **1.** (i) 891·4; (ii) No. **2.** C £8531. 3*s*. 0*d*.; D £5687. 8*s*. 8*d*.
3. 9370·0739, 570·73 sq. ft. **5.** £2049. 3*s*. 7*d*.
6. 22·5 yr. approx. **7.** 7. **8.** £10. 14*s*. 6*d*.
9. 37 ft. 6 in., 30 ft., 12 ft.

H 1. 21·13859 lb. 2. 2·34186. 3. (*a*) £173·155; (*b*) £18·891.
4. £621. 10*s*. 5. (*a*) 48·8 in.; (*b*) 20·95 in. 6. 8·31.
7. £2306. 5*s*. 0*d*. 9. 2·49 cm. approx.

I 1. 23. 2. 0·318. 3. 43, 45. 4. £10 increase.
5. 21·8 %. 6. £14. 15*s*. $4\frac{1}{2}d$. 7. $381\frac{1}{8}$ cub. in., 11*s*.
8. 1 : 17·568. 9. £$888\frac{24}{67}$, £$44\frac{28}{67}$, £$555\frac{15}{67}$.
10. 2 ac. 3 ro. 21 po. approx.

J 2. $\frac{61}{1265}$. 3. $\frac{1}{24}$, $\frac{901}{24000}$. 4. 3*s*. 5. 428·456 kilo.
6. £1333. 6*s*. 8*d*., $3\frac{7}{16}$ %. 7. 122·5 cub. in., 24·5 cub. in.
8. £487. 16*s*. 8*d*. 9. 38 class A, 62 class B approx.
10. 31 ac. 0 ro. 33 sq. po.

K 1. 2·78, 2·718282. 2. 105×10^6. 3. 1*s*. 3*d*., $1{\cdot}11 \times 10^6$ approx.
4. 4·06294. 5. (i) 0·04; (ii) 0·27. 6. 13·1 in.
7. £2604. 3*s*. 4*d*. 8. 61 : 35. 9. 10·5. 10. 60, 92·5, 123.

L 2. (*a*) 44; (*b*) £30. 5*s*. 6·6*d*; (*c*) 82·6234; (*d*) 1·5241.
3. 714 cub. in. 4. 50. 5. 0·84. 6. 4 %.
7. 49 % gold, 51 % silver. 8. 0·75 ft.
9. Diminished, 30 : 29.

M 1. 0·27959 approx. 2. 43·2. 3. £8. 16*s*. 9*d*. 4. 12.
5. 10, 49·8; $36\frac{12}{13}$ miles per hour.
6. Tax, £12. 16*s*. 0*d*.; income, £243. 4*s*. 0*d*. 7. 7·6 gal.
8. 84 years. 9. 440 approx. 10. 1 : 1 nearly.

N 1. 410 ft. 2. 10182 lb. 3. £$30526\frac{6}{19}$. 4. 461 ft.
5. 23 % nearly. 6. 11 miles per hour.
7. 23·37, 25·87, 25·7, 28·54, 33·34, 26·98. 8. £3. 13*s*. 9*d*.
9. £3750. 10. (i) 4·58, 0·22; (ii) 3·55.

O 1. £0. 1*s*. 8*d*. 2. 0·900. 3. $102\frac{37}{44}$. 4. £195, £3480
5. 9231·6 cub. ft. 6. Second expression. 7. 103 cm., 53 cm. approx
8. 26·5 per thousand. 9. 642 approx. 10. Loses £1. 0*s*. 10*d*. on £100

P 1. 92 lb. 2. 84. 3. $\frac{43}{54}$, 42018193 14. 4. 42 lb.
5. $\frac{2abx}{100}$ approx., 16·68. 6. $\frac{27abc}{4}$, 45000.
7. $33\frac{3}{4}$, 27, $22\frac{1}{2}$ days respectively. 8. £3000.
9. $\frac{7}{12}$ mile, $\frac{1}{2}$ mile. 10. 0·87 ft., 0·78 in.

Typical Examination Papers. P. 555.

PART I.

Cambridge Local (Preliminary). I.

1. 1176009, One million one hundred and seventy-six thousand and nine.
2. £14407. 8*s.* 10*d.* 3. £62. 12*s.* 0½*d.* 4. £7. 9*s.* 4*d.*
5. 77. 6. (i) $\frac{7}{8}$, (ii) $\frac{7}{12}$. 7. 7·014. 8. £0. 17*s.* 10*d.*
9. 1636 fr. 30 c. 10. £12. 3*s.* 0*d.* 11. £54. 12. 114 ac.

Preliminary. II.

1. 494078. 2. £15983. 1*s.* 6*d.* 3. £87. 8*s.* 11½*d.* 4. £336.
5. (i) $\frac{3}{5}$, (ii) $\frac{5}{7}$. 6. 2·12. 7. 1265. 8. 2 fr. 80 c.
9. £23. 8*s.* 9*d.* 10. £0. 1*s.* 7*a.*, 18·4%. 11. 2½ miles.

Preliminary. III.

1. 120005. 2. £7999. 19*s.* 8*d.* 3. £365. 12*s.* 6*d.* 4. 321.
5. £435. 1*s.* 4*d.* 6. (i) 6, (ii) $\frac{5}{14}$. 7. 75800, 0·00758. 8. £6. 15*s.* 0*d.*
9. 2588 sq. yds., 23292 sq. ft. 10. 8 kilom. 47 m. 11. 10.

Junior.

A.

1. 19. 2. 441000 sec. 3. $\frac{3}{4}$, 167960. 4. £11. 11*s.* 9*d.*
5. £73. 0*s.* 10½*d.* 6. 326 fr. 4 c. 7. £3. 17*s.* 7*d.*

B.

1. 34·56. 2. 14. 3. A 200, B 328. 4. 192.
5. 2600 dollars. 6. 92.

Oxford Local (Preliminary). I.

1. £103. 10*s.* 11¼*d.* 2. 4752. 3. 7½. 4. 0·4448.
5. 0·84375. 6. $\frac{9}{160}$. 7. £47. 7*s.* 10*d.* 8. £16. 9. 2½.

Preliminary. II.

1. £2257. 16*s.* 8¾*d.* 2. 38. 3. $6\frac{23}{42}$, $39\frac{2}{7}$.
4. £1. 14*s.* 4*d.* 5. 0·3015. 6. 0·0375.
7. £14. 16*s.* 9¾*d.* 8. £1000. 9. £146. 6*s.* 0¾*d.*

Preliminary. III.

1. £15. 7*s.* 1½*d.* 2. 2, 7, 13, 14, 26, 91, 182. 3. 4.
4. $\frac{1}{21}$. 5. 0·080325. 6. $\frac{13}{16}$. 7. £14. 11*s.* 0*d.*
8. £17. 9*s.* 6*d.* 9. 3½*d.*

Preliminary. Higher Arithmetic.

1. 0·0187. 2. Cost £1. 12*s.* 6*d.* 3. 3¼ years. 4. 36 marks 11 pfennigs.
5. Each son £1000, each daughter £600, each nephew £200.
6. Increase of £35. 7. £11. 5*s.* 0*d.*

Junior.

1. $1\frac{3}{4}$. 2. $\frac{4}{17}$. 3. £96. 14*s.* 4·125*d.*
4. $\frac{161}{72}=2{\cdot}23611\ldots$, $\sqrt{5}=2{\cdot}236068$. 5. 44 francs 8 centimes.
6. 13 men. 7. £163. 1*s.* 6*d.* 8. £0. 18*s.* 5*d.*
9. £400. 10. 5% at 132.

College of Preceptors, Third Class. I.

1. (i) 34138054, (iii) seventeen thousand four hundred and eighteen.
2. 142 tons 6 cwt. 0 qr. 24 lb. 3. 6 ac. 3 ro. 37 sq. po. 14·75 sq. yds.
4. £7475. 10*s.* 0*d.* 5. $29\frac{19}{120}$, $\frac{56}{255}$. 6. 1.
7. $\frac{1}{50}$, $\frac{9}{500}$, $2\frac{1}{4}$, $1\frac{1}{200}$. 8. 0·21402, 1200.
9. £4. 9*s.* 11¾*d.* 10. £156. 15*s.* 4½*d.* 11. 9216.

Professional Preliminary.

1. 384. 2. £1. 19*s.* 6*d.* 4. $5\frac{15}{371}$.
5. 0·861406. 6. 0·243 *nearly*. 7. 17 fr. 72 c.
8. £134. 2*s.* 8*d.* 9. 46 cm. 10. 5911 yds.
11. £11. 13*s.* 4*d.*, £16. 13*s.* 4*d.* 12. 322.

Second Class or Junior. I.

1. £1953. 2*s*. 6*d*. 2. 49. 3. 9, 57
4. 21 min. 34 sec. 5. (i) $696\frac{18}{35}$, (ii) $1\frac{11}{24}$. 6. 10·101101.
7. 42·28, 23810·03325. 8. £1776. 5*s*. 0*d*. 9. £3. 10*s*. 0*d*.
10. 0·0000035, 0·00025. 11. Sunday, 8 a.m., 540 knots.

Third Class. II.

1. 30. 2. 8 hrs. 3. £4. 13*s*. 6*d*.
4. £419. 17*s*. $2\frac{1}{4}d$. 5. $\frac{11}{18}$. 6. $\frac{3}{400}$.
7. $\frac{3}{80}$, $\frac{1}{400}$, £0. 1*s*. 6*d*. 8. 0·00400437, 0·3016.
9. £2000. 10. £0. 13*s*. 4*d*. 11. 1815 tons.

Second Class or Junior. II.

1. 2873. 2. £192. 12*s*. $6\frac{1}{4}d$., £195. 13*s*. $11\frac{3}{4}d$., 2952.
3. 1848; 1, 24, 28, 42, 44. 4. $2\frac{590}{609}$. 5. 0·144, 0·017, 0·4585.
6. 2 days 5 hr. 0 min. 49 sec. 7. £1. 8*s*. 4*d*. 8. £41. 1*s*. 9*d*.
9. (i) $\frac{73}{350000}$, (ii) 0·002086. 10. $27\frac{17}{54}$, £0. 9*s*. 10*d*. 11. 0·075 cm.

Third Class. III.

1. 129413 + 59. 2. £12. 18*s*. $7\frac{1}{4}d$. 3. 12 tons 2 cwt.
4. £62. 0*s*. 8*d*. 5. $\frac{1}{8}$. 6. $78\frac{29}{85}$. 7. $\frac{2}{21}$.
8. 0·06045, 1374. 9. 44.
10. A £5625, £1875, B £3750, £1250, C £1875, £625, D £3750, £1250.
11. 1296.

Pupil Teacher Candidates' Examination (Junior Grade).

1. $4\frac{1}{14}\%$. 2. 43, 0·526 in. 3. £0. 10*s*. $10\frac{1}{4}d$.
4. £2. 9*s*. 7*d*. 5. £0. 2*s*. $4\frac{1}{2}d$.

Pupil Teacher Candidates' Examination (Senior Grade).

7. 9 tons 13 cwt. 2 qr. 8. $\frac{19}{99}$. 9. 48. 10. 162 sq. ft.
11. £0. 12*s*. 6*d*. 12. £79. 13*s*. 0*d*.

Teachers' Certificate Preliminary Examination, Part I.

1. 3 years 38 days. 2. 96. 3. 7·35. 4. £1. 8*s*. $2\frac{3}{4}d$., £1. 8*s*. 3*d*.
5. £2. 8*s*. 9*d*. 6. 252. 7. 1566. 8. 110885897421.
10. £9916. 13*s*. 4*d*. 12. £0. 10*s*. 9*d*. 13. 3% per annum.
14. 98 lb. 7 oz.; 113. 15. (*a*) 10000, (*b*) £2395, (*c*) 3, (*d*) £4. 6*s*. $7\frac{1}{2}d$.

Typical Examination Papers. P. 571.

PART II.

Civil Service. I.

1. 864. 2. £3. 3. 60·617, 0·127, 0·945, 64 : 1, 1 : 7·4
4. 4·3, 212, 28 lb. 5. 10*d*. 6. 1 : 4·7 ; 12·7 cm.; £93 approx.

Civil Service (*Higher Arithmetic*).

1. £134. 5*s*. 0*d*. 2. 1170 approx. 3. $2\frac{1}{2}$. 4. 42 % approx.
5. £2. 15*s*. 0*d*., £3. 3*s*. 0*d*., £2. 17*s*. 0*d*., 9·5 %, 19 %. 6. 52 lb.
7. 85 gal. approx. 8. $15\frac{3}{7}$ min. from the start, 17872 ft.

Certificate Examination.

1. 1, 3, 4. 4. 70·4 ft. per sec., 88 ft. per sec., 352 ft., 440 ft.
5. 19·635 sq. cm. 6·438 sq. dm. 6. (*a*) 125, (*b*) $\frac{1}{12}$, (*c*) 2999080.
8. 166·5 miles, 10 min. past 11, to nearest minute.

College of Preceptors. 1*st Class* (*Mensuration*).

2. 11 ac. 1 ro. 3. £166. 6*s*. 0*d*. 4. 9·5 chains.
5. £7. 15*s*. $8\frac{1}{2}d$. 6. 182 yd. 7. 1130.
8. £25. 13*s*. 4*d*. 9. $864+268\sqrt{2}$. 10. 7·24 ft. lb. approx.

11.
0	351	0
84	305	
	205	129 ; 0·472095 ac.
84	71	
0	0	0

Cambridge Local. Junior (*Mensuration*).

1. £27. 10*s*. 0*d*. approx. 2. 11 in. approx. 5. 18·1 sq. ch.

London Chamber of Commerce.

1. 2·68 cm. 2. $266\frac{2}{3}$ %. 3. 175 lb.
4. (*a*) £35·31875, (*b*) £0. 17*s*. $5\frac{1}{2}d$., (*c*) £48103. 9*s*. 4*d*.
5. £502. 10*s*. 0*d*. 6. 50·175°, 61·3°, 0·05°, 0·6°.
7. 3 tons 6 cwt. 108 lb. 8. 258·7 approx. 9. £298. 10*s*. 6*d*. approx.

Oxford Local (*Senior*).

1. 2, 3, 7, 11, 66, 77, 154, 231. 2. 0·21805. 3. 225 ft.
4. 307 blocks, 48828. 5. £547. 17*s*. 2*d*. 6. 4.48 p.m.
7. 14 %. 8. 2 litres, 12 litres. 9. 5 : 13.

Cambridge Local (*Senior*).

1. £33. 3*s*. 0*d*. 2. 47. 3. $\frac{2}{5}$. 4. 1213 ft.; 0·925.
5. 141 fr. 12 centimes. 6. £122. 7*s*. 4*d*. 7. £148. 7*s*. 3*d*. 8. 24 %.
9. £42. 10. Time $5.13\frac{13}{17}$ p.m.; Distance from A $21\frac{2}{3}$ miles.

INDEX

PART I

(*The numbers refer to pages*)

Addition, 4; by steps, 8; of long columns, 10; compound, 56; of length, 79; of metric length and decimals, 91; of fractions, 166.
Approximations, 236.
Approximate or contracted multiplication, 259.
Area, 116; British measures of, 117; metric, 118; measurement of irregular, 126.
Averages, arithmetical, 40.

Capacity, measurement of, 136; liquid measures, 138; dry measures, 138; problems on, 139.
Compound addition, 56; subtraction, 58; multiplication, 61; division, 65.
Contracted division, 268; multiplication, 259.
Cubed numbers, 26.

Decimal coinage, 130.
Decimalization of money, 239; possible errors, 243.
Decimals, introduction of, 88; addition and subtraction, 91; multiplication and division by a single figure, 95; division of, general case, 107; division of, to a given number of decimal places, 110; multiplication of, 98.
Decimals as vulgar fractions, 169; recurring, 171.
Division, 29; by factors, 31; general method, 34; Italian, 35; true to nearest unit, 36; compound, 65; contracted, 268; of decimals, by a single figure, 95; of decimals, general case, 107; to a certain number of decimal places, 111; lengths, 79; metric lengths, 95; of vulgar fractions, 172.

Equivalent decimals to a vulgar fraction, 169; fractions, 162; graphic illustrations, 165.
Errors in data, 254; in decimalizing money, 243.

Factors, 37.
Fractions, 157; graphic illustrations of, 159; improper, and mixed numbers, 160; addition and subtraction of, 166; equivalent, 162; equivalent, graphic illustrations of, 165; expressed as decimals, 170; multiplication and division, 172; one quantity as a fraction of another, 161; square root of, 278.

Greatest common measure, 182; graphic illustration, 186; general method, 188.

Interest, 278; simple, 283.

Least common multiple, 191; general method, 192.

Length, addition, subtraction, multiplication and division, 79; British measures, 74; (*continued*), 113; metric, 75; reduction, 75; the measurement of, 74.

Metric system, of length, 75; reduction, 85; addition, 91; multiplication by a single figure, 95.
Money, British systems, 53; decimalization of, 239; foreign, decimal coinage, 130.
Multiplication, 18; by a single digit, 21; factors, 22; general case, 22; compound, 61; contracted, 259; of decimals, general case, 107; fractions, 172; lengths, 79; metric, 95.

Notation and numeration, 1.

Practice, 244.

Percentages, 257.
Proportion, 197.

Recurring decimals, 171.
Roots, square, 273.

Scale drawing, 220.
Signs, use of, 97.
Simple interest, 283.
Squared and cubed numbers, 26.
Square root, 273; general method, 274; of vulgar fractions, 278.
Subtraction, 12; compound, 58; of fractions, 166.

Time, measurement of, 147.

Volume, 132; tables of, 134.

Weight, measurement of, 141; apothecaries', 142; avoirdupois, 142; metric, 142; troy, 142.

PART II

(*The numbers refer to pages*)

Angles, measurement of, 327.
Approximations, in division, 309; in multiplication, 307; in square root, 314, 320.
Areas of circles, 331; conical surfaces, 436; rectilinear figures, 358; triangles, 432; spherical surfaces, 441.
Averages, 505.

Banker's discount and bills of exchange, 436.
Bills of exchange and Banker's discount, 436.

Chain, use of the, 303.
Circles, areas of, 331.
Combinations and permutations, 521.
Compound interest, 414; compound interest law (geometrical progression), 421; tables, 573.
Cones, areas of, 438; volumes of, 447.
Cylinder, 334.

Days, table o., 572.
Discount (Banker's), 436.
Division, approximations in, 309; by logarithms, 398.

Equation of payments, 478.
Evaluation of formulae by logarithms, 402.
Evolution by logarithms, 399.
Examination papers (typical), 555.
Exchange, bills of, 436; foreign, 487.
Expectation of life, 524.

Foreign bills of exchange, 487.

Geometrical progression, 421.
Graphic method of finding square roots, 318.

Interest, compound, 414; tables, 573.
Inverse proportion, 342.
Involution, by logarithms, 399.

Length, measurement of, 297.
Logarithms, introductory, 387; determination of logs, etc., 393; division, 398; evaluation of formulae, 402; evolution, 399; involution, 398; multiplication, 398; use of logarithms, 396.

Method of mean squares, 510.
Money exchange, 482.
Multiplication, approximations in, 307; by logarithms, 398.

Payments, equation of, 478.
Permutations and combinations, 521.
Prism, 334.
Probabilities, 521.
Proportion, 323; inverse, 342; examination for, 347.
Pyramid, volume of, 447.

Reciprocals, table of, 569.
Rectilinear figures, areas of, 358.
Revision exercises, 370, 459, 529.

Screw gauge, 297.
Similar figures, 338.
Slide rule, 407.
Sphere, area of surface of, 441; volume of, 450.
Spherometer, 297.
Square root, approximations in, 314, 320; graphic method of finding, 318; table of, 569.
Statistics, 505.
Stocks, 491.

Tables, compound interest, 573; logarithms, 570; reciprocals, 569; days, 572; square roots, 569.
Triangle, area of, 432.

Vernier, 299.
Volumes of, cones and pyramids, 447; frusta, 448; graphic calculation, 454; spheres, 450.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.

A
MODERN ARITHMETIC

BY

H. SYDNEY JONES, M.A.

Part I., separately, 2s. 6d.

SOME PRESS OPINIONS.

School World.—"It is stated in the preface to this book that in it an attempt has been made to provide a course of work in which the essential parts of arithmetic are dealt with in the spirit of recent recommendations. The attempt seems to us to be very successful. The expository parts of the book are clear and reasonable, while the exercises are exceedingly varied, free from unnecessary complications, and yet of sufficient difficulty to call forth the best efforts of the average pupil."

Schoolmaster.—"Teachers on the look-out for a suitable book for their elder scholars will do well to examine this. It contains all the necessary rules from the earliest work to square root and simple interest, together with graphic and practical exercises. The tests for revision are exceedingly well selected."

Teacher's Aid—"The attention of teachers is strongly drawn to a book so replete with intellectual methods and with sound common sense."

Teacher.—"A really excellent volume. . . . We know no better book of its type."

Educational News.—"This is one of the most meritorious publications in this department for some time. It aims at (and successfully) producing a clear conception of units, accuracy, quickness, and reasoning power. . . . For teachers the graphic parts of the work are of the greatest value, and the book being most moderate in price, deserves soon to be universally used."

LONDON: MACMILLAN AND CO., LTD.

A MODERN ARITHMETIC

BY

H. SYDNEY JONES, M.A.

Part II., separately, 2s. 6d.

SOME PRESS OPINIONS.

Schoolmaster.—"With the cunning hand of the master, Mr. Jones applies the finishing touches. The ordinary text-book teaches arithmetic; this shows how practical use can be made of it. From cover to cover the book is thoroughly good, and will delight many a one who has a fair knowledge of the higher rules but does not see how to apply it."

Educational News.—"With the utmost confidence we recommend this Arithmetic to all teachers of mathematics. They will find in it much that is new, and at the same time necessary for present-day requirements."

School World.—"Part II. fully maintains the high standard reached in Part I. The treatment is singularly fresh and instructive; it is thoroughly practical, and at the same time intelligent, providing an excellent discipline while avoiding the dangers of too great abstraction. . . . We very cordially recommend it to the consideration of teachers."

Guardian.—"There is evidence of careful thought and experienced discretion in the arrangement and choice of material."

School Guardian.—"The volume will be found to meet not only the requirements of the more conservative examining bodies, but also those of the Army, Civil Service, and other examiners keenly alive to the change which is taking place in mathematical teaching."

LONDON: MACMILLAN AND CO., LTD.

YB 17234

Page 121 70yd x

" 127 Irregular areas

" 199 Good problems on proportion

" 331 Area of circle

" 333 graphic repr. by circle

" 334 " "

" 342 Inverse proportion

" 415 Compound interest

" 482 Foreign money 513

" 555 Examination papers.

M306015

QA 103
J63

THE UNIVERSITY OF CALIFORNIA LIBRARY